PRACTICAL BUSINESS ARITHMETIC

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PRACTICAL BUSINESS ARITHMETIC

BY

JOHN H. MOORE

AND

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REVISED BY

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GINN AND COMPANY

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PREFACE

The revised edition of the Practical Business Arithmetic retains those features of the first edition that are so highly commended by teachers; namely, the development of each topic in such a manner as to make it practical and helpful to the student; the presentation of each subject in a logical order; the selection of problems that appeal to the needs and the interest of the student and of the community as well; the omission of complex and useless problems; a plan of grading and grouping problems which aids the student in acquiring facility and advancing his educational equipment; the elimination of subjects which have little or no connection with business interests and which have slight practical value; the inclusion of an amount of work that will contribute to real efficiency; the development of subjects inductively and the omission of set rules; the unusual amount of oral work in the different chapters.

The revised edition introduces the parcel post, the income tax, the postal savings bank, the computation of loss and gain on the selling price, tests on a time limit, a tariff schedule in accordance with the most recent legislative enactments, additional work on graphs, statistical matter based on the latest census, and an appendix on the varied uses of the adding machine.

In the first edition the authors acknowledged their indebtedness to Dr. David Eugene Smith, Professor of Mathematics, Teachers College, Columbia University, New York, who read the complete manuscript and much of the proof, and kindly made numerous suggestions for the betterment of the book; to Mr. George M. Clough of Somerville, Massachusetts, for the larger part of the material in the chapter on life insurance; to Mr. George Abbot

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In the present edition the author is indebted to Mr. Wm. B. Medlicott, Lecturer on Property Insurance at Harvard University, for his work on the chapter on property insurance; to Mr. Montgomery Rollins of Boston, author of "Money and Investments," and to Mr. Harold T. Sibley of Chicago, for suggestions on the chapter on stocks and bonds; to Mr. Alexander H. Sproul of the State Normal School, Salem, Massachusetts, and to Mr. C. D. McGregor of Des Moines, Iowa, for reading the manuscript as a whole, and for their coöperation in revising the text.

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PRACTICAL BUSINESS ARITHMETIC

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PRACTICAL BUSINESS ARITHMETIC

FUNDAMENTAL PROCESSES

CHAPTER I

INTRODUCTION

1. The student who is prepared to study business arithmetic must be familiar with the ordinary symbols used in the statement or the solution of problems; he must have the ability to read and to write numbers with facility; he must know the fundamentals, and he must be able to perform ordinary operations in United States money, and in both common and decimal fractions.

2. In this course in business arithmetic one learns many simple methods for handling numbers and solving problems, and the adaptation of arithmetic to important business operations; he also acquires skill, rapidity, and accuracy, and he learns how to prove his own work, thus developing self-reliance. Because arithmetic deals with the problems of the home as well as the business office, the study of its practical and everyday features increases one's knowledge of the usages, the phraseology, and the literature of business and commerce.

3. Much attention is given, in the text, to the fundamental processes, for these are at the foundation of all arithmetic. One must acquire a high degree of accuracy and speed in the handling of these fundamentals if he is to achieve any marked degree of success in his subsequent work.

The text contains an unusual amount of material for the student's work, and portions of it may be omitted, at the discretion of the instructor, if the advancement of the class warrants it.

CHAPTER II

NOTATION AND NUMERATION

ORAL EXERCISE

1. How many different figures are used to express numbers?

2. What is the meaning of the syllable *teen* in the numbers from 13 to 19 inclusive?

3. What is the meaning of the syllable *ty* in such numbers as 20, 30, 40, 45, 75, 87, 96?

4. What name is given to 10 tens? to 10 hundreds? to 1000 thousands? to 1000 millions?

5. In 7, 70, 700, 7000, and 70,000 how does the 7 change in value? In 7007 how do the values of the 7's compare?

6. What is the value of the cipher in any number? Why is it used? Explain the use of the ciphers in 900,905.

7. Upon what two things does the value of a figure depend? Illustrate your answer, using the number 121,000,121.

8. Mention five things that are counted in thousands; three things that are counted in millions; two things that are counted in billions. Can you think of any use for trillions?

9. Read aloud the following:

a. During a single year the coinage department of the United States government received from the superintendent 1,193,100 standard ounces of gold coin, from which was produced standard ounces of coin of the value of $6,369,090. During the same period the coiner also received 9,189,533 standard ounces of silver for coinage.

b. In the United States Bureau of Engraving and Printing there are printed yearly about 70,000,000 sheets of United States notes, certificates of deposit, bonds, and national currency to the amount of about $500,000,000. In addition to this there are printed more than 10,000,000,000 postage stamps.

THE ARABIC SYSTEM

4. This is the common system of notation. It is generally called the **Arabic system** because the numerals which it employs were introduced into Europe by the Arabs.

The Arabic numerals 1, 2, 3, and so on to 9 originated in India about 2000 years ago. When only these numerals were used, the system proved to be cumbersome, and all mathematical operations involved great difficulty. About 1200 years ago the cipher 0 was added, thus making a system sufficiently ample and simple for ordinary purposes of analysis and investigation. The Arabs introduced the system into Europe in the twelfth century, but it was not until about 300 years later that it displaced the clumsy Roman system.

5. The distinctive feature of the Arabic system is the **place value** of the numerals employed. The value of an Arabic numeral depends as much upon its place in the number as upon its simple or digit value.

Thus, in the Roman system, VII = 5 + 1 + 1. In the Arabic system, 511 = 5 *hundreds* + 1 *ten* + 1. 5 has not only the unit value *five*, but also the place value *hundreds;* and the 1 following has not only the unit value *one*, but also the place value *ten.*

6. The successive places a figure may occupy in a number are called **orders of units.**

7. *Orders of units increase from right to left and decrease from left to right in a tenfold ratio.* Therefore,

8. The Arabic system of notation is properly called a **decimal system,** from the Latin *decem*, meaning ten.

9. A comma (**separatrix**) or a greater space than that between other figures may be used to separate a number into periods.

Thus, twenty-five thousand four hundred twenty-one may be written 25,421 or 25 421.

ORAL EXERCISE

Read aloud the following numbers:

1. 92,482.
2. 77,009.
3. 375,214.
4. 278,900.
5. 8 217 000 214.
6. 7 000 421 817.

10. For convenience in reading, the successive orders of units are divided into groups of three figures each, called **periods**. The first four periods are shown in the following numeration table. The number used for illustration is sixty-seven billion, four hundred twenty-one million, five thousand, two hundred sixteen, and seven hundred fifty-one thousandths.

NUMERATION TABLE

PERIODS:	Billions			Millions			Thousands			Units				Thousandths		
ORDERS:	Etc.	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
		6	7,	4	2	1,	0	0	5,	2	1	6	.	7	5	1

11. In reading integers do not use the word *and*. In decimal fractions *and* has an office to perform, but if it is used in reading integers, misunderstandings may occur.

Thus, 400.011 is read *four hundred and eleven thousandths;* but
.411 is read *four hundred eleven thousandths;* and
411. is read *four hundred eleven.*

WRITTEN EXERCISE

Write in figures the following:

1. Six million, six thousand, five.
2. Seven hundred fifty-three billion.
3. Four million, one hundred twenty-five.
4. Three hundred twenty-one million, six.
5. Three million four dollars and five cents.
6. Ten billion, one thousand, one hundred three.
7. Twenty-seven and one hundred twenty-five thousandths.
8. Sixty-two thousand and four hundred twenty-five thousandths.
9. Three million four hundred twenty thousand one dollars and fifteen cents.

12. Integers should be read in the **shortest way possible.**

Thus, 1946 should be read *nineteen hundred forty-six*, not *one thousand nine hundred forty-six*. The space for writing the amount on a check,

Boston, Mass., June 30, 19 No. 15

First National Bank

Pay to the order of S. C. Williams $1946.00

Nineteen Hundred Forty-six 00/100 Dollars

Charles R. Sherman

note, or other business paper is generally limited to one line, and it is important that the amount be expressed in the fewest words possible.

ORAL EXERCISE

Read aloud the following:

1. Rhode Island has an area of 1250 sq. mi., and contains 800,000 A.; Ohio has an area of 41,060 sq. mi., and contains 26,278,400 A.; California has an area of 153,360 sq. mi., and contains 101,350,400 A.; Texas has an area of 265,780 sq. mi., and contains 170,099,200 A.

2. Lake Itasca, the source of the Mississippi River, is situated approximately 3160 mi. from the sea; it is 1575 ft. above sea level. The Mississippi River is navigable for a distance of about 2200 mi.; the area drained by this river and its tributaries is estimated at 1,244,000 sq. mi.

3. The area of the United States is approximately 3,025,600 sq. mi.; of Alaska, 570,000 sq. mi.; of Russia in Europe, including Poland, 2,060,940 sq. mi.; of Switzerland, 15,792 sq. mi.

4. The area of Africa is estimated at 11,500,000 sq. mi.; the coast line approximates 16,000 mi. in length; the Sahara Desert has an area of 2,000,000 sq. mi. The greatest length of Asia is about 7500 mi., and its greatest breadth is about 5160 mi.

THE ROMAN SYSTEM

ORAL EXERCISE

1. Make a list of the Roman numerals used in the headings marking the divisions of this book, and read the list so prepared.

2. What symbol ordinarily appears on a watch face for four?

13. This system of writing numbers is called **Roman notation** because it was first used by the Romans. It is now rarely used except for numbering books and their parts, for writing inscriptions on buildings, and for marking the hours on the dials of clocks and watches. It employs seven capital letters:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

14. Other numbers are expressed by a combination of these letters on the general principle that

A combination of letters arranged from left to right in the order of value is equal to the sum of the constituent letters.

15. But the use of the same letter four or more times is avoided by employing the sub-principle that

When one letter precedes another of greater value the value of the two is that of their difference.

Thus, II = 2; VIII = 8; and CCC = 300. But IV or IIII = 4; XL = 40; XC = 90; and CD = 400.

ORAL EXERCISE

1. Multiply twenty-seven by itself in Roman numerals.

2. Why is the Arabic system better than the Roman system?

3. Read the following inscription: MDCCCXLVIII — Charlestown High School — MCMVI.

Nineteen hundred was formerly written MDCCCC, but it is now often written MCM.

4. Read the following numbers of chapters in a book: XXIX, XXXVIII, LXIX, LII, LXVII, LXXVI, LXXIX, CLIII.

5. Read the following numbers of years: MDCCXCV, MCMVII, MDCCLXXVI, MCMIX, MDCCCXCVIII.

WRITTEN EXERCISE

1. Write in the Roman system: 19, 88, 99, 124, 1907, 1910.

2. Write the largest possible number using the six following numerals: 1, 0, 8, 0, 9, 5.

3. Write in Arabic numerals the following number: five billion, two hundred seventeen million, two hundred ten thousand, and fifteen thousandths.

4. Write in the Roman system the following historical years: the discovery of America; the landing of the Pilgrim Fathers at Plymouth; the declaration of independence.

5. Write in Arabic numerals the number in problem 3 increased by two hundred seventy-one and four hundred fifteen thousandths; diminished by two thousand, four hundred sixty, and eleven thousandths.

16. A **unit** is a standard quantity by which other quantities of the same kind are measured.

The simplest form of a unit is a *single entire thing* by which other similar things can be measured by integral enumeration. Thus, the unit of distance is an inch; a group of 12 in. taken in succession is a foot; 3 ft. is a yard; and so on.

17. Numbers that have units of the same kind are called **like numbers.**

Thus, $12 and $15, and 8 hr. and 3 hr., are like numbers.

ORAL EXERCISE

Name the unit in each of the following:

1. A barrel of sugar sold by the pound.
2. A car load of apples bought by the barrel.
3. A car load of lumber sold by the thousand feet.
4. Sixty-four thousand bricks sold by the thousand.
5. Forty and one-half yards of carpet sold by the yard.
6. Twenty-five hundred pounds of beef bought by the hundredweight.
7. When the value in a five-dollar gold piece is thought of, what is the unit?

CHAPTER III

UNITED STATES MONEY

ORAL EXERCISE

Read the following expressions, supplying the missing word or words:

1. The denominations of United States money used in business are dollars, ——, and ——.

2. —— mills or —— cents equal one dollar.

3. The —— is not a coin, but it is sometimes used in making calculations.

4. The first two figures at the right of dollars denote ——, and the third figure denotes ——.

5. The two figures denoting cents express —— of a dollar; the figure denoting mills expresses —— of a dollar.

6. One thousandth of a dollar is —— mill; seven mills are —— —— of a dollar.

7. Fifteen hundredths of a dollar are —— ——; nine tenths of a dollar are nine —— or —— cents.

8. \$25 = —— ¢; 3700 ¢ = \$——; \$17.85 = —— ¢; 4925 ¢ = \$——; \$79 = —— ¢.

9. State a short method of reducing dollars to cents; dollars and cents to cents; cents to dollars.

18. The following **kinds of currency** are in daily use in the United States at the present time: gold coins; silver dollars; subsidiary coins (small change); gold certificates; silver certificates; United States notes; United States Federal Reserve notes; National Bank notes.

The coins now issued by the United States government are as follows:

1. The *gold* double eagle, eagle, half eagle, and quarter eagle.
2. The *silver* half dollar, quarter dollar, and dime.
3. The *nickel* five-cent piece and the *bronze* one-cent piece.

19. Gold or silver in bars or ingots is called **bullion.**

The paper money of the United States is at present as follows:

1. *Gold certificates*, issued for gold deposited in the U. S. Treasury.
2. *Silver certificates*, issued for silver deposited in the U. S. Treasury.
3. *United States notes* (*greenbacks*), promises of the government to pay to the holder on demand a definite number of gold or silver dollars.
4. *National bank notes*, issued by national banks under the supervision of the National Government.
5. *Treasury notes*, which were issued for silver bullion deposited in the U. S. Treasury. These notes are not now issued.
6. *United States Federal Reserve notes.*

ORAL EXERCISE

1. What is meant by *money*, *currency*, *legal tender?*

In such exercises as the above the student should not try to repeat definitions, but should explain the terms in his own way.

2. Name the gold coins of the United States; the silver coins; the paper money; give the value of each of the gold coins.

3. Read in three ways: \$4.8665; \$25.87½; \$178.475.

4. Name the largest gold and silver coins that will exactly express each of the following amounts: \$27.90; \$28.20; \$75.80.

20. When it is desirable to express United States money in written words, the cents should be written in fractional form, as in the following note:

\$1050 45/100 New York, June 30, 19—

Thirty days after date I promise to pay to the order of Charles B. Kingsbury

Ten Hundred Fifty 45/100 Dollars

at First National Bank

Value received

No. 24 Due July 30, 19— F. M. Everett

CHAPTER IV

ADDITION

ORAL EXERCISE

1. Find the sum of 1, 2, 3, 7, 5, 9, 4, 8, and 6.

2. Read each of the numbers in problem 1 increased by 2; by 5; by 3; by 7; by 8; by 9; by 17; by 23.

3. Find the sum of 8, 7, 9, 5, 6, 11, and 12.

4. Read each of the numbers in problem 3 increased by 12; by 15; by 18; by 24; by 42; by 19; by 16.

5. Illustrate what is meant by like numbers.

21. *Only like numbers can be added.*

22. To secure speed and accuracy in addition name *results only* and express these in the fewest words possible.

Thus, in adding 2, 4, 7, 8, 3, 2, and 8 say *6, 13, 21, 4, 6, 34*; do *not* say *2 and 4 are 6 and 7 are 13 and 8 are 21 and 3 are 24 and 2 are 26 and 8 are 34.*

ORAL EXERCISE

Name the sum in each of the following problems:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
3	2	2	8	1	5	8	1	3	5	5	1	3	4	2
2	1	4	2	3	2	2	3	3	1	4	7	2	5	7
1	6	3	1	6	1	3	6	4	6	4	2	1	2	3
2	8	2	2	4	3	7	4	2	2	3	7	5	8	5
8	4	1	3	4	4	4	9	8	7	2	3	2	6	4
4	8	4	4	3	7	7	5	3	3	1	4	8	4	2
5	6	3	5	2	2	3	8	6	2	0	5	2	5	1
6	0	6	2	3	1	4	2	2	5	7	2	6	3	4
3	8	1	7	7	6	1	1	1	1	7	7	1	2	3
3	6	2	2	4	2	2	4	3	4	2	1	1	1	2
2	2	3	5	1	8	3	2	2	3	1	3	8	6	2
4	1	5	1	2	3	2	4	1	2	4	4	9	8	7

23. Addition is the basis of all mathematical processes. It constitutes a large part of all the computations of business life and concerns, to some extent, every citizen of to-day. Ability to add rapidly and accurately is therefore a valuable accomplishment.

24. **Rapid addition** depends mainly upon the ability to group; that is, to instantly combine two or more figures into a single number. In reading it is never necessary to stop to name the individual letters in the words. All the letters of a word are taken in at a glance; hence the whole word is known at sight. Words are then grouped in rapid succession and a whole line is practically read at a glance. This is just the principle upon which rapid addition depends. From two to four figures should be read at sight as a single number, and the group so formed should be rapidly combined with other groups until the result of any given column is determined. This can be done only by intelligent, persistent practice.

25. The following list contains **all possible groups of two figures each.**

ORAL EXERCISE

Pronounce at sight the sum of each of the following groups:

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
1.	1	1	2	2	4	1	3	3	4	3	1	4	2	4	7
	1	3	1	2	1	5	2	3	2	6	7	3	5	6	7
2.	8	9	8	5	6	4	5	5	7	1	5	6	6	8	9
	9	9	8	5	1	4	3	4	2	8	6	6	9	6	1
3.	8	7	7	4	9	7	6	7	5	3	2	4	5	7	6
	2	3	5	8	3	8	7	9	9	8	9	9	8	4	2

The above exercise may be copied on the board and each student in turn required to name the results from left to right, from right to left, from top to bottom, and from bottom to top. The drill should be continued until the sums can be named at the rate of 150 per minute. This is the first and most important step in grouping.

ORAL EXERCISE

Name the sum in each of the following problems:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
6	7	3	5	6	7	9	9	9	1	2	5	8	2	8
3	1	4	2	4	7	9	8	8	4	7	2	3	7	2
8	7	7	5	8	2	5	9	4	5	8	3	5	4	1
7	9	6	9	3	9	8	4	7	1	1	7	9	5	9
3	8	5	3	8	1	6	4	9	2	6	5	7	3	4
9	4	7	7	2	9	8	5	1	3	5	7	6	5	5
5	6	6	8	2	4	4	3	6	3	6	8	7	4	6
5	6	5	5	7	5	4	2	1	3	6	4	9	4	8
2	3	2	1	1	2	3	1	1	2	5	3	8	1	9
4	3	3	1	4	2	1	5	6	4	5	9	7	6	6

Name the results only and make groups of two figures each. Thus, in problem 1, beginning at the bottom and adding up, say *6, 16, 28, 43, 52.*

16–45. Add the numbers in the exercise on page 10 by groups of two figures each.

26. It is practically as easy to add 54 and 9, 59 and 6, etc., as it is 4 and 9, 9 and 6, etc. 4 and 9 are always equal to 1 ten and 3 units, and 9 and 6 to 1 ten and 5 units. Hence in adding 54 and 9 think of the tens as increased by 1, call the units 3, and the result is 63; in adding 59 and 6 think of the tens as 6, the units as 5, and the result as 65.

ORAL EXERCISE

Pronounce at sight the sum of each of the following groups:

1.	27	48	59	77	58	52	59	75	95	84	39	59	84	76	91
	7	8	6	8	7	8	8	6	9	7	6	5	9	8	8
2.	75	59	77	88	74	23	24	44	89	78	67	37	56	58	68
	8	9	9	5	6	8	9	9	9	9	9	7	7	4	5
3.	37	49	38	37	45	95	98	87	54	72	63	42	73	97	88
	5	8	7	6	9	8	7	7	9	9	8	9	8	5	9

27. In combining numbers between 10 and 20 think of them as one ten and a certain number of units and not as a certain number of units and 1 ten.

Thus, in combining 17 and 18 think of 28 and 7, or 35; in combining 19 and 15 think of 29 and 5, or 34; and so on.

ORÁL EXERCISE

Pronounce at sight the sum of each of the following groups:

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
1.	12	17	12	16	11	12	18	16	17	11	19	13	18	12	17
	15	17	12	13	14	11	18	12	18	19	15	13	12	14	19
2.	13	11	15	19	14	19	17	15	13	19	16	14	18	18	12
	18	16	16	14	15	16	16	13	11	18	14	14	11	15	19
3.	11	17	12	17	15	15	12	18	16	14	19	14	19	17	11
	11	14	13	13	17	15	17	16	16	13	19	18	13	11	15

The above exercise contains all combinations possible with the numbers from 11 to 19 inclusive. Drill on the exercise should be continued until results can be named at the rate of 120 per minute.

28. Numbers between 10 and 20 may be combined with numbers above 20 in practically the same manner as in § 27

Thus, in adding 62 and 12 think of 72 and 2, or 74; in adding 79 and 17 think of 89 and 7, or 96.

ORAL EXERCISE

Pronounce at sight the sum of each of the following groups:

1.	25	48	59	87	91	75	86	75	48	78	57	89	37	56	75
	17	17	16	14	18	18	19	12	16	13	16	14	17	18	14
2.	29	47	83	92	36	54	59	78	67	92	77	86	53	78	85
	13	14	19	14	19	13	18	15	13	13	19	19	17	14	14
3.	31	32	45	69	74	95	98	92	96	87	86	34	43	64	38
	19	17	19	15	8	18	14	19	15	17	19	18	18	19	17

ORAL EXERCISE

1. Count by 7's from 1 to 85.

SOLUTION. 8, 15, 22, 9, 36, 43, 50, 7, 64, 71, 8, 85.

Count by :

2. 2's from 39 to 55.
3. 5's from 11 to 86.
4. 6's from 15 to 63.
5. 5's from 2 to 107.
6. 7's from 11 to 60.
7. 8's from 25 to 89.
8. 9's from 31 to 112.
9. 8's from 32 to 192.
10. 7's from 18 to 102.
11. 6's from 72 to 126.
12. 9's from 10 to 136.
13. 9's from 17 to 152.
14. 8's from 10 to 138.
15. 7's from 19 to 152.
16. 6's from 20 to 128.
17. 6's from 15 to 111.
18. 9's from 12 to 102.
19. 8's from 17 to 113.
20. 7's from 24 to 108.
21. 6's from 27 to 117.
22. 4's from 19 to 183.
23. 11's from 14 to 102.
24. 12's from 17 to 161.
25. 13's from 17 to 121.

26. Beginning at 1 count by 4's to 17; going on from 17 count by 7's to 52; from 52 count by 9's to 133; from 133 count by 5's to 158; from 158 count by 12's to 206; from 206 count by 13's to 271.

This exercise furnishes one of the best possible drills in addition, and it should be continued until the successive results can be named at the rate of 150 per minute.

29. If the student is accurate and rapid in making groups of two figures each, he is ready for practice in groups of three figures each. In the following exercise are **all the possible groups of three figures each.**

ORAL EXERCISE

Name at sight the sum of each of the following groups:

4, 2, and 3 should be thought of as 9 just as p-e-n is thought of as pen.

1.	4	1	9	8	1	1	3	1	8	1	4	5	1	7	8
	1	3	1	2	2	3	1	7	3	3	1	4	4	1	4
	3	3	2	1	7	5	6	3	1	9	4	1	6	4	1

2.	1	6	1	4	1	2	1	1	1	1	7	6	9	8	1
	4	1	2	1	2	2	9	1	1	6	6	6	5	5	5
	9	2	5	2	3	1	1	8	7	8	1	1	1	1	7
3.	6	5	2	5	2	3	9	2	2	2	2	6	1	1	2
	1	1	3	3	3	2	2	8	7	6	5	1	1	1	2
	5	5	6	2	4	3	2	2	2	2	2	1	5	4	4
4.	3	2	1	2	2	6	2	6	5	5	7	1	1	1	1
	2	2	1	7	6	8	6	2	2	2	2	1	1	6	9
	2	2	3	7	9	2	7	6	9	8	5	2	1	9	9
5.	9	8	9	8	7	3	4	5	6	6	5	4	3	3	4
	1	1	1	1	1	5	8	7	7	7	5	4	4	4	4
	8	8	7	7	7	5	4	5	9	8	6	7	9	8	6
6.	5	6	6	9	5	7	3	4	9	6	6	8	3	3	3
	5	7	6	4	4	3	4	4	4	8	7	4	9	4	4
	5	7	9	9	4	4	6	4	8	6	6	8	9	5	4
7.	3	4	6	9	8	5	4	3	3	2	3	3	4	5	8
	8	7	6	9	9	9	7	8	3	5	3	7	7	8	8
	9	9	6	9	9	9	8	8	9	6	8	9	7	9	8
8.	8	5	4	3	3	5	2	3	3	4	5	7	7	5	4
	8	8	9	8	7	2	4	3	7	6	7	9	8	7	6
	9	5	6	7	3	5	9	6	7	8	9	9	9	8	7
9.	3	3	2	2	3	3	4	5	7	9	9	9	7	3	6
	6	3	4	4	3	6	6	7	8	7	6	5	6	3	4
	9	5	8	7	4	8	6	7	8	7	5	4	3	3	2
10.	2	2	3	4	5	7	2	2	3	4	5	7	9	6	6
	4	9	6	5	6	7	4	8	5	5	6	7	9	6	5
	5	9	6	8	8	8	4	9	9	7	7	7	6	5	4
11.	8	8	9	2	2	3	4	5	6	8	8	9	6	8	7
	5	8	3	3	7	5	5	5	8	8	5	4	5	7	3
	3	2	2	8	9	7	5	9	9	6	5	4	3	2	2

This exercise should be drilled upon until the sums of the groups, in any order, can be named at the rate of 120 per minute.

ORAL EXERCISE

1–15. Turn to the exercise on page 10 and find the sum of the numbers given.

Name results only, and make groups of three figures each. Thus, in problem 1, say *9, 23, 37, 43.*

Add from the bottom up and check the work by adding from the top down.

Find the sum in each of the following problems:

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
1	3	1	4	2	2	2	4	4	5	1	2	9	5	4
1	1	3	3	3	1	6	3	9	5	7	4	0	7	3
1	1	4	1	5	2	2	4	5	0	2	4	1	2	1
2	1	3	1	3	1	4	1	8	8	9	2	8	0	1
2	4	1	4	6	4	5	8	3	2	0	3	0	0	6
2	2	3	8	1	1	2	1	7	1	1	5	2	5	8
2	4	2	2	2	2	2	3	8	3	5	7	2	6	1
5	2	1	4	5	3	7	6	2	7	3	7	2	6	6
1	2	9	4	3	2	3	1	8	2	2	1	6	0	7
5	1	8	3	4	2	1	2	9	9	6	7	2	3	3
3	5	2	3	3	6	9	3	3	1	2	8	2	6	3
1	3	1	3	3	1	0	5	6	3	7	0	4	1	1
3	2	7	2	4	3	0	2	8	8	4	7	2	5	9
5	4	2	5	2	4	8	5	1	2	3	3	2	3	2
2	4	1	4	4	3	2	2	0	4	3	0	5	2	1
1	2	1	2	6	6	4	4	6	6	3	6	2	5	8
8	6	2	3	3	3	5	2	4	4	3	3	2	8	2
1	2	6	5	1	1	1	3	0	5	6	1	6	2	1
4	4	1	3	7	2	9	3	7	9	1	5	7	5	7
3	5	2	2	2	6	2	2	3	1	7	3	3	7	2
4	2	5	6	1	3	1	3	0	3	2	2	1	3	1
4	2	1	2	1	2	2	7	7	7	1	1	9	2	2
9	7	2	2	3	8	3	1	2	3	9	1	2	5	2
1	3	4	4	4	1	7	7	1	0	0	8	4	8	4
2	1	3	7	3	2	5	7	6	5	5	2	4	4	3
1	6	2	1	5	5	3	2	3	2	8	1	3	6	3
2	3	1	1	2	1	1	2	1	2	1	5	7	1	1

30. It is always an advantage to find **groups of figures aggregating 10 and 20** in the body of a column.

These groups should be added immediately to the sum already obtained by simply combining the tens of the two numbers. It is not a good plan, however, to take the digits in irregular order in order to form groups of 10 and 20.

ORAL EXERCISE

Find the sum in each of the following problems, taking advantage of groups of 10 and 20 wherever possible:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
1	2	7	6	5	2	5	3	4	3	7	8	2	5	9
9	8	3	4	5	5	4	3	2	5	5	4	7	8	9
7	4	5	1	8	5	6	7	8	5	5	6	3	2	1
3	6	5	9	2	7	9	8	7	4	0	2	5	8	1
2	7	7	2	4	3	1	2	3	6	9	7	5	2	5

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
4	1	9	1	4	6	5	7	1	2	4	4	1	6	1
3	1	2	6	1	2	2	2	4	2	2	3	9	3	1
3	8	9	3	5	2	3	1	5	6	4	3	2	1	8
7	1	4	6	5	4	4	7	5	1	8	7	8	7	0
2	6	1	2	2	4	3	3	5	7	6	2	3	4	9
1	3	5	2	3	2	3	4	4	2	4	5	7	5	0
6	5	9	2	4	4	8	6	6	5	3	1	8	1	1

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
2	3	9	8	6	6	6	6	5	8	6	6	2	7	6
2	8	4	5	6	7	7	5	7	4	3	9	8	9	7
9	9	7	6	8	7	8	6	8	9	8	5	9	9	4
9	7	9	9	7	9	7	8	7	7	9	6	9	2	9

46.	47.	48.	49.	50.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
38	42	25	35	46	14	21	12	18	29	57	17	13	14	15
8	1	4	2	4	5	0	3	2	5	5	4	6	2	7
1	2	1	9	7	6	6	3	4	7	6	8	6	7	2
1	7	3	9	9	0	6	8	4	8	9	8	8	5	8
2	4	6	5	2	9	8	6	7	4	5	5	2	3	6

31. When three figures are in consecutive order the sum may be found by multiplying the middle figure by 3; when five figures are in consecutive order the sum may be found by multiplying the middle figure by 5; etc.; or the sum of any number of consecutive numbers may be found by taking one half the sum of the first and last numbers and multiplying it by the number of terms.

ORAL EXERCISE

By inspection find the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
7	10	13	16	19	22	25	28	31	34	37	40	43	46	49
8	11	14	17	20	23	26	29	32	35	38	41	44	47	50
9	12	15	18	21	24	27	30	33	36	39	42	45	48	51

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
11	16	21	26	31	36	41	46	51	56	61	66	71	76	81
12	17	22	27	32	37	42	47	52	57	62	67	72	77	82
13	18	23	28	33	38	43	48	53	58	63	68	73	78	83
14	19	24	29	34	39	44	49	54	59	64	69	74	79	84

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
7	10	13	16	19	22	25	28	31	34	37	40	43	46	49
8	11	14	17	20	23	26	29	32	35	38	41	44	47	50
9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
10	13	16	19	22	25	28	31	34	37	40	43	46	49	52
11	14	17	20	23	26	29	32	35	38	41	44	47	50	53
12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
13	16	19	22	25	28	31	34	37	40	43	46	49	52	55
14	17	20	23	26	29	32	35	38	41	44	47	50	53	56
15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
16	19	22	25	28	31	34	37	40	43	46	49	52	55	58
17	20	23	26	29	32	35	38	41	44	47	50	53	56	59

32. When a figure is repeated several times the sum may be found by multiplication.

ORAL EXERCISE

By inspection find the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
4	3	4	5	3	7	8	8	15	6	7	8	15	13	9
9	7	4	5	3	7	5	7	15	6	8	7	14	13	8
9	8	4	5	9	7	5	9	15	12	7	8	15	13	8
9	8	9	5	9	8	6	9	8	12	7	7	14	7	9
9	8	9	9	8	8	6	9	8	12	7	8	15	7	8

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
3	7	4	2	7	5	12	2	4	6	8	9	8	5	16
3	7	4	2	7	5	5	2	4	6	8	9	8	5	16
3	7	4	2	4	5	5	2	4	6	8	9	8	5	16
2	2	7	8	4	4	5	3	5	4	3	5	8	5	16
2	2	7	8	2	4	5	3	5	4	3	5	8	5	20
2	2	7	8	2	4	5	3	5	4	3	5	9	8	1

33. In all written work make plain, legible figures of a uniform size, write them equal distances from each other, and be sure that the units of the same order stand in the same vertical column.

34. Many of the errors that occur in business are in simple addition. Errors in addition result from two main causes: irregularity in the placing of figures; poor figures.

35. In business it is important that figures be made rapidly; but rapidity should never be secured at the expense of legibility.

WRITTEN EXERCISE

Copy and find the sum of:

1.	2.	3.	4.	5.	6.
1745	1842	1249	4271	6229	1481
1862	1695	1810	8614	4813	1862
7529	4716	6241	9217	7142	4129
8721	8412	1728	8214	6212	2412

7.	8.	9.	10.	11.	12.
4216	2110	4142	1061	4113	4112
8912	8420	4347	1875	8217	1012
4729	1641	1012	6214	8614	1862
8624	1722	1816	1931	1692	1721
4829	1837	4112	1648	1591	1692
6212	4216	4210	1721	1686	1486
4110	4117	1618	1728	2172	4112
4210	1832	4060	1421	1754	1010

36. The simplest way to **check addition** is to add the columns in reverse order. If the results obtained by both processes agree, the work may be assumed to be correct.

37. In adding long columns of figures it is generally advisable to record the entire sum of each column separately; then if interruptions occur, it will not be necessary to re-add any portions already completed. After the total of each column has been found the entire total may be determined by combining the separate totals of the columns.

38. The best way to test the accuracy of columns added in this manner is to begin at the left and repeat the addition in reverse order. The entire total of each column should again be written and the complete total of the problem found by adding the separate totals of the several columns. If the results obtained by the two processes agree, the work may be assumed to be correct.

39. Example. Find the sum of 54,669, 15,218, 36,425, 45,325, and 68,619. Check the result.

SOLUTION. Beginning at the bottom of the right-hand column, add each column in regular order and write the entire totals as shown in (*a*). Beginning at the top of the left-hand column again add each column and write the entire totals as shown in (*b*). Next add the totals obtained by the first and second additions and compare the results. Since the total shown by (*a*) is equal to the total shown by (*b*), the result, 220,256, is assumed to be correct. All work in addition should be carefully checked.

(*b*)		(*a*)
19	54669	36
28	15218	12
21	36425	21
12	45325	28
36	68619	19
220256	220256	220256

WRITTEN EXERCISE

See how many times the following numbers can be written in one minute. Write each number in form for vertical addition.

1. 426579.
2. 123987.
3. $7983.21.
4. $4080.91.
5. $70812.34.
6. $41182.50.

Thus, in repeating the number in problem 1 write it as follows:

4 2 6 5 7 9
4 2 6 5 7 9
4 2 6 5 7 9
4 2 6 5 7 9
Etc.

Be sure that the spacing between the lines and between the columns is uniform. Increase the speed gradually until from 150 to 200 figures can be written per minute.

40. Skill in writing figures from dictation should be cultivated. The dictation should be slow at first, but it should be gradually increased until the requisite speed is acquired.

41. In calling off numbers to another great care should be taken in order that no errors may be made. In reading United States money the word *dollars* should be called with each amount. The word *cents* may be omitted in all cases except where there are no dollars.

Thus, in calling $400.37 say *four hundred dollars, thirty-seven;* in calling $25.11 say *twenty-five dollars, eleven;* in calling $1573.86 say *fifteen hundred seventy-three dollars, eighty-six;* in calling $5.31 say *five dollars, thirty-one.*

WRITTEN EXERCISE

Write from dictation and find the sum of:

1. $75.18, $123.95, $147.25, $9.50, $181.45, $172.16, $84.98, $314.95, $49.10, $69.90, $312.60, $415.90.

2. $3140.19, $310.92, $3164.96, $3162.19, $18.62, $410.95, $690.18, $10.75, $3100.40, $300.40, $200.50, $100.90, $410.80, $100.85, $310.60, $80.90, $399.80, $412.60.

WRITTEN EXERCISE

Copy, find the sum, and check:

1.	2.	3.
$ 6925 64.21	$ 624 2654.92	$ 215 1621.54
1345 678.50	125 9216.05	126 2141.67
379 0269.17	217 9203.04	213 1472.03
672 1234.23	913 1426.04	146 2141.07
421 2612.45	125 9212.46	964 8968.03
124 9212.54	132 1652.17	124 2472.42
128 2964.16	214 1621.46	986 8638.68
217 9216.54	315 1259.06	214 1262.49
692 3469.48	211 4201.58	896 9849.61
246 8627.16	317 4257.69	126 2479.14
549 2165.18	426 9237.50	341 6218.49

4.	5.	6.
$ 126 724.45	$ 1275 246.48	$ 1251 624.64
437.26	751 2047.72	721 266.44
12.91	126 056.42	411 200.54
7.69	216 414.79	1500 000.98
647 686.42	16 147.92	196 9214.54
541 64.95	721 46.54	2401 206.25
712.78	12.75	129 863.64
92.14	126.92	1264.92
569.86	2146.01	151 2692.07
11.46	326 45.91	169 2162.04
726 9.42	4260.01	1721.90
692 4.75	3001.98	416.75
127 98.69	75.06	672 1.90
124.60	1.92	14.95
1.59	4120.78	4261.74

7.	8.	9.
$1 2 6 4 1 2.80	$2 1 4 6 2 1.6 4	$1 6 2 0 1 7 7.2 5
9 8 5 3 6.4 1	9 2 6 8 4.7 5	2 5 1 6 2 1 4.2 9
6 8 1 2.0 0	3 5 7 8 9.4 4	1 2 6 1 4 2 6.4 1
2 3 2 7 1 4.1 8	1 6 9 2 9.7 5	1 0 1 9 0 2 7.6 0
9 8 2.7 5	1 8 9 2 7.9 5	9 9 1 8 3 3.4 0
1 2 5 6 8.3 0	3 7 4 2 6 1.4 2	7 2 1 1 6 4.7 4
2 2 9 9 4 2.7 8	1 0 2 6 4 0 6.4 7	3 8 9 9 4 6.3 6
2 6 0 5 4.9 0	6 2 1 4 5 7.9 2	1 9 4 1 7 2.4 5
2 2 6 4 9 1.3 7	7 4 2 4 6 7.4 8	9 1 6 9 3 8.5 5
8 3 6 2.4 0	6 9 2 4 1 6.2 4	1 2 0 1 2 4 0
4 2 6 8 3 4.7 5	5 2 1 2 0 7.8 4	6 4 1 2.7 5
8 2 9 6 4.3 5	3 7 2 9 2.2 0	1 6 1 6 9 1 2.6 2
3 5 9 4 8 2.2 0	1 2 0 4 0.0 0	7 5 1 8.9 5
7 9 6 0.0 0	7 2 4 5.6 9	1 6 2 1 9.7 5
8 6 5 2 1.4 6	3 6 0 4 7.4 6	1 4 1 7 5 2 1.6 4
7 5 4 3 7 6.5 0	6 9 4 1 1 6.5 2	1 1 2 1 4 9 0.7 5
1 0 1 6 8 2 3.2 5	7 2 1 2 0 5.1 1	6 9 8 7 6.6 2
6 2 9 4 3.1 2	6 2 1 4 7 6.0 1	7 2 4 1 3 8.7 5
4 8 3 6.9 0	7 4 1 6 2 7.0 3	1 2 6 4 2 1.2 6
9 2 7 8 5 9.4 6	6 2 1 2 6 4.7 5	2 1 7 2 2 1.6 4
2 6 2 5 8 2.7 4	6 9 1 2 8.6 0	6 9 2 1 4.7 5
1 2 6 1 2 9.4 5	1 0 1 2 0 1 6.5 9	2 1 1 1 6 9 4.4 1
9 6 8 5 2.1 7	9 9 2 0 1 7.2 9	9 9 9 9 4 0.6 9
1 4 9 1 2.1 8	1 2 1 2 0 2.6 0	7 2 1 2 6.7 3
2 5 1 9 2 6.8 6	1 0 1 4 2 7.6 5	2 5 1 3 2 7.1 4
4 6 5 3 2.4 5	2 4 1 6 5 9.6 4	7 5 9 7 8 3.9 6
2 2 6 1 4.9 2	6 9 3 9 8.9 5	1 6 1 2 6.4 7
4 6 2 2 8 4.2 0	7 0 1 2 0.9 2	7 2 1 2.4 5
9 7 0 5 2.0 4	6 9 5 4.8 7	2 7 7 2 6 9.1 8
6 8 0 5 2.6 5	1 2 6 7 0 2 6.5 4	1 8 9 2 4 7.9 0
1 2 6 1 2 5.6 5	3 2 1 6 1 4.7 0	6 9 2 4 6.7 8
3 7 5 8 0.5 4	1 6 2 3 6 7.9 0	2 4 1 2 6.2 5

42. Some accountants practice adding two columns at once when the columns are short. The method generally employed is similar to the method explained for combining groups in regular addition.

43. Example. Find the sum of 83, 72, 89.

SOLUTION. Beginning at the bottom and adding up, think of 89 and 72 as 159 and 2, or 161; of 161 and 83 as 241 and 3, or 244. In adding name results only. Thus say *159, 161, 241, 244.*

$$\begin{array}{r} 83 \\ 72 \\ 89 \\ \hline 244 \end{array}$$

ORAL EXERCISE

By inspection give the sum of each of the following groups:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
43	64	52	37	65	38	52	85	93	68	58	76	83	57	62
25	18	29	56	27	43	67	34	72	75	46	39	47	25	39

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
53	52	61	34	91	68	48	24	78	54	94	57	92	76	43
46	43	37	76	13	47	69	96	76	35	36	44	37	31	56

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
65	44	46	48	67	44	53	25	54	46	33	16	67	83	88
86	57	65	25	48	57	45	31	65	39	64	34	43	82	25
75	21	34	31	39	21	67	69	87	87	77	25	41	98	31

HORIZONTAL ADDITION

44. In some kinds of invoicing and in short-extending the items of an account numbers to be added are written in horizontal lines. Much time may be saved by adding these numbers as they stand. After careful practice it will be found possible to add numbers written in horizontal lines with as much facility as numbers written in vertical columns.

45. In adding numbers written horizontally care should be exercised to combine only units of the same order. It is generally best to add from left to right and to verify the work from right to left. Grouping may be employed to advantage in horizontal addition.

WRITTEN EXERCISE

Copy and add the following numbers horizontally. Verify the work.

Thus, in problem 1, beginning at the left, say *10, 20, 32, 52.* In verifying the work from the right say *20, 32, 42, 52.*

1. 8, 2, 1, 1, 7, 1, 4, 6, 2, 3, 8, 9.
2. 7, 9, 6, 5, 4, 8, 7, 4, 3, 7, 3, 1, 3.
3. 6, 2, 4, 8, 3, 1, 7, 6, 4, 2, 8, 9, 4, 2.
4. 15, 23, 46, 83, 29, 35, 42, 15, 21, 26.
5. 64, 48, 56, 35, 47, 87, 32, 45, 67, 91.
6. 52, 64, 86, 28, 76, 41, 15, 32, 12, 87.
7. 32, 48, 24, 62, 85, 14, 63, 54, 78, 94, 23, 45.
8. 42, 76, 49, 81, 17, 42, 17, 19, 21, 43, 64, 17.
9. 45, 48, 34, 46, 48, 53, 25, 42, 35, 56, 70, 10.
10. 291, 196, 855, 578, 210, 354, 102, 232, 241, 162.
11. 469, 388, 962, 764, 351, 899, 111, 232, 190, 175.
12. 15^{25}, 50^{25}, 16^{84}, 31^{42}, 86^{38}, 19^{10}, 23^{12}, 10^{13}, 64^{80}, 40^{10}.

It is frequently desirable to express dollars and cents without the dollar sign and the decimal point. This may be done by slightly raising the cents of the amount. Thus, \$ 17.17 may be written 17^{17}; \$ 2.08 may be written 2^{08}.

13. 15^{25}, 8^{93}, 4^{88}, 21^{84}, 16^{35}, 18^{46}, 29^{14}, 44^{60}, 62^{90}, 84^{60}, 40^{50}.
14. 76^{75}, 84^{97}, 67^{05}, 95^{74}, 68^{63}, 52^{21}, 13^{25}, 42^{18}, 60^{95}, 80^{13}, 90^{62}.

46. It is important that the student acquire the ability to carry a series of numbers in mind. The following exercises are suggestive of what may be done to cultivate ability in this direction.

The dictation suggested should not be slower than at the rate of one hundred twenty words per minute. Nothing should be written by the students until all of the numbers of a problem have been called by the instructor; then one student may be sent to the blackboard and required to write the numbers from memory. If the numbers are correctly written, the instructor may require another student to give the sum of them without using pen or pencil. The numbers may be written on the board in either vertical or horizontal order, as the instructor may direct.

ORAL EXERCISE

From the instructor's dictation find mentally the sum of each of the following problems:

1. 6, 9, 8, 4, 8, 6.
2. 14, 17, 20, 5, 9.
3. 24, 17, 16, 9, 5.
4. 5, 6, 7, 1, 3, 8.
5. 6, 2, 8, 1, 7, 4.
6. 364, 436, 657, 25.
7. 438, 212, 750, 64.
8. 859, 441, 769, 71.
9. 2140, 3160, 4000.
10. 200, 415, 600, 95.
11. 857, 643, 237, 500.
12. $4150, $4050, $850.
13. $5.15, $2.15, $6.70.
14. $167.14, $232.86, $9.
15. 147, 253, 179, 121.
16. 423, 517, 81, 49.
17. 255, 45, 89, 121.
18. 25, 65, 27, 133.
19. $48, $32, $138.
20. $135, $275, $418.
21. $23, $67, $281.
22. $284, $36, $245.
23. 133, 167, 29, 61.
24. 2319, 1681, 2335.
25. 3310, 2790, 1565.
26. 2740, 1365, 2135.
27. 2273, 1237, 1145.
28. 1432, 1058, 1210.

WRITTEN REVIEW EXERCISE

1. Find the sum of all the integers from 2165 to 2260 inclusive.

2. Find the sum of all the integers from 1137 to 1200 inclusive.

3. Complete the following sales sheet. Add by columns and by lines and check the work by adding the vertical and horizontal totals.

SUMMARY OF SALES FOR WEEK ENDING AUG. 25

	PINE	OAK	MAPLE	SPRUCE	WALNUT	CHERRY	TOTAL
Monday	1216 18	16161 47	649 58	860 40	315 64	186 50	
Tuesday	5160 40	3214 90	316 40	160 50	513 80	216 54	
Wednesday	6152 18	2150 18	163 59	430 17	968 52	756 14	
Thursday	1216 18	2160 50	130 98	115 67	413 60	314 75	
Friday	4160 80	1215 40	315 16	218 90	411 50	132 75	
Saturday	3165 80	2115 72	218 50	165 37	118 50	17 05	
Total							

4. Add the following by columns and by lines, and check the work by adding the vertical and horizontal totals:

21162 49	962 18	124 76	54168 97	52 19		
176 19	1278 95	52698 13	7529 87	95162 87		
2164 89	7524 16	47612 87	6842 23	5948 23		
76 95	87 14	2150 49	172 93	1745 86		
51276 92	18187 95	75 19	162 14	5290 18		
9834 18	92923 15	25 91	162 18	14 95		
754 95	2167 92	2584 16	9176 92	3164 82		
1356 05	1314 93	7125 95	2167 18	2645 97		
756 92	142 18	167 42	926 44	3167 18		
75162 19	82195 78	72162 18	9165 97	168 44		
7162 95	4167 18	7156 95	172 18	1 56		
2 15	6843 82	3954 05	60 65	9 18		
8 85	9162 19	5144 65	8162 18	91684 57		
2416 45	1829 32	4217 64	1492 95	8647 64		
168 94	257 16	417 86	952 17	347 18		

5. Complete the following sales sheet. Add by columns and by lines and then check the work by adding the vertical and horizontal totals.

SUMMARY OF CLERKS' DAILY SALES

NAMES OF CLERKS	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL FOR WEEK
J. E. Snow	167 18	194 67	98 46	241 80	175 66	314 90	
W. B. Moore	78 20	65 14	50 42	60 93	51 19	64 86	
T. B. Welch	112 40	118 64	192 40	146 18	110 50	140 12	
E. H. Ross	164 90	143 18	192 64	214 10	110 60	190 18	
Minnie Davis	165 19	214 78	120 42	167 18	164 27	140 51	
Ada Benton	68 49	90 81	64 75	120 14	142 16	60 90	
Elmer S. Frey	240 18	920 41	718 52	167 59	840 72	143 86	
Joseph White	22 49	72 86	51 47	62 14	91 26	72 15	
Margaret Dix	47 26	91 18	21 64	18 42	61 19	64 86	
F. O. Beck	127 16	95 27	114 82	162 15	102 15	112 61	
L. O. Avery	214 91	218 46	920 41	172 14	152 86	142 71	
B. W. Snyder	162 14	153 46	118 64	162 14	182 15	69 58	
Ella Harding	21 27	18 92	17 65	28 64	59 18	72 41	
Carrie Simpson	21 18	45 30	16 98	42 41	20 68	75 98	
W. F. Baldwin	162 10	114 80	115 90	116 84	117 41	200 60	
E. O. Burrill	84 90	90 10	116 80	114 30	65 20	300 75	
Total							

6. On the page following are a number of inventory extensions; find the footing of each.

Each column should be added in approximately three minutes.

a.	*b.*	*c.*	*d.*
$1628.45	$1743.19	$2065.32	$1156.78
176.22	78.91	145.55	11.69
453.26	1011.45	28.49	189.24
1102.65	125.60	217.86	338.54
45.22	101.25	207.41	24.75
143.51	74.24	1078.44	8.95
55.68	212.35	45.60	1250.05
425.70	338.90	256.85	74.33
119.45	227.83	194.61	258.97
7.15	46.21	112.45	364.20
89.52	117.25	34.65	126.70
48.75	57.81	133.27	82.56
170.25	9.11	24.13	219.63
613.81	764.35	495.34	360.42
1203.05	88.75	6.17	24.45
327.16	401.19	98.75	175.26
654.32	1145.22	856.21	1364.12
108.17	366.18	1408.95	374.11
75.45	201.13	376.23	211.06
9.58	64.33	112.50	21.08
142.53	8.78	56.40	312.74
189.70	89.56	76.50	118.44
20.65	192.35	244.56	94.65
673.13	98.75	339.54	951.15
120.06	320.16	87.40	73.07
65.34	135.20	467.80	239.15
753.25	846.18	515.05	1856.22
371.10	1904.76	287.01	475.02
69.16	128.44	301.04	602.13
231.06	473.08	1654.23	342.50
503.44	95.16	270.15	89.65
78.42	143.65	69.40	439.20
250.21	54.30	355.11	175.06
2859.89	1756.84	742.13	1914.56

WRITTEN EXERCISE

Each of these groups should be added in approximately one minute.

1.	2.	3.
6,354,276,742	6,917,408,583	8,632,714,509
5,116,433,854	5,880,734,112	6,875,352,272
8,446,750,284	6,538,922,553	8,539,168,753
2,141,991,648	7,543,276,445	8,335,674,912
4,653,752,816	6,441,819,263	5,321,841,174
3,256,851,539	2,574,438,911	2,435,627,819
5,462,175,255	4,156,717,549	1,546,721,973
6,435,621,953	1,817,908,667	5,167,534,217
4,717,880,945	6,745,243,517	4,576,819,054
5,601,523,764	3,546,798,212	4,151,762,492

4.	5.	6.
2,334,515,637	5,432,317,892	6,453,615,809
5,734,516,772	6,534,317,865	4,576,879,253
7,354,618,227	4,531,841,962	2,432,653,274
3,542,618,906	3,132,341,264	7,819,457,836
5,115,616,874	4,651,272,335	1,918,776,425
7,175,437,256	3,256,728,143	5,327,731,224
1,735,273,540	1,830,925,165	4,532,551,243
8,134,556,773	2,543,175,213	5,231,605,228
3,115,617,221	3,413,214,605	7,615,257,818
3,255,617,711	5,443,216,448	5,663,338,119

7.	8.	9.
4,334,561,228	3,653,212,716	4,176,234,562
1,765,371,442	7,661,823,394	5,934,736,548
9,717,632,461	5,749,287,716	6,254,817,259
5,616,727,640	3,264,723,445	1,218,735,143
2,615,431,817	2,845,367,621	7,342,235,907
4,256,713,332	4,256,443,486	6,543,217,194
3,078,395,668	7,254,563,823	2,546,718,911
7,845,446,772	9,117,224,383	3,664,293,189
8,250,114,337	1,493,786,332	9,226,847,821

A WRITTEN REVIEW TEST

Write the following problems from dictation, and then complete the work. Time, approximately, thirty minutes, including the dictation. Add the following:

1.		2.	
23,418	17,546	28,356	43,572
43,621	23,811	32,718	21,471
71,892	32,264	10,918	37,509
22,457	31,542	27,354	42,311
34,256	45,623	45,612	20,175
41,243	54,819	33,452	75,604
44,172	22,716	31,174	70,219
34,523	45,613	44,530	28,332
31,113	41,414	45,517	37,743
20,125	13,064	50,197	22,508
62,157	93,845	29,875	45,612
?	?	?	?

In problems 3 and 4 add by columns and then by lines, and check the work by adding the vertical and horizontal totals.

3.			4.		
542	236	?	123	235	?
437	653	?	947	834	?
572	246	?	174	215	?
445	354	?	313	208	?
236	716	?	364	312	?
423	394	?	252	733	?
347	616	?	243	317	?
455	494	?	203	406	?
337	651	?	543	392	?
453	283	?	527	618	?
624	912	?	624	483	?
538	496	?	713	626	?
712	324	?	235	439	?
?	?	?	?	?	?

CHAPTER V

SUBTRACTION

ORAL EXERCISE

State the number that, added to the smaller number, makes the larger one in each of the following:

1.	3	4	4	5	6	7	8	8	9	9	9	9	8	8	7
	1	2	1	3	2	3	3	2	3	1	6	4	4	1	2
2.	12	11	12	11	12	11	12	11	10	11	10	11	10	12	10
	9	2	3	9	8	3	4	8	4	7	6	4	7	5	3
3.	18	17	16	17	16	15	14	15	14	13	13	16	15	14	13
	9	8	7	9	8	6	9	7	8	4	7	9	8	5	9
4.	13	14	14	15	16	17	18	18	19	19	19	19	18	18	17
	11	12	11	13	12	13	13	12	13	11	16	14	14	11	12
5.	22	21	22	21	22	21	22	21	20	21	20	21	20	22	20
	19	12	13	19	18	13	14	18	14	17	16	14	17	15	13
6.	38	27	26	37	26	35	44	25	34	53	43	36	45	54	73
	29	18	17	29	18	26	39	17	28	44	37	29	38	45	69
7.	42	51	72	81	92	71	32	41	70	61	90	81	30	62	50
	39	42	63	79	88	63	24	38	64	57	86	74	27	55	47

47. A **parenthesis** ( ) signifies that the numbers included within it are to be considered together. A **vinculum** $\overline{\quad}$ has the same signification as a parenthesis.

Thus, $15 - (4 + 2)$, or $15 - \overline{4 + 2}$ signifies that the sum of 4 and 2 is to be subtracted from 15.

48. Examples. 1. Find the difference between 849 and 162.

SOLUTION. 2 from 9 leaves **7**. 6 cannot be subtracted from 4, but 6 from 14 leaves **8**. Since 1 of the 8 hundreds has been taken, there are but 7 hundreds remaining. 1 from 7 leaves **6**.

CHECK. $687 + 162 = 849$.

$$\begin{array}{r} 849 \\ 162 \\ \hline 687 \end{array}$$

The above is a common method of subtraction. For practical computation, however, the "making change" method is best. It is easily understood and is much more rapid when once learned. The "making change" method is illustrated in the following example and solution.

2. Find the difference between 7246 and 4824.

SOLUTION. Think "$4 + 2 = 6$," and write 2; "$2 + 2 = 4$," and write 2; "$8 + 4 = 12$," and write 4; "1 and $4 + 2 = 7$," and write 2.

CHECK. $2422 + 4824 = 7246$.

$$\begin{array}{r} 7246 \\ 4824 \\ \hline 2422 \end{array}$$

ORAL EXERCISE

1. $16 + 23 + ? = 54?$
2. $27 + 14 + ? = 72?$
3. $17 + 36 + ? = 62?$
4. $19 + 17 + 12 + ? = 57?$
5. $25 + 14 + 11 + ? = 75?$
6. $18 + 17 + 16 + ? = 70?$
7. $16 + 18 + 16 = 25 + ?$
8. $72 + 17 + 11 = 37 + ?$
9. $14 + 18 + 38 = 42 + ?$
10. $12 + 16 + 12 + 14 + ? = 75?$
11. $16 + 15 + 19 + 15 + ? = 93?$
12. $18 + 17 + 15 + 29 + ? = 98?$

WRITTEN EXERCISE

1. Without copying the individual problems, find quickly the sum of the twenty differences in the following:

\$2140.50	\$4157.50	\$5000.24	\$9000.72	\$3145.62
714.23	1236.80	249.17	1246.18	2000.79
\$5500.89	\$1624.14	\$1985.72	\$1379.54	\$1742.18
2799.14	957.80	645.92	923.18	842.16
\$9275.17	\$2446.80	\$3169.14	\$3156.19	\$4756.83
842.99	1321.44	874.36	1400.72	2738.44
\$7514.85	\$7291.80	\$1756.92	\$8721.13	\$1872.14
721.92	1642.95	921.74	2049.79	742.12

2. Copy the following table and show (*a*) the total exports for each year given; (*b*) the excess of exports for each year given; (*c*) the total exports and imports for the ten years; (*d*) the total excess of exports for the ten years. Check.

IMPORTS AND EXPORTS IN THE UNITED STATES FOR TEN YEARS

YEAR ENDING JUNE 30	EXPORTS		TOTAL EXPORTS	IMPORTS	EXCESS OF EXPORTS
	Domestic	Foreign			
1904	$1 435 179 017	$25 648 254		$991 090 978	
1905	1 491 744 641	26 817 025		1 117 513 071	
1906	1 717 953 382	25 911 118		1 226 562 446	
1907	1 853 718 034	27 133 044		1 434 421 425	
1908	1 834 786 357	25 986 357		1 194 341 792	
1909	1 638 355 593	24 655 511		1 311 920 224	
1910	1 710 083 998	34 900 722		1 556 947 430	
1911	2 013 549 025	35 771 174		1 527 226 105	
1912	2 170 319 828	34 002 581		1 653 264 934	
1913	2 428 506 358	37 377 791		1 812 978 234	
Total					

Under the term *domestic exports* are included exports of merchandise, the growth, produce, or manufacture of the United States; under *foreign exports* are included articles of merchandise previously imported into the United States and subsequently reëxported. Under the term *imports* are included imports of all merchandise of whatever origin received into the United States.

49. The common method of **making change** is to add to the price of the goods purchased a sum that will equal the amount offered in payment.

Thus, if a person buys groceries amounting to 74¢ and tenders $1 in payment, the mental process of the clerk in making the change is as follows: "74¢ + 1¢ + 25¢ = $1"; the customer should receive as change a 1-cent piece and a quarter of a dollar.

Change may be made in a number of ways. In the above example two dimes and a 5-cent piece might be given instead of the quarter of a dollar. In the following exercise name the largest coins and bills that could be used.

ORAL EXERCISE

1. Name the coins and the amount of change to be given from $1 for each of the following purchases: 17¢; 24¢; 31¢; 38¢; 45¢; 52¢; 59¢; 66¢; 73¢; 80¢; 87¢; 18¢; 25¢.

2. Name the coins and the amount of change to be given from \$2 for each of the following purchases: \$1.19; \$1.26; \$1.33; \$1.40; \$1.47; \$1.54; \$1.61; \$1.68; \$1.75; \$1.82; \$1.89; \$1.20; \$1.27; \$1.34; \$1.41; \$1.48; \$1.55; \$1.62; \$1.69; \$1.76; \$1.83; \$1.90.

3. Name the bills and coins and the amount of change to be given from \$5 for each of the following purchases: \$1.21; \$1.28; \$1.35; \$1.42; \$2.22; \$2.29; \$2.36; \$4.43; \$3.49; \$4.50; \$3.51; \$3.56; \$4.57; \$2.58; \$1.63; \$2.64; \$1.65; \$1.70; \$2.71; \$3.72; \$2.77; \$3.84; \$1.91; \$2.85; \$2.92.

4. Name the bills and coins and the amount of change to be given from \$10 for each of the following purchases: \$4.93; \$3.86; \$7.70; \$2.44; \$8.37; \$5.30; \$3.23; \$5.17; \$4.24; \$3.31; \$8.38; \$2.45; \$6.52; \$4.59; \$3.66; \$5.73; \$4.80; \$3.87; \$2.88; \$7.81; \$9.74; \$5.67; \$3.60; \$4.53; \$2.46; \$3.29; \$8.32; \$7.25; \$2.18; \$7.49; \$9.42; \$3.67; \$1.93.

50. It is frequently necessary to find the difference between a minuend and several subtrahends. If the "making change" method of subtraction is employed, the operation is a simple one.

51. Example. From a farm of 578 A. I sold at one time 162 A., at another 98 A., and at another 121 A. How many acres remained unsold?

SOLUTION. Arrange the numbers as shown in the margin. Eleven $(1+8+2)$ and **seven** are 18; write 7. Three (1 carried $+2$), eighteen $(3+9+6)$ and **nine** are twenty-seven; write 9. Four (2 carried $+1+1$) and **one** are 5; write 1.

CHECK. $197+121+98+162=578$.

578 A.
162 A.
98
121
197 A.

WRITTEN EXERCISE

Find the amount each person has remaining on deposit:

1. A. Deposit, \$900; checks, \$210, \$175, \$198.
2. B. Deposit, \$875; checks, \$157, \$218, \$157.
3. C. Deposit, \$750; checks, \$120, \$117, \$121, \$118.
4. D. Deposit, \$960; checks, \$128, \$109, \$118, \$117.

5. E. Deposit, $967; checks, $192, $102, $117, $128, $146.

6. F. Deposit, $998; checks, $119, $117, $105, $123, $173.

Do not neglect to check all work. The bank clerk who makes an error a day in work like the above, and who fails to discover and correct this error, will not long retain his position.

7. Copy the following, supplying the missing terms and checking the results:

$$\begin{array}{rrrr}
\$148.90 + & \$149.75 + & \$421.77 = & \$???.?? \\
118.60 + & 172.12 + & ???.?? = & ???.?? \\
242.30 + & ???.?? + & 210.96 = & ???.?? \\
???.?? + & 168.72 + & 130.41 = & ???.?? \\
\hline
\$718.95 + & \$698.75 + & \$978.60 = & \$????.??
\end{array}$$

The following problem shows a portion of a bank discount register. In the first column are recorded the amounts of several notes that have been discounted; in the second, the discount charges; and in the third, the collection and exchange charges. The proceeds of any note is the difference between the amount (face) of the note and the total charges upon it.

8. Copy and complete the following bank record. Check the work. ($j + i + h$ should equal g.)

Face of Paper		Discount		Coll. & Exch.		Proceeds	
729	14	7	29		73	a	
862	29	4	31		86	b	
725	74	7	26		73	c	
832	16	12	48	1	26	d	
426	19	6	39		43	e	
378	36	3	78		38	f	
g		h		i		j	

52. The **complement** of a number is the difference between the number and a unit of the next higher order.

Thus, 2 is the complement of 8, 23 is the complement of 77, and 152 is the complement of 848. 3 and 7, 24 and 76, 250 and 750, are complementary numbers. Observe that *when two numbers of more than one figure each are complementary, the sum of the units' figure is 10 and the sum of the figures in each corresponding higher order is 9.*

53. Since numbers are read from left to right, in finding the complement of a number, begin at the left to subtract.

54. In beginning at the left to subtract take 1 from the highest order in the minuend and regard the other orders as 9's, except the last, which regard as 10.

55. Example. A man gave a 100-dollar bill in payment for an account of $77.52. How much change should he receive?

SOLUTIONS. (*a*) Begin at the left. 7 from 9 leaves **2**; 7 from 9 leaves **2**; 5 from 9 leaves **4**; 2 from 10 leaves **8**. Or

(*b*) 7 and **2** are 9; 7 and **2** are 9; 5 and **4** are 9; 2 and **8** are 10. $22.48.

$100.00	
77.52	
$22.48	

This method of finding the *amount of change* is used by many clerks and cashiers. The work is in all cases proved by counting out to the customer the bills and coins necessary to make the amount of the purchase equal to the amount offered in payment.

ORAL EXERCISE

Find the total of each group of numbers, then find the difference between the two totals:

1. (24 26 32 30 35 25) – (18 13 19 12 20 30).
2. (13 27 45 25 21 19) – (15 14 21 32 18 22).
3. (11 29 35 15 24 16) – (27 13 18 22 25 20).
4. (17 14 29 32 22 26) – (23 13 16 24 20 16).
5. (45 25 30 40 32 18) – (25 35 33 17 20 30).
6. (34 24 35 30 15 32) – (21 39 14 15 11 30).
7. (15 25 33 27 14 36) – (13 30 16 14 20 16).
8. (14 16 30 10 40 50) – (11 19 18 12 20 30).
9. (19 10 11 20 30 32) – (15 11 14 30 32 18).
10. (33 17 22 11 17 50) – (21 19 31 12 17 40).
11. (25 30 15 40 15 20) – (15 16 19 21 29 30).
12. (17 23 25 26 15 44) – (24 20 26 27 13 20).
13. (11 39 52 18 10 20) – (12 18 40 22 28 12).
14. (35 15 27 23 34 16) – (21 17 12 42 13 15).
15. (22 18 34 26 60 10) – (35 15 20 11 19 31).
16. (33 17 22 18 40 60) – (14 26 23 17 40 12).

ORAL EXERCISE

State the amount of change in each of the following problems:

	Cost of Items Purchased	Amount Paid		Cost of Items Purchased	Amount Paid
1.	17¢, 13¢, 42¢	$2	14.	$1.25, $0.75, $2.18	$20
2.	27¢, 23¢, 14¢	$2	15.	$1.50, $2.70, $1.18	$20
3.	45¢, 55¢, 13¢	$5	16.	$4.60, $1.40, $2.13	$20
4.	64¢, 16¢, 87¢	$5	17.	$1.50, $1.20, $2.30	$10
5.	23¢, 14¢, 27¢	$2	18.	$3.17, $4.11, $4.98	$50
6.	63¢, 17¢, 59¢	$5	19.	$4.25, $0.75, $3.18	$20
7.	49¢, 84¢, 37¢	$5	20.	$1.29, $2.17, $1.50	$20
8.	78¢, 42¢, 67¢	$5	21.	$1.64, $1.66, $2.50	$20
9.	52¢, 69¢, 88¢	$5	22.	$1.59, $23.41, $118	$200
10.	75¢, 86¢, 54¢	$5	23.	$24.17, $20.83, $15	$100
11.	89¢, 46¢, 72¢	$5	24.	$11.48, $10.52, $50	$100
12.	76¢, 54¢, 29¢	$5	25.	$18.91, $12.09, $45	$100
13.	75¢, 25¢, 89¢	$10	26.	$21.27, $2.73, $50.50	$100

56. $19-7=9$ (the minuend minus 10) $+3$ (the complement of the subtrahend); $191-17=91$ (the minuend minus 100) $+83$ (the complement of the subtrahend); $1912-178=912$ (the minuend minus 1000) $+822$ (the complement of the subtrahend), and so on.

57. This principle makes it a simple matter to find the difference between a subtrahend and several minuends.

58. Examples. The following examples illustrate the application of the principle:

	1.	2.	3.
	316	299	311
	+478	+488	+111
	−118	−111	−219
	=676	=676	=203

Solutions. **1.** 2 (the complement of 8), 10, 16; $16-10=6$. 9 (the complement of 1), 16, 17; $17-10=7$. 9, 13, 16; $16-10=6$.

2. 9, 17, 26; $26-10=16$; that is, 6 and 1 to add to the minuends. 9, 18 (9+8+1), 27; $27-10=17$; that is, 7 and 1 to add to the minuends. 9, 14, 16; $16-10=6$.

3. 1, 2, 3. $3-10$ is impossible, so subtract 1 ten from the minuend (or add 1 ten to the subtrahend). 9, 10. $10-10=0$. 8, 9, 12. $12-10=2$.

59. Example. The following problem shows a concrete application of the foregoing principle:

DEPOSITORS' LEDGER

DEPOSITOR	BALANCE	CHECKS	DEPOSITS	BALANCE
A	$ 74	$ 25	$ 86	$ 135
B	$ 86	$ 11	$ 99	$ 174
C	$ 92	$ 79	$ 81	$ 94

SOLUTION. Here is a depositors' ledger. The data in the first three columns being given, it is required to find the new balance.

The process is as follows: A. 6, 11, 15, **5**; 8, 16, 23, **13**; balance, $135. B. 9, 18, **24**, **4** and 1 to add to the minuend. 10, 19, 27, **17**; balance, $174. C. 1, 2, **4** and 1 to take away from the minuend. 7, 10, 19, **9**; balance, $94.

WRITTEN EXERCISE

Find the new balances, the total old balance, the total checks, the total deposits, the total new balances, and check the work:

1.

DEPOSITOR	BAL.	CHECKS	DEPOSITS	BAL.
A	$ 758	$ 128	$ 421	*a*
B	921	154	175	*b*
C	934	214	122	*c*
D	862	162	218	*d*
E	478	187	126	*e*
F	921	215	124	*f*
G	756	157	137	*g*
H	864	128	142	*h*
I	926	214	121	*i*
J	752	221	124	*j*
K	878	162	218	*k*
	l	*m*	*n*	*o*

2.

DEPOSITOR	BAL.	CHECKS	DEPOSITS	BAL.
A	$ 428	$ 125	$ 718	*a*
B	726	128	296	*b*
C	832	279	318	*c*
D	456	154	421	*d*
E	298	275	568	*e*
F	728	178	188	*f*
G	762	218	215	*g*
H	837	316	176	*h*
I	493	121	219	*i*
J	862	128	188	*j*
K	925	125	211	*k*
	l	*m*	*n*	*o*

60. $48 - 29 = \overline{48 + 1}$ (30, the next higher order of units than 29, $- 29) - 30$, or 19; $128 - 59 = \overline{128 + 1} - 60$, or 69.

61. This principle may be applied to advantage in billing items in which the gross weights and the tares are recorded.

The **gross weight** is the weight of merchandise, together with bag, cask, or other covering; the **tare** is the weight of the bag, cask, or other covering

of merchandise; the **net weight** is the difference between the gross weight and the tare.

62. Example. The gross weights and tares, in pounds, of 3 bbl. of sugar are: 332 – 19, 337 – 18 335 – 18. Find the total net weight.

SOLUTION. The numbers would be written on the bill horizontally, as shown in the margin. Adding the units of the tare, the result is 25; 30 (the next higher order of units than 25) minus 25 equals 5; 5 added to the units of the gross weight equals 19; 19 – 30 is impossible, so write **9** and subtract 2 tens (the difference between the tens in 30 and 19) from the gross weight or add 2 tens to the tens of the tare. Adding 2 tens to the tens of the tare, the result is 5; 10 – 5 = 5; 5 added to the tens of the gross weight equals 14; 14 – 10 = **4**. Adding the hundreds in the gross weight, the result is **9**. Net weight is 949 lb.

332 – 19 337 – 18 335 – 18 949#

WRITTEN EXERCISE

Copy the following bills. Verify the net weights given and supply all missing terms.

1.

CHICAGO, ILL., June 12, 19

Messrs. McGraw, Sprague & Co.
Tonawanda, N. Y.

Bought of PHILIP ARMOUR & CO.

Terms 60 da.

	3	tubs Lard						
		70-14 69-14 71-15 167	13#	21	71			
	6	casks Shoulders						
		422-64 428-70 424-65						
		420-64 426-70 424-70 2141	14#	299	74			
	5	casks Hams						
		410-70 411-70						
		408-70 412-71 416-69 ****	18#	307	26	****	**	

2.

Chicago, Ill., July 20, 19

Messrs. A. M. THOMPSON & CO.
Rochester, N.Y.

Bought of **Nelson, Morris & Co.**

Terms 30 da.

6	tubs Lard 72-17 70-14 69-14 71-14 71-15 70-16	***	$0.13	43	29	
6	casks Shoulders 421-65 426-70 424-72 422-64 427-72 421-60	****	.14	299	32	
6	casks Hams 409-72 412-70 414-71 410-73 412-70 416-71	****	.18	368	28	*****

3. The gross weights and tares of 6 casks of shoulders are as follows: 428 – 68, 419 – 70, 423 – 65, 432 – 72, 436 – 69, 434 – 65 lb. Find the total net weight.

4. The gross weight and tares of 12 tubs of lard are as follows: 71 – 14, 70 – 15, 69 – 14, 71 – 15, 72 – 17, 73 – 17, 69 – 15, 71 – 16, 72 – 15, 73 – 16, 74 – 17, 75 – 17 lb. Find the total net weight.

5. The gross weights and tares of 10 bbl. of sugar are as follows: 319 – 18, 331 – 19, 329 – 17, 334 – 20, 338 – 21, 325 – 18, 326 – 16, 325 – 19, 327 – 19, 321 – 17 lb. Find the total net weight.

BUSINESS TERMS AND RECORDS

63. A **debit** is an expression of value received; a **credit** is an expression of value delivered.

A buys of B 100 bu. wheat for $100 cash; the value received (debit) by A is *100 bu. wheat* and the value parted with (credit), *$100*. A sells C 50 bu. wheat for $75, C agreeing to pay for the same in 10 da.; the value received by A is *C's express* or *implied promise* to pay for the wheat in 10 da. and the value parted with is *50 bu. wheat*.

64. An **account** is a collection of related debits and credits.

65. Some of the **common accounts** kept in business are the cash account; personal accounts; the merchandise account; the expense account; the proprietary account.

66. A **resource** is any property on hand or any amount owed to a person or concern; a **liability** is any amount owed by a person or concern. The excess of resources over liabilities is the **net capital** or **present worth**; the excess of liabilities over resources, the **net insolvency**.

67. A **gain** is any sum realized in excess of the cost of a business or of business transactions; a **loss** is any sum spent or incurred in excess of the returns of a business or of business transactions. The excess of gains over losses is the **net gain**; the excess of losses over gains, the **net loss**.

68. The **cash account** is kept for the purpose of showing the receipts and payments of cash and the amount of cash on hand.

Cash

19–				19–			
Jan.	1	L. O. White	1200 –	Jan.	6	Office expense	169 50
	5	B. E. Smith	175 –		12	Rent of store	75 –
	9	Mdse.	69 50		12	Balance	1200 –
			1444 50				1444 50
Jan.	12	Balance	1200 –				

The receipts are entered on the left, or debit side, the payments, on the right, or credit side. The excess of debits is the balance or cash on hand.

In these exercises the use of red ink is not imperative.

69. Personal accounts are kept for the purpose of showing whether persons owe us or we owe them, and how much in either case.

J. E. King & Co.

19—				19—				
Feb.	3	Mdse.	721 60	Feb.	15	Discount 7 22 / Cash 714 38		721 60
Mar.	2	Mdse.	114 90	Apr.	1	Cash		114 90
Apr.	15	Mdse.	610 80	Apr.	25	Discount 6 11 / Cash 604 69		610 80
May	10	Mdse.	421 —					

On the left (debit) side of these accounts are placed the amounts which the persons owe us or which we pay them; on the right (credit) side, the amounts which we owe them or which they pay us. When the debits of an account are in excess of the credits, the account owes us for the amount of the excess; when the credits are in excess of the debits, we owe the account for the amount of the excess.

70. The **merchandise account** is kept for the purpose of showing the cost of goods purchased, the proceeds of goods sold, and the gain or loss resulting from such dealings.

Merchandise

19—					19—				
Mar.	1	Unsold goods	960 —		Mar.	5	Cash		430 60
	6	W. E. Haynes	160 70			7	F. C. Hill		529 40
	9	L. W. Lawton	720 05			14	L. W. King		360 15
	12	W. E. Haynes	160 85			16	R. O. Cook		360 —
	18	Gain	275 05			18	Cash		65 90
			2276 65			18	Unsold goods		530 60
									2276 65
Mar.	18	Unsold goods	530 60						

On the left (debit) side is entered the cost of goods purchased and on the right (credit) side the proceeds of goods sold. When the goods are all disposed of the excess of credits is a gain; the excess of debits, a loss. When it is desired to show the gain or loss on merchandise before the goods are all disposed of, it is necessary to first enter in the credit side of the account the present market value of the unsold goods.

71. The **expense account** is kept for the purpose of showing the cost of outlays incurred in carrying on the business.

Expense

Apr.	1	Stationery		16	50					12	40
	6	Coal		19	40					5	30
	6	Clerk hire		20	00						
	9	Drayage		10	00						
				67	70					67	70
Apr.	9	Coal on hand		12	40						

Such outlays are entered on the left (debit) side of the account. Ordinarily there are no credit entries. When the expense items are all used the debit of the account is a loss. When it is desired to show the loss or gain on expense and there are unused expense items on hand, it is first necessary to enter in the credit side of the account the present value of such items.

72. The **proprietary account** is kept for the purpose of showing how much the proprietor invests in the business and how much he withdraws from the business.

F. W. Simpson, Proprietor

May	10	Cash		160	—	May	1	Cash		6000	—
		Investment		6000	—		31	Net Gain		169	90
				6169	90					6169	90
						June	1	Present Worth		6009	90

On the right (credit) side are entered all sums invested and the net gain, and on the left (debit) side all sums withdrawn and the net loss. The excess of credits is the present worth of the business.

ORAL EXERCISE

1. In the cash account on page 41 what are the total receipts? the total payments? the balance of cash on hand?

2. At the top of page 42 is your account with J. E. King & Co. On what dates did you sell the firm merchandise? When and how were payments made on account? What was the balance of the account May 10?

3. In the account with merchandise, page 42, what is the cost of the purchases? the proceeds of the sales? How would the value of the unsold goods be determined in business? Verify the amount of the gain. Is it correct?

4. Verify the amount of the loss in the expense account, page 43. Is it correct?

5. What are the total withdrawals in the account with F. W. Simpson, Proprietor, page 43? the total investment?

WRITTEN EXERCISE

1. Copy the cash account on page 41 and continue it with the following items: Jan. 12, receive cash of Jones & Co., $75; Jan. 14, pay cash for groceries, $165.62; Jan. 15, receive cash for groceries, $189.75; Jan. 18, pay cash to office help, $129.74; Jan. 20, pay cash for stationery, $11.75; Jan. 22, receive cash for groceries, $126.94; Jan. 24, receive casn of H. W. Conant, $200.67. Balance the account as shown in the model.

2. Copy the purchases and sales of the merchandise account, page 42. Assuming that the value of the unsold goods is $327.61, find the gain and close the account.

3. Copy the purchases and sales of the merchandise account, page 42. Assuming that the value of the unsold goods is $50, find the gain or loss and close the account. Assuming that all of the goods are sold, find the gain or loss and close the account.

4. Arrange the following data in the form of your account with Benj. F. Butler. June 1, buy of Benj. F. Butler on account (without making payment) dry goods amounting to $627.96; June 10, pay him for invoice of June 1 less $6.28 discount; June 28, buy of him dry goods amounting to $472.69 and pay cash to apply on the bill, $172.69; July 15, buy of him on account dry goods amounting to $369.71; July 31, pay him cash to apply on bill of July 15, $79.79; Aug. 2, sell him lace amounting to $14.60. Find the balance of the account and tell whether such balance is a resource or a liability.

5. Using the above data, write Benj. F. Butler's account of his dealings with you. Balance the account.

6. Copy the account with F. W. Simpson, Prop., page 43. Continue the account through June, using the following items: June 6, make an additional investment of $1000; June 25, withdraw for personal use $160; June 30, the net gain for the month, which is to remain as an additional investment, is $369.75. Find the present worth and close the account.

WRITTEN EXERCISE

Copy the following statements, supplying the missing terms:

1.

Statement of Losses and Gains, July 31, 19–

	Gains				
Merchandise	sales		3470	60	
	purchases	4160.10			
	on hand	960.10			
	cost of sales		3200	–	
	gain				6670 60
	Losses				
Expense	cost		1178	60	
	office safe on hand		400	–	
	loss				778 60
F. O. Ely's net gain					1920 00

2.

Statement of Resources and Liabilities, July 31, 19–

	Resources			
Merchandise	goods on hand	960	10	
Real Estate	store and lot	4000	–	
Cash	balance on hand	1290	60	
H. E. Gale	owes the business	610	90	
J. L. Bush	" " "	200	40	
F. C. Miller	" " "	150	–	7201 00
	Liabilities			
W. M. Cary	business owes him	614	20	
L. S. Martin	" " "	300	80	
C. H. Jones	" " "	429	22	1345 60
F. O. Ely's present worth				5855 40

3. A merchant purchased a stock of hardware amounting to $45,112.18 and sold from this stock goods amounting to $31,136.85. He then took an account of stock and found that the value of the hardware on hand was $18,438.50. Find the amount of his gain.

4. C. E. Cyr's resources and liabilities at the close of a month were as follows: dry goods on hand, $1629.40; store and lot, $3000; cash in bank, $1400.60; C. O. Bond owes the business $400; L. E. Young, $390.10; and J. O. Snow, $209.90. The business owes Roe & Co. $750; and Doe & Co. $90.75. Make a statement of resources and liabilities.

5. At the close of the same month C. E. Cyr's business accounts show the following results: stock of dry goods on hand at the beginning of the month, $1270.40; purchases of dry goods for the month, $3229.60; sales of dry goods for the month, $3762.90; market value of the dry goods on hand at the close of the month, $1629.40; expense for the month, $413.95; value of expense items on hand, $250. Make a statement of losses and gains.

6. A real estate agent had property on hand Jan. 1 to the amount of $8155.60. During the year he bought property costing $4150.60, added buildings at a cost of $6190.40, and paid in taxes $250.90. April 15 a house valued at $1690 was destroyed by fire, and for this loss the insurance company paid him $1300. During the year he sold property for $9260.50 and received for rents $840.80. If the expenses of the sales aggregated $240.19, and the value of the property on hand Dec. 31 was $11,250.60, what was his net gain or loss for the year?

7. A real estate agent bought a piece of land for $6254.25. He made the following expenditures: for grading, $328.65; for draining, $28.30; for taxes, $98.45; for the construction of a dwelling house, $4580. In the course of the year he sold a house and lot for $6685.40; 5 lots for $845.25 each. If his expenses including insurance were $175.50, and he still owned 2 lots valued at $950 each, what was his gain on the whole transaction?

WRITTEN EXERCISE

Copy each of the following examples; complete the work, and check the result. Time, approximately, thirty minutes. Face of paper, minus discount and collection and exchange, equals proceeds.

	Face of Paper	Discount	Coll. and Exch.	Proceeds
1.	$376.25	$3.76	$0.38	?
	255.78	1.28	0.26	?
	176.44	1.76	—	?
	259.86	2.60	0.26	?
	492.71	2.46	0.49	?
	149.52	0.75	0.15	?
	643.15	6.43	0.64	?
	319.21	0.80	—	?
	?	?	?	?

	Face of Paper	Discount	Coll. and Exch.	Proceeds
2.	$186.73	$1.87	$0.19	?
	237.50	2.38	0.24	?
	412.88	4.13	—	?
	337.95	3.38	0.34	?
	360.90	0.90	—	?
	245.91	1.23	0.25	?
	307.55	3.08	0.31	?
	250.40	3.76	0.25	?
	?	?	?	?

	Face of Paper	Discount	Coll. and Exch.	Proceeds
3.	$421.95	$2.11	$0.42	?
	112.70	1.13	0.11	?
	324.50	1.62	—	?
	245.60	2.46	0.25	?
	194.75	0.92	0.19	?
	217.50	1.09	0.22	?
	311.59	3.12	0.31	?
	289.72	1.45	0.29	?
	?	?	?	?

WRITTEN EXERCISE

Copy each of the following examples; find the new balance, the total old balance, the total checks, the total deposits, and check the work:

1.

Name	Bal.	Checks	Deposits	Bal.
A	$516	$423	$313	?
B	203	507	398	?
C	195	461	412	?
D	204	165	—	?
E	335	515	296	?
F	411	309	156	?
G	135	257	145	?
H	—	295	512	?
I	500	316	104	?
J	450	225	117	?
K	650	751	211	?
L	512	337	206	?
M	242	176	105	?
	?	?	?	?

2.

Name	Bal.	Checks	Deposits	Bal.
A	$309	$423	$267	?
B	476	379	112	?
C	277	407	321	?
D	255	—	126	?
E	167	217	178	?
F	213	453	329	?
G	208	196	114	?
H	126	240	114	?
I	255	153	—	?
J	—	212	317	?
K	455	235	103	?
L	275	230	115	?
M	315	275	144	?
	?	?	?	?

3.

Name	Bal.	Checks	Deposits	Bal.
A	$311	$242	$301	?
B	235	115	118	?
C	178	—	212	?
D	142	188	206	?
E	—	268	315	?
F	447	397	109	?
G	214	375	226	?
H	154	106	—	?
I	256	400	144	?
J	512	613	199	?
K	245	237	155	?
L	146	—	114	?
M	565	743	248	?
	?	?	?	?

4.

Name	Bal.	Checks	Deposits	Bal.
A	$267	$133	$145	?
B	—	247	315	?
C	256	119	228	?
D	116	—	177	?
E	215	411	196	?
F	423	375	116	?
G	198	213	132	?
H	245	157	109	?
I	233	334	168	?
J	443	337	155	?
K	704	856	266	?
L	231	250	119	?
M	178	252	107	?
	?	?	?	?

A WRITTEN REVIEW TEST

Write problems 1 and 2 from dictation, and then complete the work. Time, approximately, forty minutes, including the dictation.

1. Add the following and check the results:

3412	4571	3582	?
6243	8132	2738	?
1729	2642	9810	?
4572	7235	1345	?
2643	6254	4165	?
3414	8149	3542	?
7145	6127	4713	?
3328	2170	2312	?
5331	4262	1745	?
3473	1208	8052	?
?	?	?	?

2. Find the new balance, the total old balance, the total checks, the total deposits, and check the work:

NAME	BALANCE	CHECKS	DEPOSITS	BALANCE
A	$235	$142	$217	?
B	312	254	122	?
C	288	300	212	?
D	542	250	106	?
E	—	350	500	?
F	600	325	—	?
G	750	—	252	?
H	219	200	151	?
I	224	100	216	?
J	116	330	214	?
K	255	225	110	?
?	?	?	?	?

3. The text, page 46, problem 3.
4. The text, page 46, problem 6.
5. The text, page 46, problem 7.

CHAPTER VI

MULTIPLICATION

ORAL EXERCISE

1. Which of the following numbers are **concrete**; that is, refer to some particular kind of object or measure? 12; $5\frac{1}{2}$; 12 ft.; 2.5 da.; 15 yd.; 18 men; 200; \$12; $172\frac{4}{5}$.

2. Which of the above numbers are **abstract**; that is, do not refer to any particular kind of object or measure?

3. $5+4+2+8+9=?$

4. $9+9+9+9+9=?$ 5 times $9=?$

5. Could the sum of the numbers in problem 3 be found by any shorter process?

6. What is the first process in problem 4 called? the second?

7. 9 times $27=?$ 9 times 29 bu. $=?$

8. If 1 bu. of rye weighs 56 lb., what will 12 bu. weigh?

73. In problems 7 and 8 it is seen that *the multiplier is always an abstract number;* and *the multiplicand and product are like numbers.*

74. Three 5's are equal to five 3's; \$3 multiplied by 5 is equal to \$5 multiplied by 3; 4 trees multiplied by 125 is equal to 125 trees multiplied by 4.

75. It is therefore seen that *the product is not affected by changing the order of the factors regarded as abstract numbers.*

76. The multiplicand and multiplier together are called **factors** (makers) of the product; the product of two abstract integers is sometimes called a **multiple** of either of the factors.

77. Sometimes a number is used several times as a factor. Numbers so used are indicated by a small figure, called an **exponent**, written above and at the right of the factor.

Thus, 4 used *twice* as a factor is written 4^2, 5 used *four times* as a factor is written 5^4, and 6 used *five times* as a factor is written 6^5.

78. The product arising from using a number two or more times as a factor is called a **power** of that number.

Thus, 4 is the second power of 2; 64 is the third power of 4 and the sixth power of 2.

Too much attention should not be given to the definitions like the above. They are valuable only as they help to make clear the matter in the exercises. They are rarely heard in business and therefore should not be memorized.

ORAL EXERCISE

1. Multiply at sight each number below by 2; by 3; by 4; by 5; by 6; by 7; by 8; by 9.

Name the products by lines from left to right and from right to left; also by columns from left to right and from right to left. Name results only. Thus, to multiply lines by 4 say 20, 36, 8, 24, 40, 12, 28, 44, 16, 48, 32, 52, 68, 84, and so on up to 100; and backwards, 100, 80, 96, 64, and so on back to 20. To multiply columns by 4 say 20, 68, 36, 84, and so on to 52, 100; and backwards 100, 52, 80, 32, and so on to 68, 20. Continue the work until results can be named at the rate of 120 or more per minute.

5	9	2	6	10	3	7	11	4	12	8	13
17	21	14	18	22	15	19	23	16	24	20	25

2. Multiply as instructed in problem 1 and add 8 (carried) to each product. Also multiply as instructed and add 6, 4, 7, 2, 5, 3, and 9 to each product.

Name results only. Thus, to multiply by lines say 20, 28; 36, 44; 8, 16; and so on.

3. Multiply by 2: 27, 35, 81, 36, 28, 32, 47, 93, 56, 39, 54, 45, 52, 86, 75, 67, 59. Also by 4, 3, 5, 8, 6, 7, 9.

4. Find the cost of each of the following: 20 lb. crackers at 8¢; 9 lb. coffee at 34¢; 7 lb. tea at 57¢; 11 lb. beef at 17¢; 120 lb. sugar at 4¢; 134 lb. sugar at 5¢.

5. Find the cost of each of the following: 44 yd. at 9¢; 37 yd. at 8¢; 123 yd. at 6¢; 214 yd. at 4¢; 52 yd. at 12¢; 29 yd. at 8¢; 8 yd. at $1.03; 7 yd. at $1.01; 5 yd. at $1.35.

6. Beginning at 0 count by 9's to 81; by 10's to 150; by 11's to 154; by 12's to 108; by 13's to 117; by 14's to 126; by 15's to 135; by 16's to 144; by 17's to 153; by 18's to 162; by 19's to 171; by 20's to 180.

79. Examples. 1. Find the cost of 2150 lb. at 5¢.

SOLUTION. Since 1 lb. costs 5¢, 2150 lb. will cost 2150 times 5¢; but 2150 times 5¢ is equal to 5 times 2150¢. 5 times $21.50 (2150¢) equals $107.50, the required result.

$$\begin{array}{r} \$21.50 \\ 5 \\ \hline \$107.50 \end{array}$$

2. Multiply 224 by 46.

SOLUTION. In multiplying one number by another, there is no practical advantage in beginning with the lowest order of units of the multiplier; in fact, in some multiplications there is a decided advantage in beginning with the highest order. The arrangement of work for both methods is shown in the margin.

$$\begin{array}{r} 224 \\ 46 \\ \hline 1344 \\ 896 \\ \hline 10304 \end{array} \qquad \begin{array}{r} 224 \\ 46 \\ \hline 896 \\ 1344 \\ \hline 10304 \end{array}$$

CHECK. The work may be checked by multiplying first by one method and then by the other, or by interchanging the multiplier and multiplicand and re-multiplying. (See also pages 89 and 90.)

3. Multiply 2004 by 1275.

SOLUTION. When one of two numbers to be multiplied contains a number of zeros or ones, it is always easier to take that number as the multiplier. Since the product of any number multiplied by 0 is 0, the product of 1275 multiplied by the tens and hundreds of the multiplier need not be written.

$$\begin{array}{r} 1275 \\ 2004 \\ \hline 5100 \\ 2550 \\ \hline 2555100 \end{array} \qquad \begin{array}{r} 1275 \\ 2004 \\ \hline 2550 \\ 5100 \\ \hline 2555100 \end{array}$$

CHECK. The problem may be checked the same as problem 2.

When two numbers are to be multiplied, it is generally easier to take as the multiplier the number having the least number of places. Thus, to find the cost of 1647 A. of land at $27 per acre, take 27 as the multiplier.

If one of the two numbers to be multiplied has two or more digits alike, it is easier to take that number as the multiplier. Thus, to multiply together 6729 and 7777, it is easier to take 7777 as the multiplier.

ORAL EXERCISE

1. Find the value of 51 T. of hay at $17 per ton.
2. Find the cost of 175 lb. of sugar at 5¢ per pound.
3. How much will a boy earn in 87 hr. at 9¢ an hour?
4. What is the cost of a flock of 52 sheep at $7 per head?
5. At the rate of 47 mi. an hour, how far will a person travel in 12 hr.?
6. What is the cost of 12 pr. of shoes at $4.50 per pair, and 8 pr. of boots at $3.50 per pair?

7. What must be paid for handling 12 loads of freight at $2.25 per load?

8. In an orchard there are 13 rows of trees, each containing 21 trees. How many trees in the orchard?

9. If you buy 5 pencils at 9¢ each and 9 penholders at 5¢ each, and some stationery costing 25¢, how much change should you receive from a two-dollar bill? from a ten-dollar bill?

10. I bought 6 cd. of wood at $5.75 per cord. If a fifty-dollar bill is offered in payment, how much change should be received?

11. I bought 12 bu. of wheat at $1.05. If I gave in payment two ten-dollar bills, what change should I receive?

12. My average marketing expenses per day are $2.10. If I offer a twenty-dollar bill in payment for 7 days' expenses, what change should I receive?

13. I sold 16 chairs at $7 each, and 5 tables at $9 each. If two one-hundred-dollar bills are offered in payment, how much change should I return? If a one-hundred-dollar bill, a fifty-dollar bill, and a twenty-dollar bill are offered in payment, how much change should I return?

WRITTEN EXERCISE

In the following problems find the missing numbers by multiplying across and adding down. Check the results by comparing the sum of the line products with the sum of the multiplicands multiplied by one of the multipliers.

1.	2.	3.
$15 \times 211 = ?$	$9 \times 1475 = ?$	$12 \times \$16.50 = ?$
$15 \times 346 = ?$	$9 \times 2618 = ?$	$12 \times \$27.75 = ?$
$15 \times 318 = ?$	$9 \times 1575 = ?$	$12 \times \$14.95 = ?$
$15 \times 721 = ?$	$9 \times 1792 = ?$	$12 \times \$29.86 = ?$
$15 \times 936 = ?$	$9 \times 4936 = ?$	$12 \times \$49.88 = ?$
$15 \times 849 = ?$	$9 \times 7289 = ?$	$12 \times \$39.62 = ?$
$15 \times 218 = ?$	$9 \times 8728 = ?$	$12 \times \$86.99 = ?$
$15 \times \quad ? \quad = ?$	$9 \times \quad ? \quad = ?$	$12 \times \quad ? \quad = ?$

4.	5.	6.
$12 \times 192 = ?$	$98 \times 2178 = ?$	$16 \times \$18.10 = ?$
$12 \times 721 = ?$	$98 \times 1692 = ?$	$16 \times 17.20 = ?$
$12 \times 836 = ?$	$98 \times 2536 = ?$	$16 \times 21.40 = ?$
$12 \times 456 = ?$	$98 \times 2892 = ?$	$16 \times 25.85 = ?$
$12 \times ? = ?$	$98 \times ? = ?$	$16 \times ? = ?$

Problems such as the above are very helpful. They afford a variety of work and suggest a simple method by which the student may test the correctness of his results. The instructor should add as many more problems as circumstances require.

7. A produce dealer bought 2145 bu. of potatoes at 83¢ a bushel, and sold them at $1.05 a bushel. What did he gain?

8. A drover bought 125 head of cattle at $15.75 per head. He sold 65 head at $23.40, 15 head at $13.75, and 45 head at $17.75. Did he gain or lose, and how much?

9. A grocer bought 14 bu. of apples at $1.35 per bushel and 12 bu. of potatoes at 84¢ per bushel. He sold the apples at 40¢ a peck and the potatoes at 25¢ a peck. What did he gain?

10. A bought 1247 bbl. of apples at $3.10 per barrel. After holding them for three months he sold them at $4.80 per barrel. If he paid $74.82 for storage, and his loss by decay was 37 bbl. of apples, what was his gain?

11. The gross weight in pounds, and tare in pounds, of 25 tubs of lard are as follows: 71 – 14, 70 – 15, 69 – 14, 72 – 16, 71 – 14, 72 – 15, 70 – 15, 69 – 14, 71 – 15, 70 – 15, 69 – 14, 71 – 16, 71 – 15, 71 – 14, 70 – 15, 68 – 14, 73 – 16, 73 – 15, 70 – 14, 70 – 14, 71 – 15, 73 – 16, 74 – 18, 71 – 13, 73 – 16. Find the cost at 13¢ per pound.

12. The gross weight in pounds, and the tare in pounds, of 25 casks of hams are as follows: 400 – 78, 420 – 68, 420 – 71, 403 – 71, 409 – 71, 418 – 68, 412 – 72, 407 – 67, 423 – 69, 419 – 67, 426 – 68, 403 – 70, 399 – 69, 400 – 69, 425 – 71, 413 – 72, 399 – 67, 412 – 72, 418 – 68, 409 – 71, 408 – 70, 412 – 68, 402 – 71, 421 – 71, 403 – 71. Find the cost at 18¢ per pound.

SHORT METHODS IN MULTIPLICATION

80. There are many short methods in multiplication, but of these only a few are practical, either because they apply generally to problems that in themselves are not practical or because they have been supplanted by the elaborate use of tables and mechanical devices. A great many practical and helpful tables are in use for figuring pay rolls, interest, discount, and the like. These tables are great time savers. (See pages 229 and 321.)

81. The machines that are used for adding, subtracting, multiplying, dividing, and for setting forth results in interest and discount are now in such common use that a chapter is devoted to their consideration in Appendix A at the close of this volume. These machines are found in business offices, especially where extensive operations are to be performed. Both in accuracy and in the saving of time they are most valuable.

82. The short methods given herewith have a wide application. They are not dependent upon formal rules, and are suggestive of other ways in which the student may exercise his own ingenuity to shorten his work in multiplication.

Multiplication by Powers and Multiples of Ten

ORAL EXERCISE

1. 40 is how many times 4? 60 is how many times 6? 100 is how many times 10? 150 is how many times 15?

2. Give a short method for multiplying an integer by 10.

3. 400 is how many times 4? 600 is how many times 6? 1000 is how many times 10? 1500 is how many times 15?

4. Give a short method for multiplying an integer by 100; by 1000; by 10000.

5. How does the product of 40×66 compare with the product of $4 \times 66 \times 10$? the product of 400×59 with the product of $4 \times 59 \times 100$?

6. Give a short method for multiplying an integer by any number of 10's, 100's, or 1000's.

7. Multiply 270 by 300.

SOLUTION. In the accompanying illustration it will be seen that $270 \times 300 = 27 \times 3 \times 1000$ or 81,000.

$$\begin{aligned} 270 &= 27 \times 10 \\ 300 &= 3 \times 100 \\ \hline 81000 &= 81 \times 1000 \end{aligned}$$

8. Formulate a rule for finding the product when there are zeros on the right of both factors.

9. \$7 is how many times \$0.70? \$90 is how many times \$0.90? \$500 is how many times \$0.50?

10. State a short method for multiplying United States money by 10; by 100; by 1000.

11. Read aloud the following, supplying the missing words:

(*a*) Annexing a cipher to an integer multiplies the integer by ——; annexing two ciphers to an integer —— the integer by ——.

(*b*) Removing the decimal point in United States money one place to the right —— the number by 10; removing the decimal point two places to the right —— the number by ——.

12. Multiply \$14.70 by 10; by 100; by 1000.

83. In the above exercise it is clear that

Annexing a cipher to an integer multiplies the integer by 10; and

Removing the decimal point one place to the right multiplies the number by 10.

ORAL EXERCISE

1. Read aloud the following numbers multiplied by 10; by 100; by 1000: 17; 285; 3712; \$413.45; \$1926.75; 4165.95.

2. Read each of the following numbers multiplied by 20; by 400; by 600; by 5000: 16; 19; 37; 49¢; 64¢; \$122; \$2.60.

3. By inspection find the cost of:

a. 750 lb. coffee at 30¢.
b. 500 lb. cocoa at 40¢.
c. 650 lb. chocolate at 50¢.
d. 300 bbl. lump salt at \$3.
e. 200 bbl. oatmeal at \$4.50.
f. 170 bx. wool soap at \$3.
g. 650 yd. silk at \$1.20.
h. 140 bu. beans at \$3.50.
i. 500 bu. beans at \$2.50.
j. 240 gro. jet buttons at \$3.
k. 500 doz. half hose at \$5.50.
l. 800 yd. taffeta silk at \$1.20.

84. When the multiplier is a number a little less than 10, 100, or 1000, the multiplication may be shortened as shown in the following examples.

85. Examples. 1. Multiply 123 by 99.

SOLUTION. *99 is 100 diminished by 1; hence, multiply 123 by 100 and then by 1 and subtract the results.* The product is 12,177. Check by retracing the steps in the process.

$$\begin{array}{r} 12300 \\ 123 \\ \hline 12177 \end{array}$$

2. Multiply 145 by 96.

SOLUTION. *96 is 100 diminished by 4; hence, multiply 145 by 100 and then by 4 and subtract the results.* The product is 13,920. Check by retracing the steps in the process.

$$\begin{array}{r} 14500 \\ 580 \\ \hline 13920 \end{array}$$

WRITTEN EXERCISE

1. Find the total cost of:

5260 bu. rye at 99¢.
1521 bu. rye at 92¢.
1640 bu. wheat at 98¢.
2994 bu. millet at 97¢.
1112 bu. wheat at 97¢.
2160 bu. millet at 96¢.
834 bu. millet at 95¢.
246 bu. wheat at 92¢.
998 bu. millet at $1.04.
998 bbl. apples at $1.05.
893 bkt. peaches at $1.05.
993 bu. clover seed at $3.35.

MULTIPLICATION BY 11 AND MULTIPLES OF 11

86. Example. Multiply 237 by 11.

SOLUTION. To multiply by 11 is to multiply by $10 + 1$. Hence, annex a cipher to 237 and add 237; or, better still, add the digits as follows: **7**; $3 + 7 = 10$; $3 + 2 + 1$ (carried) $= 6$; bring down **2**; therefore, the result is 2607.

ORAL EXERCISE

1. Multiply each of the following by 11:

14; 26; 45; 19; 16; 34; 36; 49; 64; 125; 112; 115; 128; 192; 175; 116; 142; $4.95; $9.62; $4.41; $6.82; $5.21; $3.65; $4.31; $21.12; $14.21; $18.32; $3.26.

2. Find the cost of 11 yd. at 27¢; at 91¢; at 86¢; at 95¢; at $1.49; at $1.23; at $2.17; at $2.31; at $2.40; at $2.50; at $2.75; at $4.35; at $3.15; at $3.10; at $8.13.

87. Examples. 1. Multiply 46 by 22.

SOLUTION. 22 is 11 times 2. Multiply 46 by 11 and by 2, as follows: $2 \times 6 = 12$; write 2 and carry 1. $4 + 6 = 10$; $2 \times 10 + 1$ (carried) $= 21$; write 1 and carry 2. $2 \times 4 + 2$ (carried) $= 10$; write 10. The result is 1012.

$$\begin{array}{r} 46 \\ 22 \\ \hline 1012 \end{array}$$

2. Find the cost of 122 bu. of potatoes at 66¢ per bu.

SOLUTION. $6 \times 2 = 12$; write 2 and carry 1. $2 + 2 = 4$; $6 \times 4 + 1$ (carried) $= 25$; write 5 and carry 2. $1 + 2 = 3$; $6 \times 3 + 2$ (carried) $= 20$; write 0 and carry 2. $6 \times 1 + 2$ (carried) $= 8$. Write 8. The result is $80.52.

$$\begin{array}{r} 122 \\ .66 \\ \hline 80.52 \end{array}$$

WRITTEN EXERCISE

In the following problems make all the extensions mentally.

1. Find the total cost of:

11 lb. coffee at 42¢.
14 doz. eggs at 21¢.
64 lb. cheese at 22¢.
33 bu. carrots at 56¢.
11 bu. potatoes at 85¢.
88 bu. wheat at 88¢.
115 bu. rye at 99¢.
215 bu. peas at 77¢.
344 bu. oats at 44¢.
300 bu. grain at 85¢.
115 bu. barley at 88¢.
400 bbl. apples at $3.25.

2. Find the total cost of:

77 bu. peaches at $1.85.
151 bu. corn at 66¢.
265 bu. onions at 80¢.
135 bu. apples at 82¢.
241 bu. turnips at 44¢.
112 bu. tomatoes at 55¢.
820 bu. rye at 88¢.
327 bu. oats at 33¢.
314 bu. peas at 66¢.
110 bu. pears at $1.66.
880 bu. barley at $1.17.
100 bu. quinces at $1.60.

A careful computer checks his work *at every step*. The student who forms the habit of doing this in all his computations will soon find himself in no need of printed answers to problems involving only numerical calculation.

Checks for multiplication have already been mentioned. To guard against large errors, it is also important to form a rough estimate of an answer before beginning the solution. Thus, in finding the cost of 211 yd. of lining at 32¢, at once see that the result will be a little more than $63.00 (210 times 30¢); this will do away with such absurd results as $6752, $675.20, or $6.75.

3. Copy and find the amount of the following bill:

Boston, Mass., July 21, 19

Mrs. GEORGE W. MUNSON
168 Huntington Ave., City

Bought of S. S. PIERCE COMPANY

Terms Cash

15	cs.	Horse-radish	$0.66			
25	lb.	Huyler's Cocoa	.44			
31	gal.	N. O. Molasses	.63			
55	lb.	Japan Tea	.48			
212	"	Raisins	.11			

Multiplication by 25, 50, and 75

88. Annexing two ciphers to an integer multiplies it by 100. Removing the decimal point two places to the right multiplies the decimal by 100.

89. Example. Multiply 76 by 100.

Solution. $76 \times 100 = 7600$. (Annexing the two ciphers gives the required result without the necessity for a written solution.)

90. Example. Multiply 148 by 25.

Solution. $148 \times 100 = 14{,}800$. $14{,}800 \div 4 = 3700$.

Hence, to multiply an integer by 25, annex two ciphers to the multiplicand and then divide by 4.

91. Example. Multiply 278 by 50.

Solution. $278 \times 100 = 27{,}800$. $27{,}800 \div 2 = 13{,}900$.

Hence, to multiply an integer by 50, annex two ciphers to the multiplicand and then divide by 2.

92. Example. Multiply 48 by 75.

Solution. $48 \times 100 = 4800$. $4800 \div 4 = 1200$; $3 \times 1200 = 3600$.

Hence, to multiply an integer by 75, annex two ciphers to the multiplicand, divide that product by 4, and then multiply by 3.

ORAL EXERCISE

State the product of:

1. 36×25.
2. 27×50.
3. 28×75.
4. 97×25.
5. 248×25.
6. 126×50.
7. 164×25.
8. 48×50.
9. 52×75.
10. 67×50.
11. 89×50.
12. 186×50.
13. 146×25.
14. 204×50.
15. 64×75.
16. 63×25.
17. 69×25.
18. 56×75.
19. 240×75.
20. 184×75.
21. 144×75.

WRITTEN EXERCISE

In the following problems make all the extensions mentally.

1. Find the total cost of:

42 lb. cocoa at 40¢.
45 lb. cocoa at 50¢.
50 lb. coffee at 28¢.
25 lb. raisins at 15¢.
23 lb. tea at 40¢.
27 bx. salt at 50¢.
23 lb. coffee at 25¢.
21 lb. candy at 75¢.
33 lb. chocolate at 50¢.
85 lb. Oolong tea at 45¢.

2. Find the total cost of:

36 yd. wash silk at 25¢.
25 doz. whalebones at 92¢.
97 yd. cloth at 75¢.
25 gro. buttons at 35¢.
29 yd. gunner's duck at 19¢.
87 yd. flannel at 50¢.
21 yd. cottonade at 18¢.
25 yd. denim at 19¢.
17 yd. dress goods at 50¢.
23 yd. cheviot at 21¢.

Multiplication by an Even Number of Hundreds

93. Example. Multiply 468 by 300.

Solution. $468 \times 100 = 46{,}800$; $46{,}800 \times 3 = 140{,}400$.

Hence, to multiply an integer by an even number of hundreds, annex two ciphers to the multiplicand and then multiply by the significant figure in the multiplier.

94. The value of many short methods is that they enable one to write results quickly without performing the mechanical operations.

95. Many short methods in multiplication are not practical because they require one to remember so many things, or they apply to so few numbers that it is impossible for an ordinary person to remember them. The short methods given in this text are practical.

ORAL EXERCISE

Find the product of:

1. 234×200.	7. 753×300.	13. 964×200.
2. 175×600.	8. 845×400.	14. 554×300.
3. 335×800.	9. 453×200.	15. 181×700.
4. 216×900.	10. 256×400.	16. 312×800.
5. 648×100.	11. 145×800.	17. 237×600.
6. 452×500.	12. 333×700.	18. 122×900.

Multiplication by Numbers from 101 to 109 Inclusive

96. Examples. 1. Find the cost of 64 bu. of wheat at \$1.02.

Solution. $2 \times 64 = 128$; write 28 and carry 1. $1 \times 64 + 1 = 65$; write 65. The result is \$65.28.

Some persons may prefer to work this problem as follows: 64 bu. at \$1 = \$64; 64 bu. at 2¢ = \$1.28; \$64 + \$1.28 = \$65.28.

$$\begin{array}{r} 64 \\ 1.02 \\ \hline 65.28 \end{array}$$

2. Find the cost of 251 bu. of barley at \$1.04.

Solution. $4 \times 51 = 204$; write 04 in the product and carry 2. $4 \times 2 + 2$ (carried) $+ 1$ (the right-hand figure of the multiplicand) $= 11$; write 1 and carry 1. $1 \times 25 + 1$ (carried) $= 26$; write 26. The result is \$261.04.

$$\begin{array}{r} 251 \\ 1.04 \\ \hline 261.04 \end{array}$$

97. Similarly multiply by such numbers as 201, 302, and 405.

98. Example. Find the cost of 124 bu. of beans at \$2.05.

Solution. $5 \times 24 = 120$. Write 20 and carry 1. $5 \times 1 + 1$ (carried) $+ 2 \times 4$ (the right-hand figure of the multiplicand) $= 14$; write 4 and carry 1. $2 \times 12 + 1$ (carried) $= 25$; write 25. The result is \$254.20.

$$\begin{array}{r} 124 \\ 2.05 \\ \hline 254.20 \end{array}$$

Some persons may prefer the following solution: 124 bu. at \$2 = \$248; 124 bu. at 5¢ = \$6.20; \$248 + \$6.20 = \$254.20. The student should try to exercise his own ingenuity in all this work.

WRITTEN EXERCISE

Find the value of:

1. 215 T. coal at $6.05.
2. 224 bu. rye at $1.02.
3. 215 bu. wheat at $1.02.
4. 318 bu. barley at $1.05.
5. 124 bbl. apples at $2.05.
6. 116 bbl. onions at $1.08.
7. 232 bbl. potatoes at $2.05.
8. 302 bu. peas at 74¢.
9. 104 bu. corn at 89¢.
10. 103 bu. beets at 85¢.
11. 205 bu. turnips at 54¢.
12. 215 bu. pears at $1.05.
13. 411 bu. plums at $1.08.
14. 206 bu. parsnips at 93¢.

Miscellaneous Short Methods

99. When one part of the multiplier is contained in another part a whole number of times, the multiplication may be shortened as shown in the following examples.

100. Example. Multiply 412 by 357.

SOLUTION. 35 is 5 times 7. $7 \times 412 = 2884$, which write as the first partial product. $5 \times 2884 = 14{,}420$, which write as the second partial product.

CHECK. Interchange the multiplier and multiplicand and remultiply. $4 \times 357 = 1428$; $3 \times 1428 = 4284$. Add. Since the results by both multiplications agree, the work is probably correct.

412	357
357	412
2884	1428
14420	4284
1470.84	147084

101. Example. Multiply 214 by 756.

SOLUTION. 56 is 8 times 7. $7 \times 214 = 1498$, which we write as the first partial product. $8 \times 1498 = 11{,}984$, which we write as the second partial product. The sum (161,784) of these partial products is the entire product.

Check as in problem 1. (See also pages 89 and 90.)

214
756
1498
11984
161784

WRITTEN EXERCISE

Find the product of:

1. 319×248.
2. 927×279.
3. 728×287.
4. 848×369.

102. The above short methods are practical in a limited number of problems.

WRITTEN REVIEW EXERCISE

1. Use 6 as a multiplier for each column. Check. (See page 53.)

a.	*b.*	*c.*	*d.*	*e.*
56	74	39	126	215
48	63	58	232	175
73	52	82	311	243
49	65	72	135	223
45	55	85	144	183
66	47	19	225	253
77	88	92	245	127

2. Use 8 as a multiplier for each column. Check.

3. I bought 15 A. of land at $275 per acre and laid it out in 100 city lots. After expending $6750 for grading and taxes, $257 for ornamental trees, and $250 for advertising, I sold 15 lots at $625 each, 35 lots at $415 each, and exchanged the remainder for a farm of 120 A., which I immediately sold at $195 per acre. Did I gain or lose, and how much?

4. Copy and find the amount of the following bill:

Rochester, N.Y., July 26, 19

Mr. F. C. GORHAM
120 Spring Street, City

Bought of C. E. Ferguson & Son

Terms 30 da.

37 bu.	Oats	$0.40
50 "	Corn	.67
76 "	Wheat	1.02
75 "	Rye	1.04
95 "	Beans	4.00
16 "	Clover Seed	3.50
26 "	Millet	.99

WRITTEN REVIEW

Copy these examples; add the checks in the *Checks in Detail* column and enter the totals in the *Total Checks* column; find the new balance, the total old balance, the total checks, the total deposits, and check the work.

1.

Name	Balance	Checks in Detail	Total Checks	Deposits	Balance
		$180.55			
A	$313.25	211.15	?	$278.40	
		165.43			
		208.19			
B	285.67	100.55		327.44	
		145.97			
C	186.53	200.12		198.45	
		45.67			
		118.95			
D	276.65	205.18		210.50	
E	———	612.40		918.75	
		64.25			
F	347.85	103.86		———	
		6.84			

2.

Name	Balance	Checks in Detail	Total Checks	Deposits	Balance
A	$195.63	$214.70		$174.25	
		71.20			
		8.50			
B	98.40	102.45		115.68	
		74.65			
		123.52			
C	153.30	10.55		89.48	
		75.20			
		55.34			
D	386.54	7.35		275.40	
		172.38			

A WRITTEN REVIEW TEST

Write the following problems from dictation, and complete the work. Time, approximately, forty minutes, including the dictation. Mental extensions only.

1. Write in one column, and find the total value:

78 yd. at 11¢	55 yd. at 55¢
69 yd. at 25¢	91 yd. at 50¢
60 yd. at 85¢	89 yd. at 99¢
45 yd. at 98¢	75 yd. at 90¢
37 yd. at 97¢	76 yd. at 70¢
112 yd. at 99¢	125 yd. at 98¢

2. Write in one column, and find the total value:

76 yd. at $1.10	82 yd. at $1.05
55 yd. at $1.06	65 yd. at $1.20
108 yd. at $1.11	130 yd. at $1.09
56 yd. at $1.25	83 yd. at $1.50
88 yd. at $1.04	97 yd. at $1.03
67 yd. at $1.02	137 yd. at $1.01

3. Write in one column; use 11 as the multiplier, and check the results:

49, 16, 34, 78, 57, 73, 85, 94, 59, 64, 56, 81.

4. Write in one column; use 6 as the multiplier, and check the results:

125, 212, 350, 175, 162, 224, 319, 452, 133, 145, 121, 142.

5. Write in one column; use 8 as the multiplier, and check the results:

45, 75, 62, 29, 76, 61, 19, 34, 85, 92, 27, 77.

6. Write in one column; square each number, and total the products:

25, 55, 15, 45, 75, 35, 85, 65, 95.

7. The text, page 63, problem 3, in the Written Review Exercise.

CHAPTER VII

DIVISION

ORAL EXERCISE

1. What is the product of 12 times 15? How many times is 15 contained in 180? What is $\frac{1}{12}$ of 180?

2. How much is 11 times \$17? How many times is \$17 contained in \$187? What is $\frac{1}{11}$ of \$187?

3. What is the product of 9 times 12 ft.? How many times is 12 ft. contained in 216 ft.? What is $\frac{1}{15}$ of 225 ft.?

4. When one factor and the product are given, how is the other factor found?

103. The process of finding either factor when the product and the other factor are given is called **division.**

104. The known product is called the **dividend**; the known factor, the **divisor**; the unknown factor, when found, the **quotient.**

105. The part of the dividend remaining when the division is not exact is called the **remainder.**

While definitions such as the above should not be memorized, the ideas which they express should be thoroughly understood.

106. Since 6 times 7 ft. = 42 ft., 42 ft. ÷ 7 ft. = 6, and 42 ft. ÷ 6 = 7 ft. It is therefore clear that

1. *If the dividend and divisor are concrete numbers, the quotient is an abstract number;* and

2. *If the dividend is concrete and the divisor abstract, the quotient is a concrete number like the dividend.*

In §106 it will be seen that there are **two kinds of division**: 42 ft. ÷ 7 ft. = 6 is sometimes called *measuring*, because 42 ft. is measured by 7 ft.; 42 ft. ÷ 6 = 7 ft. is sometimes called *partition*, because 42 ft. is divided into 6 equal parts.

ORAL EXERCISE

1. Divide by 2: 18, 32, 78, 450, 642, 964, 893.
2. Divide by 3: 27, 57, 72, 423, 642, 963, 845.
3. Divide by 4: 64, 88, 92, 488, 192, 396, 728.
4. Divide by 5: 65, 85, 95, 135, 275, 495, 725.
5. Divide by 6: 84, 96, 54, 246, 546, 672, 846, 636.
6. Divide by 7: 63, 84, 91, 217, 497, 714, 791, 921.
7. Divide by 8: 72, 56, 88, 248, 640, 128, 144, 152.
8. Divide by 4: 56, 96, 77, 241, 168, 128, 920, 848.
9. Divide by 6: 78, 96, 56, 272, 848, 190, 725, 966.
10. Divide by 9: 98, 72, 49, 279, 819, 720, 189, 918.

ORAL EXERCISE

1. 16 ft. ÷ 2 = ? 24 ft. ÷ 8 ft. = ?
2. $25 ÷ 5 = ? $29.75 ÷ 5 = ? $129.78 ÷ 9 = ? 13.40 ÷ 4 = ?
3. 126 yd. ÷ 3 yd. = ? $125 ÷ 25 = ? $6.25 ÷ $1.25 = ?
4. If 9 T. of coal cost $49.50, what is the cost per ton?

SOLUTION. $49.50 ÷ 9 = $5; subtracting 9 times $5, the result is $4.50 undivided; $4.50 ÷ 9 = $0.50. Therefore the quotient is $5.50.

$$9\overline{)\$49.50}\quad \$5.50$$

5. At $1.75 a yard, how many yards can be bought for $35?

SOLUTION. The divisor contains cents and it is therefore better to first change both dividend and divisor to cents. It is found that $35 would buy 20 times as many yards as $1.75, or 20 yd.

$$175\overline{)3500}\quad 20$$

6. If 5 T. of coal cost $31.25, what is the cost per ton?
7. At $2.50 per yard how many yards can be bought for $550?

ORAL EXERCISE

1. How many weeks in 98 da.?
2. What is $\frac{1}{25}$ of 2250 bbl. of apples? $\frac{1}{15}$? $\frac{1}{5}$? $\frac{1}{45}$?
3. The quotient is 8 and the dividend 128. What is the divisor?
4. How many times can 18 be subtracted from 75, and what will remain?

5. At 15¢ per pound, how many pounds of beef can be bought for $6.30?

6. The quotient is 5, the divisor 23, and the remainder 2. What is the dividend?

7. If 5 men earn $17.50 a day, how much can 8 men earn in 2 da. at the same rate?

8. What is the nearest number to 150 that can be divided by 9 without a remainder?

9. If there are 960 sheets in 40 qr. of paper, how many sheets in 5 qr.? in 11 qr.?

10. If 6 bbl. of apples are worth $21, what are 24 bbl. worth at the same rate? 36 bbl.?

11. If 17 bbl. of flour cost $85, what will 25 bbl. cost at the same rate? 32 bbl.? 48 bbl.? 34 bbl.?

12. If 8 be added to a certain number, 7 can be subtracted from that number 7 times. What is the number?

13. If 20 yd. of cloth cost $60, for how much per yard must it be sold to gain $25? to gain $15?

14. A grocer sold 250 oranges at 5¢ each and gained $5. How much did he pay a dozen for the oranges?

15. A grocer pays $3 for 20 doz. of eggs. At what price per dozen must he sell them in order to gain $1.50?

16. At $2.50 per yard, how many yards of cloth can be bought for $75? for $150? for $2500? for $750?

17. How many days' labor at $3.50 per day will pay for 2 T. of coal at $7 a ton and 5 lb. of tea at 70¢ per pound?

18. A clothier pays $96 for a dozen overcoats. At how much apiece must he retail them to gain $48 on the lot?

19. A man exchanged 1140 bu. of wheat at $1 per bushel for flour at $6 per barrel. How many barrels did he receive?

20. It was found that after 15 had been subtracted 5 times from a certain number the remainder was 4. What was the number?

21. A man contracts a debt of $175 which he promises to pay in weekly installments of $3.50 each. After paying $35, how many more payments has he to make?

107. Examples. 1. Divide 4285 by 126.

Complete Operation		Required Work
$34\frac{1}{126}$		$34\frac{1}{126}$
126)4285		126)4285
378	= 3 times 126	378
505	undivided	505
504	= 4 times 126	504
1	undivided	1

Check. $34 \times 126 + 1 = 4285$

The remainder cannot always be written as a part of the quotient. Thus in the problem, "At \$7 per head how many sheep can be bought for \$37," we cannot say, "$5\frac{2}{7}$ sheep," but "5 sheep and \$2 remaining."

2. A farmer received \$283.25 in payment for 275 bu. of wheat. How much was received per bushel for the wheat?

Solution. \$283.75 ÷ 275 = \$1 and \$8.25 undivided. \$8.25 ÷ 275 = \$0.03. \$1.03 per bushel was therefore received for the wheat.

Check. 275 times \$1.03 = \$283.25.

\$1.03
275)\$283.25
275
8 25
8 25

108. Work in division may be abridged by omitting the partial products and writing only the partial dividends.

109. Example. Divide \$614.80 by 232.

Solution. Omit writing the products; subtract mentally and write the remainder only: 2 × 232 = 464; 464 subtracted from 614 equals 150; omit the writing of the 464. Proceed as follows: 2 times 2 plus 0 = 4; 2 times 3 plus 5 = 11. 2 times 2 + 1 = 5, and 5 plus 1 = 6. Bring down 8. 6 times 2 plus 6 = 18; 6 times 3 plus 1 = 19, and 19 + 1 = 20; 6 times 2 plus 2 = 14, and 14 plus 1 = 15. Bring down 0 and proceed as before.

\$2.65
232)\$614.80
150 8
11 60
0 00

WRITTEN EXERCISE

1. Find the cost of 8800 lb. of oats at 45¢ per bushel of 32 lb.

2. How many automobiles, at \$650 each, can be purchased for \$4,225,000?

3. By what number must 8656 be multiplied to make the product 8,223,200?

4. If 120 bbl. of flour cost $660, what will 829 bbl. cost at the same rate?

5. The product of two numbers is 1,928,205. If one of them is 621, what is the other?

6. If 380 T. of coal can be bought for $3040, how many tons can be bought for $3600?

7. How many cords of 128 cu. ft. in a pile of wood containing 235,820 cu. ft.? What is it worth at $4.50 per cord?

8. A speculator sold a quantity of apples that cost $2500 for $4750. If his gain per barrel was $1.12½, how many barrels did he buy?

9. A man received a legacy of $11,375 which he invested in railroad stock. He paid a broker $125 to buy stock at $112.50 per share. How many shares were bought?

10. A dealer bought 250 T. of coal by the long ton of 2240 lb. at $6.50 per ton. He retailed the same at $8.25 per short ton of 2000 lb. What was the total gain?

11. In a recent year there were produced in the United States 730,627,000 bu. of wheat on 45,814,000 A. What was the yield per A.? What was the total yield worth at 90¢ per bu.?

12. Copy and complete the following table of corn statistics. Check the work. (The total yield multiplied by the price per bushel should equal the total valuation.)

PRINCIPAL CORN-GROWING STATES IN A RECENT YEAR

STATE	YIELD IN BUSHELS	FARM PRICE PER BUSHEL	FARM VALUATION
Illinois	426 320 000	62¢	264 318 400
Iowa	?	62¢	267 853 020
Nebraska	?	62¢	113 221 920
Missouri	?	62¢	151 220 480
Indiana	199 364 000	62¢	?
Kansas	174 225 000	62¢	?
Total	?	62¢	?

13–15. Make and solve three self-checking problems in division

SHORT METHODS IN DIVISION

Powers and Multiples of 10

ORAL EXERCISE

1. How many times is 10 contained in 50? 100 in 800? 1000 in 9000?

2. Cutting off a cipher in 30 divides it by what number?

3. Cutting off two ciphers in 800 divides it by what number?

4. Cutting off three ciphers in 11,000 divides it by what number?

5. Read aloud, supplying the missing words:

a. The number of 10's in any number may be found by cutting off the units' figure; the number of 100's by cutting off the —— and —— figures; the number of —— by cutting off the hundreds' and tens' and units' figures.

b. In 4561 there are 456 tens and 1 unit, or $456\frac{1}{10}$ tens; 45 —— and 61 units, or $45\frac{61}{100}$ hundreds; and —— thousands and 561 units, or $4\frac{561}{1000}$ thousands.

6. How many times is \$0.10 contained in \$1? \$0.01 in \$1? \$0.001 in \$1?

7. What is $\frac{1}{10}$ of \$1? $\frac{1}{100}$ of \$1? $\frac{1}{1000}$ of \$1?

8. Read aloud, supplying the missing words: \$0.60 is —— of \$6; \$0.06 is —— of \$6; \$0.006 is —— of \$6.

9. Formulate a short method for dividing United States money by 10; by 100; by 1000.

10. By inspection find the quotient of:

a. $736 \div 10$.	*e.* \$271 ÷ 100.	*i.* 2140 lb. ÷ 100.
b. $1531 \div 100$.	*f.* \$519.50 ÷ 10.	*j.* 3145 lb. ÷ 100.
c. $16351 \div 1000$.	*g.* \$84.50 ÷ 100.	*k.* 3416 ft. ÷ 1000.
d. $311219 \div 10000$.	*h.* \$2150 ÷ 1000.	*l.* 1279 posts ÷ 100.

11. Read aloud, supplying the missing amounts:

a. $6400 \div 1600 =$ ——; $640 \div 10 =$ ——.

b. $27000 \div 9000 =$ ——; $2700 \div 900 =$ ——; $270 \div 90 =$ ——; $27 \div 9 =$ ——.

c. $18801 \div 90 =$ —— $\div 9$; $214200 \div 700 = 2142 \div$ ——.

12. How is the quotient affected by like changes in both the dividend and divisor?

13. Divide 1323 by 400.

SOLUTION. Cut off the two ciphers in the divisor and two digits in the right of the dividend, thus dividing both dividend and divisor by 100. 4 is contained in 13 three times with a remainder 1 *hundred*. Adding to this remainder the 23 units remaining in the dividend after dividing by 100, the true remainder is 123, which write in fractional form.

$$4|00\overline{)}13|23 = 3\tfrac{123}{400}$$
$$12$$
$$\overline{123}$$

14. Read aloud, supplying the missing amounts: $1611 \div 400 = \text{——}$; $2847 \div 700 = \text{——}$; $1531 \div 300 = \text{——}$; $16139 \div 4000 = \text{——}$.

15. Formulate a rule for dividing a number by any multiple of ten.

16. State the quotient of:

a. $1231 \div 30$.	*f*. $96131 \div 400$.	*k*. $63571 \div 3000$.
b. $9647 \div 40$.	*g*. $84199 \div 700$.	*l*. $16657 \div 4000$.
c. $6551 \div 50$.	*h*. $64137 \div 800$.	*m*. $36119 \div 6000$.
d. $4273 \div 70$.	*i*. $45117 \div 900$.	*n*. $18177 \div 9000$.
e. $8197 \div 90$.	*j*. $25121 \div 500$.	*o*. $42113 \div 7000$.

ORAL REVIEW EXERCISE

The diagram on the opposite page is a portion of the New York Central time-table giving the distances between many of the stations from New York City to Suspension Bridge, and the time taken by two different trains to travel this route.

1. How many miles between New York City and Poughkeepsie? between Poughkeepsie and Utica? between Utica and Syracuse? between Syracuse and Rochester? between Rochester and Buffalo? between Buffalo and Niagara Falls?

2. What is the distance between New York City and Syracuse? between Poughkeepsie and Niagara Falls? between Rochester and Suspension Bridge?

3. How many miles between Ludlow and each station below it? between Poughkeepsie and each station below it? between Tarrytown and each station below it?

4. How many miles between Montrose and each station below it? between Oscawana and each station below it?

5. At 2¢ per mile, what is the fare from New York to Niagara Falls? from Poughkeepsie to Syracuse? from Buffalo to Utica? from Troy to Yonkers?

6. At 2¢ per mile, what is the fare from Rochester to Syracuse? from Rensselaer to Suspension Bridge? from Albany to Niagara Falls? from Syracuse to Buffalo? to Albany?

7. How long does it take train No. 93 to travel the first 30 mi. toward Poughkeepsie? the first 74 mi. toward Albany?

8. How long is train No. 93 in making the run from Fishkill Landing to Camelot? This is approximately how many miles an hour?

9. How long does it take train No. 73 to make the run from Utica to Syracuse? How long does it take train No. 73 to make the run from Fishkill Landing to Chelsea? This is approximately how many miles an hour?

10. Add each number in the column marked "Miles" to the one immediately below it.

Miles	NORTH AND WEST BOUND	Midnight Express	Poughkeepsie Local
		73	93
	New York		
0	**Grand Cent. Sta.** .....Lv.	12 10	6 01
4	**125th St. Sta.** ..... "	12 23	6 13
5	**138th St. Sta.** ..... "		6.15
7	High Bridge ..... "		6.21
8	Morris Heights..... "		6.25
10	Kings Bridge..... "		6.29
11	Spuyten Duyvil..... "		6.33
13	Riverdale ..... "	See Note A	.....
14	Mt. St. Vincent..... "		.....
15	Ludlow ..... "		.....
16	Yonkers..... "	12.46	6.43
18	Glenwood ..... "	.....	6.46
20	Hastings-on-Hudson "	.....	6.52
21	Dobbs' Ferry..... "	.....	6.59
22	Ardsley-on-Hudson "	.....	7.01
23	Irvington..... "	.....	7.05
26	Tarrytown ..... "	1.09	7.12
30	Scarborough..... "	.....	7.19
31	Ossining..... "	1.25	7.25
35	Croton-on-Hudson .. "	.....	7.31
37	Oscawana..... "	.....	7.34
38	Crugers..... "	.....	7.37
39	Montrose ..... "	.....	7.41
42	Peekskill ..... "	1.47	7.49
47	Highlands..... "	.....	7.59
50	Garrison ..... "	x	8.06
53	Cold Spring..... "	x	8.12
56	Storm King..... "	.....	8.16
58	Dutchess Junc..... "	.....	8.21
59	Fishkill Landing..... "	2.24	8.27
63	Chelsea..... "	2.31	8.34
65	New Hamburg..... "	.....	8.40
69	Camelot ..... "	.....	8.46
74	**Poughkeepsie**.....Ar.	2.53	8 55
74	**Poughkeepsie**.....Lv.	3.05	.....
80	Hyde Park..... "	.....	.....
84	Staatsburgh..... "	.....	.....
89	Rhinecliff (Rh'b'k).. "	.....	.....
95	Barrytown..... "	.....	.....
99	Tivoli..... "	.....	.....
105	Germantown ..... "	.....	.....
109	Linlithgo ..... "	.....	.....
111	Greendale ..... "	.....	.....
115	Hudson..... "	4.47	.....
119	Stockport ..... "	.....	.....
122	Newton Hook..... "	.....	.....
125	Stuyvesant ..... "	.....	.....
131	Schodack Landing.. "	.....	.....
135	Castleton ..... "	.....	.....
142	Rensselaer ..... "	.....	.....
143	**Albany**.....Ar.	5.50	.....
149	**Troy** ..... "	6 50	.....
238	**Utica**.....Ar.	8 40	.....
291	**Syracuse**..... "	9.55	.....
371	**Rochester**..... "	11.38	.....
440	**Buffalo**..... "	1 15	.....
463	**Niagara Falls**.....Ar.	2 13	.....
464	**Suspension Bridge**.... "	2 20	.....

Thus, 9, 12, 16, 24, 34, 45, 58, etc. In adding 89 and 95 think of 179 and 5, or 184; in adding 143 and 149 think first of 243 and 49 and then of 283 and 9, or 292.

11. Multiply each number in the column marked "Miles" by 5; by 8; by 3; by 7; by 6; by 4; by 9.

The numbers in the portion of the time-table illustrated may be used for such other exercises as may seem necessary at this point. Students should be impressed with the importance of being able to add, subtract, multiply, and divide numbers in any relative position.

12. Five parts of 120 are 15, 18, 32, 10, and 20. Find the sixth part, and multiply it by 15.

13. From a flock of 170 sheep I sold at different times 12, 18, 32, and 9. How many sheep remained?

14. Multiply by 11 each of the following numbers: 21, 32, 43, 54, 65, 76, 87, 98, 61, 28, 37, 14, 21, 62.

15. At 22¢ per yard, what will 18 yd. cost? 21 yd.? 36 yd.? 56 yd.? 29 yd.? 73 yd.? 94 yd.? 72 yd.?

16. Multiply each number in problem 15 by 33; by 44.

WRITTEN REVIEW EXERCISE

1. Find the total cost of the articles in problem 3 of the oral exercise, page 56. Find the total of the products in the oral exercise, page 60.

2. A mechanic earns $125 per month and his monthly expenses average $72. If he saves the remainder, how long will it take him to save $4352?

3. I spent $24,800 for apples at $2.50 per barrel. The loss from decay was equal to 74 bbl. What was my gain, if the remainder of the apples sold for $3.75 per barrel, and my expenses for storage were $675.80?

4. During a certain week a contractor employed help as follows: 34 hands, 8 hr. per day, for 5 da., at 15¢ per hour; 16 hands, 9 hr. per day, for 6 da., at 25¢ per hour; 29 hands, 10 hr. per day, for 6 da., at 18¢ per hour. Find the amount due.

5. In a recent year there were produced on 37,917,000 A. in the United States 1,418,337,000 bu. oats, valued on the farm at 31.3¢ per bushel. What was the average yield per acre? What was the value of the year's crop?

6. Without copying find (*a*) the total number of railway employees in the United States in 1910 and (*b*) the total number per one hundred miles of line in the same year.

RAILWAY EMPLOYEES IN THE UNITED STATES

CLASS	1910			1911		
	TOTAL NUMBER	NUMBER PER 100 MI.	AVERAGE DAILY WAGES	TOTAL NUMBER	NUMBER PER 100 MI.	AVERAGE DAILY WAGES
General officers	5,476	2	$13.27	5,628	2	$12.99
Other officers	9,392	4	6.22	10,196	4	6.27
General office clerks	76,329	32	2.40	76,513	31	2.49
Station agents	37,379	16	2.12	38,277	16	2.17
Other station men	153,104	64	1.84	153,117	62	1.89
Engineers	64,691	27	4.55	63,390	26	4.79
Firemen	68,321	28	2.74	66,376	27	2.94
Conductors	48,682	20	3.91	48,200	20	4.16
Other trainmen	136,938	57	2.69	133,221	54	2.88
Machinists	55,193	23	3.08	55,207	22	3.14
Carpenters	68,085	28	2.51	65,989	27	2.54
Other shopmen	225,196	94	2.18	226,785	92	2.24
Section foremen	44,207	18	1.99	44,466	18	2.07
Other trackmen	378,955	157	1.47	363,028	147	1.50
All other employees	229,806	95	2.01	227,779	93	2.08

7. Without copying find (*a*) the total number of railway employees in the United States in 1911 and (*b*) the total number per one hundred miles of line in the same year.

8. Find the total salaries paid to railway employees in 1910; in 1911.

9. Find the average daily wages paid to railway employees in 1910; in 1911.

10. In a recent year four leading railway systems had outstanding bonds as follows:

a. $761,963,000. *c.* $1,096,773,410.

b. $576,300,000. *d.* $428,649,000.

Find the average amount of the bonds outstanding.

WRITTEN REVIEW

In these problems divide across, and then add the dividend column and the quotient column. Check: divide the total of the dividend column by the divisor, and this quotient should equal the sum of the individual quotients. Time, approximately, fifteen minutes.

1.	2.	3.
$36 \div 4 = ?$	$45 \div 5 = ?$	$66 \div 6 = ?$
$48 \div 4 = ?$	$75 \div 5 = ?$	$72 \div 6 = ?$
$56 \div 4 = ?$	$95 \div 5 = ?$	$84 \div 6 = ?$
$24 \div 4 = ?$	$65 \div 5 = ?$	$78 \div 6 = ?$
$84 \div 4 = ?$	$35 \div 5 = ?$	$48 \div 6 = ?$
$44 \div 4 = ?$	$55 \div 5 = ?$	$36 \div 6 = ?$
$64 \div 4 = ?$	$15 \div 5 = ?$	$54 \div 6 = ?$
$76 \div 4 = ?$	$40 \div 5 = ?$	$42 \div 6 = ?$
$\overline{?} \div \overline{4} = \overline{?}$	$\overline{?} \div \overline{5} = \overline{?}$	$\overline{?} \div \overline{6} = \overline{?}$

4.	5.	6.
$98 \div 7 = ?$	$88 \div 2 = ?$	$48 \div 3 = ?$
$84 \div 7 = ?$	$76 \div 2 = ?$	$54 \div 3 = ?$
$63 \div 7 = ?$	$58 \div 2 = ?$	$69 \div 3 = ?$
$49 \div 7 = ?$	$64 \div 2 = ?$	$72 \div 3 = ?$
$91 \div 7 = ?$	$82 \div 2 = ?$	$84 \div 3 = ?$
$56 \div 7 = ?$	$94 \div 2 = ?$	$93 \div 3 = ?$
$77 \div 7 = ?$	$52 \div 2 = ?$	$87 \div 3 = ?$
$42 \div 7 = ?$	$74 \div 2 = ?$	$54 \div 3 = ?$
$\overline{?} \div \overline{7} = \overline{?}$	$\overline{?} \div \overline{2} = \overline{?}$	$\overline{?} \div \overline{3} = \overline{?}$

7.	8.	9.
$126 \div 2 = ?$	$144 \div 4 = ?$	$129 \div 3 = ?$
$152 \div 2 = ?$	$124 \div 4 = ?$	$114 \div 3 = ?$
$134 \div 2 = ?$	$152 \div 4 = ?$	$108 \div 3 = ?$
$168 \div 2 = ?$	$148 \div 4 = ?$	$147 \div 3 = ?$
$184 \div 2 = ?$	$176 \div 4 = ?$	$189 \div 3 = ?$
$156 \div 2 = ?$	$184 \div 4 = ?$	$165 \div 3 = ?$
$172 \div 2 = ?$	$136 \div 4 = ?$	$195 \div 3 = ?$
$138 \div 2 = ?$	$180 \div 4 = ?$	$138 \div 3 = ?$
$\overline{?} \div \overline{2} = \overline{?}$	$\overline{?} \div \overline{4} = \overline{?}$	$\overline{?} \div \overline{3} = \overline{?}$

110. All mailable matter for transmission by the United States mails within the United States or to Cuba, Mexico, Hawaii, Porto Rico, Canada, and the Philippine Islands is divided into four classes: first-, second-, third-, and fourth-class matter.

First-class matter includes letters, postal cards, and anything sealed or otherwise closed against inspection. The rate for first-class matter is 2¢ per ounce or fraction thereof; for a postal card, 1¢; for a reply postal card, 2¢. Written or typewritten matter is of the first class, whether sealed or unsealed.

Second-class matter includes newspapers and periodicals entirely in print. When sent by publishers or news agents, the rate is 1¢ per pound or fraction thereof; when sent by others, 1¢ for each 4 oz. or fraction thereof.

Third-class matter includes books and catalogues (weighing 8 oz. or less), circulars, pamphlets, proof sheets and manuscript copy accompanying the same, and engravings. The rate is 1¢ for each 2 oz. or fraction thereof. The limit of weight is 4 lb.

All postal matter of the first, second, or third class may be *registered* at the rate of 10¢ for each package in addition to the regular rates of postage.

The rates on *special delivery letters* are 10¢ per letter in addition to the regular postage. Any matter on which a special delivery stamp is affixed is entitled to special delivery within certain limits.

Foreign rates of postage are as follows: letters, 5¢ per ounce for the first ounce, and 3¢ for each additional ounce. (Double rate is charged at delivery office for any deficiency in prepayment.) Postal cards, 2¢ each; newspapers and other printed matter, 1¢ for 2 oz.

Some foreign countries, as Germany and Great Britain, come under the letter rate of 2¢ per ounce.

All *fourth-class matter* is now included in the domestic parcel post, by a law which became effective January 1, 1913. The following are some of the principal features of this law:

The country is divided into zones, the rate of postage being dependent on the zone where the parcel is to be delivered. The zone center is the point of mailing. Regular postage stamps are used on parcel-post packages.

Parcels weighing 4 oz. or less are mailable at the rate of 1¢ for each ounce or fraction of an ounce, regardless of distance. Parcels weighing more than 4 oz. are mailable at the pound rates shown in the table, a fraction of a pound being considered as a full pound.

Books and catalogues weighing in excess of 8 oz. may be sent by parcel post.

The weight limit for zones 1 and 2 is 50 lb.; for *all* the other zones, 20 lb.

The local rate applies to parcels to be delivered at the office of mailing, or on a rural route starting from that office.

A parcel may be insured against loss to the amount of its actual value not exceeding $50.

A special delivery of a parcel will be made on the payment of an additional 10¢, at the mailing office.

In the table, the rates are complete up to 11 lb. From this point on only illustrative weights and rates are given; omissions are indicated by stars.

The local post office can furnish a parcel-post map of the United States showing the regions included in the different zones.

		50 mi.	50–150 mi.	150–300 mi.	300–600 mi.	600–1000 mi.	1000–1400 mi.	1400–1800 mi.	All over 1800 mi.
Weight	First Zone		Second Zone Rate	Third Zone Rate	Fourth Zone Rate	Fifth Zone Rate	Sixth Zone Rate	Seventh Zone Rate	Eighth Zone Rate
	Local Rate	Zone Rate							
1 lb.	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 lb.	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 lb.	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 lb.	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 lb.	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 lb.	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 lb.	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 lb.	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 lb.	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10 lb.	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 lb.	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
15 lb.	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
20 lb.	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
* * *	* * *	* * *	* * *						
30 lb.	.20	.34	.34						
* * *	* * *	* * *	* * *						
40 lb.	.25	.44	.44						
* * *	* * *	* * *	* * *						
50 lb.	.30	.54	.54						

ORAL EXERCISE

1. What is the postage on a letter weighing $\frac{1}{2}$ oz.? $4\frac{1}{2}$ oz.? $1\frac{1}{2}$ oz.? $3\frac{1}{4}$ oz.? $2\frac{1}{4}$ oz.? $4\frac{1}{4}$ oz.?

2. What will be the cost of postage on the following articles at your post office to points within the United States: an ordinary letter weighing $2\frac{3}{4}$ oz.; a registered letter weighing $1\frac{1}{2}$ oz.; a bundle of papers weighing 10 oz.?

3. Find the total cost of postage on the following to points within the United States: a special delivery letter weighing $1\frac{1}{4}$ oz.; some printers' proofs weighing 18 oz.; some separate matter for the printer weighing 12 oz.; a pamphlet weighing 6 oz.

4. Use a zone map and find the cost of mailing each of the following articles:

Article	Weight	Destination
A pair of opera glasses	2 lb. 8 oz.	Kansas City, Mo.
A pair of ladies' gloves	6 oz.	Indianapolis, Ind.
A copy of Star-Land	1 lb. 8 oz.	Macon, Ga.
A copy of Whittier's Poems	1 lb. 12 oz.	Pittsburgh, Pa.
A copy of Lowell's Poems	1 lb. 10 oz.	Denver, Colo.
A box of merchandise	3 lb. 8 oz.	Chicago, Ill.
A box containing a pair of shoes	3 lb. 6 oz.	Austin, Tex.
A piece of hardware	6 lb. 9 oz.	Detroit, Mich.

5. A publisher sends 20,000 copies of his magazine by mail. If each magazine and wrapper weighs $14\frac{1}{2}$ oz. and the total number is weighed at the post office in bulk, what will the publisher have to pay for postage?

6. A subscriber mailed two copies of the above magazine to a friend. What was the cost for postage?

7. 25,000 copies of a monthly magazine weighing $14\frac{1}{4}$ oz. were sent by mail. What was the cost to the publisher for postage?

8. Find the total cost for mailing the following: printers' proof weighing $18\frac{1}{4}$ oz.; manuscript and printers' proof in one package, weighing $28\frac{1}{2}$ oz.; a special delivery letter, weighing $\frac{3}{4}$ oz.

PRICE LISTS AND INVENTORIES

Price Lists

These price lists are to be used in making out the inventories which are found on the four pages following:

Article	1.	2.	3.	4.	5.
Bedsteads	$6.25	$8.50	$11.75	$5.25	$14.50
Bookcases	42.00	38.00	25.00	35.00	32.00
Bureaus	18.50	27.25	22.50	16.50	34.20
Cabinets:					
China	22.00	20.00	27.50	32.50	37.50
Medicine	2.25	1.75	2.50	3.25	2.75
Music	10.00	11.00	9.00	8.50	10.50
Parlor	25.50	21.50	25.00	36.50	38.00
Chairs:					
Easy	12.50	10.50	15.00	22.25	18.50
Morris	10.50	12.25	12.00	9.75	8.25
Piano	5.00	6.50	7.25	9.00	12.00
Typewriter	3.50	4.50	4.00	5.50	5.00
Cheval Mirrors	20.00	17.25	18.50	16.50	21.25
Chiffoniers	24.00	18.20	31.75	27.50	40.00
Davenports	62.50	50.00	60.00	45.50	37.50
Desks:					
Flat-top	21.20	17.60	16.80	22.30	18.60
Roll-top	23.50	21.25	25.50	27.25	30.50
Typewriter	11.25	12.25	14.25	10.25	13.25
Dinner Trays	5.50	6.25	7.25	5.25	8.25
Footrests	1.75	1.50	1.60	2.00	2.25
Hall Racks	14.25	13.50	17.25	18.50	12.25
Lounges	25.00	17.50	21.50	32.00	23.50
Mattresses	11.40	12.50	13.25	17.50	21.00
Ottomans	5.25	4.50	4.25	5.50	4.75
Parlor Suites	63.00	52.50	75.00	67.50	62.50
Pillows	2.50	3.00	3.50	2.00	3.25
Sideboards	60.00	50.00	45.00	37.50	72.50
Tables:					
Dining	21.50	17.50	22.50	24.50	25.00
Dressing	37.50	32.25	21.50	35.25	24.75
Serving	13.50	11.00	14.50	10.50	12.25
Work	10.00	9.25	9.00	10.25	11.00
Wardrobes	20.50	25.25	15.50	21.75	25.00
Washstands	6.00	7.50	5.50	8.00	10.50

Each inventory may be worked out *five* times, using the above price lists. This work may be done by copying the data and making the extensions, or by making the extensions only and then totaling each inventory.

INVENTORIES

1.	2.
7 Bureaus	8 Bureaus
19 Bedsteads	17 Bedsteads
12 Chiffoniers	15 Chiffoniers
5 Dressing Tables	3 Dressing Tables
21 Washstands	18 Washstands
8 Mattresses	11 Mattresses
23 Pillows	21 Pillows
6 Bookcases	5 Bookcases
3 Davenports	4 Davenports
13 Lounges	12 Lounges
17 Easy Chairs	20 Easy Chairs
7 Morris Chairs	3 Morris Chairs
5 Parlor Suites	6 Parlor Suites
8 Music Cabinets	10 Music Cabinets
12 Piano Chairs	11 Piano Chairs
15 Parlor Cabinets	21 Parlor Cabinets
5 Sideboards	2 Sideboards
13 Dining Tables	16 Dining Tables
8 China Cabinets	5 China Cabinets
4 Serving Tables	6 Serving Tables
9 Work Tables	7 Work Tables
12 Dinner Trays	15 Dinner Trays
4 Medicine Cabinets	8 Medicine Cabinets
8 Wardrobes	5 Wardrobes
11 Cheval Mirrors	14 Cheval Mirrors
15 Ottomans	12 Ottomans
12 Footrests	17 Footrests
5 Hall Racks	7 Hall Racks
6 Roll-top Desks	4 Roll-top Desks
3 Flat-top Desks	2 Flat-top Desks
9 Typewriter Desks	10 Typewriter Desks
8 Typewriter Chairs	6 Typewriter Chairs

3.	4.
8 Bureaus	9 Bureaus
15 Bedsteads	13 Bedsteads
14 Chiffoniers	11 Chiffoniers
4 Dressing Tables	7 Dressing Tables
19 Washstands	17 Washstands
12 Mattresses	9 Mattresses
18 Pillows	23 Pillows
8 Bookcases	6 Bookcases
5 Davenports	3 Davenports
16 Lounges	21 Lounges
22 Easy Chairs	18 Easy Chairs
6 Morris Chairs	9 Morris Chairs
7 Parlor Suites	4 Parlor Suites
11 Music Cabinets	9 Music Cabinets
14 Piano Chairs	13 Piano Chairs
12 Parlor Cabinets	18 Parlor Cabinets
6 Sideboards	4 Sideboards
10 Dining Tables	12 Dining Tables
7 China Cabinets	9 China Cabinets
6 Serving Tables	5 Serving Tables
8 Work Tables	11 Work Tables
11 Dinner Trays	14 Dinner Trays
7 Medicine Cabinets	5 Medicine Cabinets
9 Wardrobes	7 Wardrobes
13 Cheval Mirrors	17 Cheval Mirrors
19 Ottomans	15 Ottomans
12 Footrests	18 Footrests
6 Hall Racks	3 Hall Racks
8 Roll-top Desks	5 Roll-top Desks
4 Flat-top Desks	6 Flat-top Desks
14 Typewriter Desks	11 Typewriter Desks
10 Typewriter Chairs	7 Typewriter Chairs

5.	6.
12 Bureaus	14 Bureaus
23 Bedsteads	21 Bedsteads
13 Chiffoniers	16 Chiffoniers
8 Dressing Tables	11 Dressing Tables
16 Washstands	14 Washstands
14 Mattresses	15 Mattresses
22 Pillows	17 Pillows
4 Bookcases	7 Bookcases
4 Davenports	7 Davenports
17 Lounges	19 Lounges
14 Easy Chairs	25 Easy Chairs
12 Morris Chairs	11 Morris Chairs
10 Parlor Suites	9 Parlor Suites
14 Music Cabinets	12 Music Cabinets
17 Piano Chairs	21 Piano Chairs
22 Parlor Cabinets	16 Parlor Cabinets
7 Sideboards	9 Sideboards
14 Dining Tables	18 Dining Tables
4 China Cabinets	6 China Cabinets
11 Serving Tables	9 Serving Tables
5 Work Tables	10 Work Tables
19 Dinner Trays	13 Dinner Trays
6 Medicine Cabinets	10 Medicine Cabinets
12 Wardrobes	15 Wardrobes
20 Cheval Mirrors	16 Cheval Mirrors
11 Ottomans	19 Ottomans
21 Footrests	14 Footrests
4 Hall Racks	8 Hall Racks
12 Roll-top Desks	9 Roll-top Desks
5 Flat-top Desks	8 Flat-top Desks
13 Typewriter Desks	7 Typewriter Desks
12 Typewriter Chairs	15 Typewriter Chairs

7	8.
19 Bureaus	17 Bureaus
21 Bedsteads	20 Bedsteads
11 Chiffoniers	16 Chiffoniers
9 Dressing Tables	6 Dressing Tables
15 Washstands	21 Washstands
11 Mattresses	15 Mattresses
24 Pillows	25 Pillows
9 Bookcases	10 Bookcases
6 Davenports	2 Davenports
22 Lounges	16 Lounges
12 Easy Chairs	8 Easy Chairs
8 Morris Chairs	13 Morris Chairs
9 Parlor Suites	3 Parlor Suites
14 Music Cabinets	10 Music Cabinets
9 Piano Chairs	7 Piano Chairs
17 Parlor Cabinets	22 Parlor Cabinets
3 Sideboards	5 Sideboards
13 Dining Tables	9 Dining Tables
4 China Cabinets	10 China Cabinets
11 Serving Tables	7 Serving Tables
13 Work Tables	17 Work Tables
24 Dinner Trays	21 Dinner Trays
6 Medicine Cabinets	9 Medicine Cabinets
13 Wardrobes	11 Wardrobes
14 Cheval Mirrors	18 Cheval Mirrors
6 Ottomans	9 Ottomans
8 Footrests	10 Footrests
11 Hall Racks	12 Hall Racks
3 Roll-top Desks	2 Roll-top Desks
7 Flat-top Desks	9 Flat-top Desks
16 Typewriter Desks	10 Typewriter Desks
9 Typewriter Chairs	11 Typewriter Chairs

CHAPTER VIII

AVERAGE

ORAL EXERCISE

1. A earns $3, B earns $4, and C earns $5 per day. What do the three earn in 1 da.? If $12 were paid to these men in equal parts, how much would each receive?

2. What sum is intermediate between 6, 7, and 8? between 6, 8, and 10? between 6, 12, and 18?

111. The process of finding a number that is intermediate between two or more other numbers is called **average.**

112. **Example.** What is the average weight of 3 bales of cotton weighing 460, 449, and 475 lb., respectively?

SOLUTION. The aggregate of the 3 bales of cotton is 1384 lb. 1384 lb. divided into three equal parts shows the mean or average weight to be $461\frac{1}{3}$ lb.

To find the average of consecutive numbers, add the highest number to the lowest, and divide by 2.

$$\begin{array}{r} 460 \\ 449 \\ 475 \\ \hline 3)\overline{1384} \\ 461\frac{1}{3} \end{array}$$

WRITTEN EXERCISE

1. A tapering board is 14 in. wide on one end and 18 in. on the other. What is the average width of the board?

2. A manufacturing pay roll shows that 15 hands are employed at $1.25 per day, 12 hands at $1.75 per day, 16 hands at $2.25 per day, 32 hands at $2.50 per day, and 5 hands at $6.50 per day. Find the average daily wages.

3. A merchant's sales for a year were as follows: January, $12,156; February, $14,175; March, $16,152; April, $12,175; May, $12,465.95; June, $12,476.05; July, $15,145.40; August, $12,431.46; September, $17,245.90; October, $18,256.45; November, $19,250.65; December, $19,654.20. What were his average sales per month?

4. In a certain school of 300 pupils, 85 are 14 yr. of age; 50, 15 yr. of age; 25, 16 yr. of age; 75, 17 yr. of age; 50, 18 yr. of age; 15, 19 yr. of age. What is the average age of the school?

5. The attendance for a certain school for a week was as follows: Monday, 727 pupils; Tuesday, 732 pupils; Wednesday, 756 pupils; Thursday, 761 pupils; Friday, 734 pupils. What was the average daily attendance for the week?

6. What should a ground feed made from 50 bu. of oats worth 38¢ per bushel, 30 bu. of barley worth 78¢, and 60 bu. of corn worth 69¢ sell for in order to make 10¢ per bushel on each ingredient used to make the mixture?

7. Find the aggregate weight and the average weight per box of 100 bx. of cheese weighing 65, 64, 62, 60, 61, 65, 62, 64, 61, 62, 61, 60, 60, 61, 62, 60, 68, 65, 66, 64, 62, 61, 65, 66, 62, 64, 67, 58, 62, 59, 59, 60, 62, 64, 66, 67, 58, 60, 65, 58, 62, 69, 62, 65, 68, 69, 61, 65, 62, 61, 65, 68, 59, 62, 64, 58, 62, 65, 71, 70, 58, 67, 58, 62, 64, 58, 62, 64, 65, 69, 65, 65, 62, 64, 60, 60, 65, 60, 65, 65, 62, 60, 62, 64, 60, 72, 64, 70, 61, 62, 60, 60, 59, 65, 60, 70, 58, 62, 61, 64 lb., respectively.

8. Counting 8 hr. to a day, find the total amount and the average daily wages in the following contractor's time sheet:

TIME SHEET FOR WEEK ENDING JUNE 30

NAME	M.	T.	W.	T.	F.	S.	HOURS	DAYS	DAILY WAGES	AMOUNT	
C. E. Ames	8	8	8	8	8	8			$1.75		
W. O. Bye	9	10	9	10	10	8			2.00		
M. E. Carey	10	9	9	10	8	10			2.00		
W. D. Frey	6	8	9	10	7	8			2.25		
G. W. Jones	10	10	10	8	10	8			2.25		
D. O. Munn	4	4	4	6	8	6			2.50		
E. H. Post	6	6	6	6	4	4			3.00		
L. C. Roe	10	10	10	10	4	4			3.25		
J. H. Small	6	8	8	10	12	12			3.25		
H. M. Young	8	8	8	8	8	8			3.50		
Total											

CHAPTER IX

CHECKING RESULTS

113. It has been seen in the preceding exercises on statistics, time sheets, etc., that various ruled forms provide for practical and convenient methods of checking results. While it is possible to give a great variety of these problems it is also necessary to give a great many problems that do not furnish such a check.

114. It is very important that all results be checked. The most common methods of checking addition, subtraction, and division have already been mentioned. Multiplication may be proved by dividing the product by either factor, or as explained on page 52.

The properties of 9 and 11 may also be applied to advantage in checking results, especially results in multiplication and division.

PROPERTIES OF 9 AND 11

Properties of 9

115. Any number of 10's is equal to the same number of 9's plus the same number of units; any number of 100's is equal to the same number of 99's plus the same number of units; any number of 1000's is equal to the same number of 999's plus the same number of units; and so on.

Thus, $10 =$ one $9 + 1$; $40 =$ four 9's $+ 4$; $100 =$ one $99 + 1$; $300 =$ three 99's $+ 3$; $500 =$ five 99's $+ 5$.

116. Any number may be resolved into one less than as many multiples of 10 as it contains digits.

Thus, $946 = 900 + 40 + 6$; $42175 = 40000 + 2000 + 100 + 70 + 5$.

117. The excess of 9's in any power of 10 or in any multiple of a power of 10 is the same as the significant figure (unless that figure is 9, then there is no excess) in that number. Hence,

The excess of 9's in any number is equal to the excess of 9's in the sum of its digits.

Thus, the excess of 9's in $241 = 2 + 4 + 1$, or 7. The excess of 9's in $946 = 9 + 4 + 6$, or 19; but 19 contains 9, and the excess of 9's in $19 = 1 + 9$, or 10; but 10 contains 9, and the excess of 9's in $10 = 1 + 0$, or 1; the excess of 9's in 946 is therefore shown to be 1.

118. In finding the excess of 9's in any number, omit all 9's and all combinations of two or three digits which it is seen at a glance will make 9 or some multiple of 9.

Thus, in finding the excess of 9's in 9458, begin at the left, reject the first digit 9, the sum of the next two digits, 9, and the single 8 will be the excess of 9's in the entire number.

Properties of 11

119. Any number of 10's is equal to the same number of 11's minus the same number of units; any number of 100's is equal to the same number of 99's plus the same number of units; any number of 1000's is equal to the same number of 1001's minus the same number of units; and so on.

Thus, 40 = four 11's − 4; 500 = five 99's + 5; 7000 = seven 1001's − 7.

120. It is therefore clear that even powers of 10 are multiples of 11 plus 1 and odd powers of 10 are multiples of 11 minus 1.

Thus, 10^2 or 100 = nine 11's + 1; 10^3 or 1000 = ninety-one 11's − 1; 10^4 or 10,000 = nine hundred nine 11's + 1.

121. From the foregoing it is evident that:

The excess of 11's in any number is equal to the sum of the digits in the odd places (increased by 11 or a multiple of 11 if necessary) minus the sum of the digits in the even places.

Thus, the excess of 11's in 45 is 1 $(5 - 4)$; the excess of 11's in 125 is 4 $(\overline{5 - 2} + \overline{1 - 0})$; the excess of 11's in 2473 is 9 $(\overline{3 + 4} + 11 - \overline{7 + 2} = 9)$; the excess of 11's in 14,206 is 5.

Checking Addition and Subtraction

122. Examples. 1. By casting out the 9's, show that the sum of 935, 651, 782, and 465 is 2833.

Solution. The sum of the digits in 935 is 17; but since 17 contains 9, find the sum of the digits in 17 and the result, 8, is the excess of 9's in the entire number. In like manner find the excess of 9's in 651, 782, and 465. Since 935 is a multiple of $9+8$, 651 a multiple of $9+3$, 782 a multiple of $9+8$, 465 a multiple of $9+6$, the sum of these numbers, 2833, should equal a multiple of $9+(8+3+8+6)$, or $9+25$. 25 is a multiple of $9+7$, and 2833 is a multiple of $9+7$; hence, the addition is probably correct.

$$\begin{array}{r} 935 = \overline{8} \\ 651 = \overline{3} \\ 782 = \overline{8} \\ 465 = \overline{6} \\ \hline 2833 = \overline{7} \end{array}$$

2. By casting out the 11's, show that the sum of 648, 217, 451, and 688 is 2004.

Solution. $\overline{8-4}+\overline{6-0}=10$, the excess of 11's in 648. $\overline{7-1}+\overline{2-0}=8$, the excess of 11's in 217. $12\ (11+1)-5+\overline{4-0}=11$; but 11 contains 11, hence, the excess of 11's in 451 is 0. $\overline{8-8}+\overline{6-0}=6$, the excess of 11's in 688. Since 648 is a multiple of $11+10$, 217 a multiple of $11+8$, 451 a multiple of 11, and 688 a multiple of $11+6$, the sum of these numbers, 2004, should be a multiple of $11+(10+8+6)$, or $11+24$. 24 is a multiple of $11+2$ and 2004 is a multiple of $11+2$; hence, the addition is probably correct.

$$\begin{array}{r} 648 = \overline{10} \\ 217 = \overline{8} \\ 451 = \overline{0} \\ 688 = \overline{6} \\ \hline 2004 = \overline{2} \end{array}$$

123. Subtraction may be proved either by casting out the 9's or 11's in practically the same manner as addition.

The difference between the excess of 9's or 11's in the minuend and subtrahend should equal the excess of 9's or 11's in the remainder; or the sum of the excess of 9's or 11's in the subtrahend and remainder should equal the excess of 9's or 11's in the minuend.

These methods are but little used for checking addition and subtraction. Addition is generally checked as explained on page 20, and subtraction as explained on page 32. On the other hand, long multiplications and divisions are almost always checked by applying the properties of 9 or 11.

Checking Multiplication and Division

124. Examples. 1. By casting out the 9's show that the product of 64×95 is 6080.

Solution. The excess of 9's in 95 is 5, and in 64, 1. Since 95 is a multiple of $9+5$ and 64 a multiple of $9+1$, the product of 64×95 should be a multiple of 9 plus (1×5). 1×5 or 5 equals the excess of 9's in 6080; hence, the work is probably correct.

$$\begin{array}{r} 95 = \overline{5} \\ 64 = \overline{1} \\ \hline 6080 = \overline{5} \end{array}$$

2. By casting out the 11's show that the product of 46×95 is 4370.

SOLUTION. The excess of 11's in 95 is 7, and in 46, 2. Since 95 is a multiple of $11 + 7$ and 46 a multiple of $11 + 2$, the product of 46×95 should be a multiple of 11 plus (2×7) or 14; but 14 is a multiple of $11 + 3$. Since the product 4370 is a multiple of $11 + 3$, the work is probably correct.

$$\begin{array}{r} 95 = 7 \\ 46 = 2 \\ \hline 4370 = 3 \end{array}$$

125. Division may be proved either by casting out the 9's or 11's in practically the same manner as multiplication. The excess of 9's or 11's in the quotient multiplied by the excess of 9's or 11's in the divisor should equal the excess of 9's or 11's in the dividend, minus the excess of 9's or 11's in the remainder, if any.

Casting out the 9's will not show an error caused by a transposition of figures; but casting out the 11's will show such an error. The method of casting out the 11's is therefore considered the better proof.

WRITTEN EXERCISE

1. Determine without dividing whether \$2.64 is the quotient of \$1375.44 ÷ 521.

2. Determine without multiplying whether \$1807.50 is the product of 482 times \$3.75.

3. Determine without adding whether 4231 is the sum of 296, 348, 924, 862, 956, and 845.

4. Multiply 34,125 by 729 in two lines of partial products and verify the work by casting out the 9's.

5. Find the cost of 173,000 shingles at \$4.27 per thousand, in two lines of partial products, and verify the work by casting out the 11's.

6. Find the cost of 126,000 ft. of clear pine at \$24.60 per thousand, in two lines of partial products, and verify the work by casting out the 9's.

7. Find the cost of 2,191,000 ft. of flooring at \$32.08 per thousand, in two lines of partial products, and verify the work by casting out the 11's.

FRACTIONS

CHAPTER X

DECIMAL FRACTIONS

ORAL EXERCISE

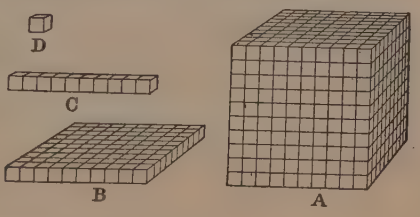
1. In the number \$7.62 what figure stands for the dollars? the tenths of a dollar? the hundredths of a dollar?

2. What name is given to the point which separates the whole number of dollars from the part of a dollar?

3. Read: 3.5 dollars; 3.5 ft.; 27.5 lb.; .7 of a dollar; .5 of a ton; 16.6; .9; 9.25 dollars; 7.25 ft.; 8.75 rd.; .95 of a dollar; .85 of a pound sterling; .57.

4. What is the first place at the right of the decimal point called? the second place?

5. In the accompanying diagram what part of A is B? What part of B is C? What part of C is D?



6. What part of A is C? What part of A is D?

7. If A is a cubic inch, what is B? C? D?

8. In a pile of 10,000 bricks one brick is what part of the whole pile? 10 bricks is what part of the whole pile? 100 bricks is what part of the whole pile? 1000 bricks is what part of the whole pile?

9. How may one tenth be written besides $\frac{1}{10}$? one hundredth besides $\frac{1}{100}$? one thousandth besides $\frac{1}{1000}$?

126. Units expressed by figures at the right of the decimal point are called **decimal units.**

127. A number containing one or more decimal units is called a **decimal fraction** or a **decimal.**

NOTATION AND NUMERATION

ORAL EXERCISE

1. Read: 0.7; 0.03; 0.25. How many places must be used to express completely any number of hundredths?

2. Read: 0.004; 0.025; 0.725. How many places must be used to express completely any number of thousandths?

3. Read: .0005; .00007; .000009; .0037; .00045; .000051; .0121; .00376; .000218; .1127; .01525; .004531; .16067.

4. How many places must be used to express completely any number of ten-thousandths? any number of hundred-thousandths? any number of millionths?

128. In **reading decimals** pronounce the word *and* at the decimal point and omit it in all other places.

Thus, in reading 0.605 or .605 say *six hundred five thousandths*; in reading 600.005 say *six hundred and five thousandths.*

129. The relation of integers and decimals with their increasing and decreasing orders to the left and to the right of the decimal point is shown in the following

NUMERATION TABLE

PERIODS:	Millions			Thousands			Units				Thousandths			Millionths		
ORDERS:	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
	9	8	7,	6	5	4,	3	2	1	.	2	3	4	5	6	7

130. Hundredths are frequently referred to as **per cent**, a phrase originally meaning *by the hundred.*

131. The symbol % stands for hundredths and is read *per cent.*

Thus 45% = .45; 48% of a number = .48 of it.

ORAL EXERCISE

Read:

1. 0.073.
2. 0.00073.
3. 3004.025.
4. 300.4025.
5. 532.002.
6. 60.0625.
7. 63.3125.
8. 126.8125.
9. 31.08%.
10. 126.75%.
11. 2150.1875.
12. 3165.00625.
13. 131.3125 T.
14. 240.0125 A.
15. A tax of 1.0625 mills.
16. A tax of 9.1875 mills.
17. Read the number in the foregoing numeration table.
18. Read the following, using the words "per cent": .17; .28; .85; .67; .425; .37½.
19. Read the following as decimals, not using the words "per cent": 25%; 75%; 87%; 62½%; 27.15%.
20. Read aloud the following:

a. The value of a pound sterling in United States money is $4.8665.

b. A meter (metric system of measures) is equal to 39.37079 in.; a kilometer, to 0.62137 mi.

c. 1 metric ton is equal to 1.1023 ordinary tons; 1.5 metric tons are equal to 1.65345 ordinary tons.

d. A flat steel bar 3 in. wide and 0.5 in. thick weighs 5.118 lb.

e. The circumference of a circle is 3.14159 times the length of its diameter.

WRITTEN EXERCISE

Write decimally:

1. Five tenths; fifty hundredths; five hundred thousandths.
2. Nine hundred and eleven ten-thousandths; nine hundred eleven ten-thousandths; five hundred and two thousandths.
3. One hundred seventy-four millionths; one hundred seventy-four million and seven millionths; seven million and one hundred seventy-four millionths.
4. Seven thousand and seventy-five ten-thousandths; two hundred fifty-seven ten-millionths; two hundred and forty-six millionths; two hundred forty-six millionths.

5. Four million ten thousand ninety-seven ten-millionths; four million ten thousand and ninety-seven ten-millionths; five hundred millionths; five hundred-millionths.

6. Six hundred six and five thousand one hundred-thousandths; six hundred six and fifty-one hundred-thousandths; fifty-six and one hundred twenty-eight ten-billionths.

7. Seventeen thousand and eighteen hundred seventy-six millionths; seventeen thousand and eighteen hundred seventy-six ten-thousandths; twenty-one hundred sixteen hundredths.

132. In the number 2.57 there are 2 integral units, 5 tenths of a unit, and 7 hundredths of a unit. In the number 2.5700 there are 2 integral units, 5 tenths of a unit, 7 hundredths of a unit, 0 thousandths of a unit, and 0 ten-thousandths of a unit. 2.5700 is therefore equal to 2.57. That is,

Decimal ciphers may be annexed to or omitted from the right of any number without changing its value.

ORAL EXERCISE

Read the following (a) as printed and (b) in their simplest decimal form:

1. 0.700.	3. 16.010.	5. 0.50.	7. 0.7000.
2. 5.2450.	4. 18.210.	6. 0.00950.	8. 12.9010.

ADDITION

ORAL EXERCISE

1. What is the sum of 0.4, 0.05, 0.0065?
2. What is the sum of 0.3, 0.021, 0.008?
3. Find the sum of seven tenths, forty-four hundredths, and two; of four tenths, twenty-one hundredths, and six thousandths.

133. Example. Find the sum of 12.021, 256.12, and 27.5.

SOLUTION. Write the numbers so that their decimal points stand in the same vertical column. Units then come under units, tenths under tenths, and so on. Add as in integral numbers and *place the decimal point in the sum directly under the decimal points in the several numbers added.*

$$\begin{array}{r} 12.021 \\ 256.12 \\ 27.5 \\ \hline 295.641 \end{array}$$

WRITTEN EXERCISE

Find the sum of:

1. 7.5, 165.83, 5.127, 6.0015, and 71.215.
2. 257.15, 27.132, 5163, 8.000125, and 4100.002.
3. 0.175, 5.0031, .00127, 70.2116001, and 21.00725.
4. 51.6275, 19.071, 0.000075, 21.00167, and 40,000.01.
5. 2.02157, 2.1785, 2500.00025, 157.2165, and 7.0021728.
6. Copy, find the totals as indicated, and check:

$1241.50	$9215.45	$1421.12	$1421.32	?
1.52	1275.92	1.46	1618.40	?
349.21	3725.41	2.18	1920.41	?
2975.47	7286.95	7.96	10.20	?
27.14	8276.92	14.21	41.64	?
9218.49	7271.44	1240.80	126.18	?
5.17	8926.95	7216.80	24.17	?
12627.85	8972.76	4.75	240.20	?
721.92	7214.25	8.16	960.80	?
11.41	8142.76	.47	1860.45	?
1.21	8436.14	.92	9270.54	?
.72	8435.96	9.26	75.86	?
14178.21	7926.14	1490.75	45.95	?
2172.14	9214.72	1860.54	75.86	?
726.95	1241.16	9265.80	72.18	?
85.21	4214.71	625.50	9260.14	?
75.92	8726.19	240.75	1.20	?
72604.25	2140.12	60.50	7.40	?
124.61	7146.14	120.41	8.32	?
2114.62	7214.86	4101.08	2860.14	?
?	?	?	?	?

7. Find the sum of twenty-one hundred sixty-five and one hundred sixty-five ten-thousandths, thirty-nine and twelve hundred sixty-five millionths, twenty-seven hundred thirty-six and one millionth, four and six tenths, six hundred and six thousandths, and six hundred sixty-five thousandths.

SUBTRACTION

ORAL EXERCISE

1. From the sum of 0.7 and 0.4 take 0.5.

2. From the sum of 0.07 and 0.21 take 0.006.

3. From seventy-four hundredths take six thousandths.

4. To the difference between .43 and .03 add the sum of .45 and .007.

5. Goods on hand at the beginning of a week, $24.50; goods purchased during the week, $35.50; goods sold during the week, $36; goods on hand at the close of the week, $36.50. What was the gain or loss for the week?

134. Example. From 14.27 take 5.123.

SOLUTION. Write the numbers so that the decimal points stand in the same vertical column. The minuend has not as many places as the subtrahend; hence suppose decimal orders to be annexed until the right-hand figure is of the same order, then subtract as in integers and *place the decimal point in the remainder directly under the decimal points in the numbers subtracted.*

$$\begin{array}{r} 14.27 \\ 5.123 \\ \hline 9.147 \end{array}$$

WRITTEN EXERCISE

Find the difference between:

1. 7.2154 and 2.8576.
2. 17.2157 and 1.0002.
3. 1.0005 and .889755.
4. $1265.45 and $87.99.
5. 9 and 5.2675.
6. 16 and 5.0000271.
7. .0002 and .000004.
8. 24.503 and 17.00021.

9. The sum of two numbers is 166.214. If one of the numbers is 40.21, what is the difference between the numbers?

10. The minuend is 127.006 and the remainder 15.494. What is the sum of the minuend, subtrahend, and remainder?

11. From the sum of ninety-nine ten-thousandths, one hundred fifty-one and five thousandths, two hundred fifty-two and twenty-five millionths, six tenths, and eighteen and one hundred seventy-five thousandths take the sum of twelve hundred fifteen millionths, and one hundred eighty-eight thousandths.

12. From the sum of two hundred fifty-seven thousandths and eight and one hundred twenty-six millionths take the sum of five hundred ten thousandths and two and one hundred twenty-four ten-thousandths.

13. A merchant had, at the beginning of a year, goods amounting to $8165.95. During the year his purchases amounted to $5265.90 and his sales to $9157.65. At the close of the year he took an account of stock and found that the goods on hand were worth $7216.56. What was his gain or loss for the year?

14. A provision dealer had on hand Jan. 1, goods worth $4127.60. His purchases for the year amounted to $4165.95 and his sales to $6256.48. Dec. 31 of the same year his inventory showed that the goods on hand were worth $3972.50. If the amount paid for freight on the goods bought amounted to $237.50, what was his gain or loss on provisions?

15. I had on hand Jan. 1, lumber amounting to $4210.60. During the year my purchases amounted to $3126.50, and my sales to $4165.85. I lost by fire lumber valued at $506.75, for which I received from an insurance company $500. Dec. 31, my inventory showed the lumber to be worth $5209.08. How much did I gain or lose on lumber during the year?

16. At the beginning of a year my resources were as follows: cash on hand, $1262.50; goods in stock, $1742.85; account against A. M. Eaton, $146.50. At the same time my liabilities were as follows: note outstanding, $156.85; account in favor of Robert Wilson, $521.22. During the year I made an additional investment of $1250.65, and withdrew for private use $275. I sold for cash during the year goods amounting to $1250.75, and bought for cash goods amounting to $530.90; I also paid Robert Wilson $320 to apply on account. At the close of the year my inventory showed goods in stock valued at $750.48. What was my gain or loss for the year and my present worth at the close of the year?

Do not fail to check all problems. No phase of arithmetic is more important.

MULTIPLICATION

ORAL EXERCISE

1. How many times .4 is 4? .77 is 7.7? .999 is 9.99?

2. 44 is how many times .44? 22 is how many times .022? 1 is how many times .001? .01 is how many times .0001?

3. Read aloud the following, supplying the missing terms: Removing the decimal point one place to the right multiplies the value of the decimal by —— ; two places, —— the value by —— ; three places, —— the value by ——.

4. Multiply 12.1252 by 1000; by 100; by 100,000.

5. Multiply $9.375 by 100; by 10,000; by 100,000.

6. Multiply 5.15 by 10; by 100; by 1000; by 10,000.

7. Multiply .000016 by 1000; by 100,000; by 1,000,000.

8. Multiply $67.50 by 10; by 100; by 1000; by 10,000.

9. Multiply .0037 by 10; by 100; by 1000; by 10,000,000.

10. What part of 1 is .1? of 7 is .7? of 29 is 2.9?

11. What part of 84 is .84? of 129 is 1.29? of 1275 is 12.75?

12. What part of .6 is .006? of .64 is .0064?

Read aloud the following, supplying the missing terms:

a. Each removal of the decimal point one place to the left —— the value of the decimal by 10.

b. To divide a decimal by —— is to find one tenth (.1) of it, or to —— it by .1.

13. Give a short method for multiplying a number by .1; by .01; by .001; by .0001.

14. Multiply .009 by .1; by .01; by .001.

15. Multiply 217.59 by .1; by .01; by .001.

16. Multiply 54.65 by .01; by .00001; by .000001.

17. Multiply 2.375 by .1; by .01; by .001; by .0001.

18. Multiply 25.215 by .1; by .01; by .001; by .0001.

19. Multiply 2111 by .01; by .001; by .0001; by .00001.

20. Compare $2400 \times \$0.06$ with $100 \times 24 \times \$0.06$ or with $24 \times \$6$.

21. Compare 3000×612.251 with $1000 \times 3 \times 612.251$, or with 3×612251.

22. Multiply 21.25 by 2400.

SOLUTION. 2400 is 24 times 100. Multiply by 100 by removing the decimal point two places to the right. The result is 2125. 24 times 2125 equals 51,000, the required product.

In multiplying begin with either the lowest or the highest digit in the multiplier as shown in the margin.

2125	2125
24	24
8500	4250
4250	8500
51000	51000

23. Formulate a brief rule for multiplying a decimal by any number of 10's, 100's, 1000's, etc.

24. Find the cost of:

a. 500 lb. at 18¢. *d*. 600 lb. at 29¢. *g*. 900 lb. at 34¢.
b. 150 lb. at 14¢. *e*. 300 lb. at 41¢. *h*. 700 lb. at 51¢.
c. 200 lb. at 26¢. *f*. 400 lb. at $12\frac{1}{2}$¢. *i*. 1400 lb. at 5¢.

135. Examples. 1. Multiply 41.127 by 4.

SOLUTION. 41.127 is equal to 41,127 thousandths. 41,127 thousandths multiplied by 4 equals 164,508 thousandths, or 164.508. That is, thousandths multiplied by a whole number must equal thousandths.

41.127
4
164.508

2. Multiply 41.127 by .04.

SOLUTION. The multiplier, .04, is equal to 4 times .01; therefore, multiply by 4 and by .01. Multiplying by 4, as in problem 1, the result is 164.508. Multiplying by .01, by simply moving the decimal point in the product two places to the left, the result is 1.64508.

41.127
.04
1.64508

It will be seen that *the number of decimal places in the product is equal to the decimal places in the multiplicand and multiplier.*

It should not be necessary to memorize the above rule. The student should know at a glance that the product of tenths and tenths is hundredths, of tenths and hundredths is thousandths, and so on.

ORAL EXERCISE

1. In multiplying 24.05 by 3.14 can you tell before multiplying how many integral places there will be in the product? how many decimal places? Explain.

2. How many integral places will there be in each of the following products: 2.5×4.015? 27.51×3.1416? 321.1×201.51? 1.421×42.267? $126.5 \times .01$? 1020×5.01? $.105 \times 6$? 2.41×10.05? How many decimal places will there be in each of the above products?

3. What are 400 bbl. of apples worth at \$2.12 per barrel? at \$1.27½ per barrel?

4. I bought 60 lb. of sugar at \$0.04½ and gave in payment a five-dollar bill. How much change should I receive?

5. A and B are partners in a manufacturing business, A receiving 52% and B 48% of the yearly profits. The profits for a certain year are \$5000. Of this sum how much should A and B, respectively, receive?

WRITTEN EXERCISES

Find the product of:

1. 3.121×152.
2. $3121 \times .152$.
3. 31.21×15.2.
4. 12.14×265.
5. $9.004 \times .021$.
6. $.3121 \times .0152$.
7. 2.531×31000.
8. $.1724 \times 18000$.
9. $.15539 \times 2002$.

10. A man owned 75% of a gold mine and sold 50% of his share. What is the remainder worth if the value of the whole mine is \$425,000?

11. A man bought a farm of 240 A. at \$137.50 per acre. He sold 75% of it at \$150 per acre, and the remainder at \$175 per acre. What was his gain?

12. Copy and complete the following table of statistics. Check the results. (The total yield multiplied by the price per bushel should equal the total valuation.)

LARGEST WHEAT-GROWING STATES IN A RECENT YEAR

STATE	YIELD IN BUSHELS	FARM PRICE PER BUSHEL	FARM VALUATION
North Dakota	143,820,000	92.4¢	?
Kansas	92,290,000	92.4¢	?
Minnesota	67,038,000	92.4¢	?
South Dakota	52,185,000	92.4¢	?
Total	?	?	?

13–15. Make and solve three self-checking problems in multiplication of decimals.

DIVISION

ORAL EXERCISE

1. Divide by 8: 64 ft., .64, .064, 6.4.
2. Divide by 9: 63 in., .63, .063, 6.3.
3. Divide by 16: \$640, \$6.40, 6.4, .64, .064.
4. Divide by 15: \$15.75, \$7.50, \$0.75, 30.45, 3.045, .3045.
5. Divide 337.5 by 45.

$$
\begin{array}{r}
7.5 \\
45\overline{)337.5} \\
315 \quad = 45 \text{ times } 7 \\
\hline
22.5 \text{ undivided} \\
22.5 = 45 \text{ times } .5
\end{array}
$$

CHECK. 45 times 7.5 = 337.5; hence, the work is correct.

136. In the above exercise it is clear that *when the divisor is an integer, each quotient figure is of the same order of units as the right-hand figure of the partial dividend used to obtain it.*

ORAL EXERCISE

1. 500 is how many times 50? \$75 is how many times \$7.50?
2. Divide 50 by 5; 500 by 50. How do the quotients compare?
3. Divide 7.50 by 15; \$75 by 150. How do the quotients compare?
4. 720 is how many times 72? 9 is how many times .9?
5. Divide 720 by 9; 72 by .9; 7.2 by .09; .72 by .009.

137. It has been seen that multiplying both dividend and divisor by the same number does not change the quotient.

138. Therefore, to divide decimals when the divisor is not an integer:

Multiply both dividend and divisor by the power of 10 that will make the divisor an integer, and divide as in United States money.

139. Divide 0.3375 by 0.45.

$.3375 \div .45 = 33.75 \div 45$. $33.75 \div 45 = .7$, with a remainder of 2.25. $2.25 \div 45 = .05$. The quotient is therefore .75.

$$45\overline{)33.75}\quad .75$$
$$31\,5$$
$$2\,25$$
$$2\,25$$

Observe that *the divisor may always be made an integer if the decimal point in the dividend is carried to the right as many places as there are decimal places in the divisor.*

Should there be a remainder after using all the decimal places in the dividend, annex decimal ciphers and continue the division as far as is desired.

ORAL EXERCISE

Divide:

1. 1 by 1.
2. 1 by .1.
3. 1 by 10.
4. .1 by .1.
5. 1 by .01.
6. 1 by 100.
7. 1 by .001.
8. .10 by .10.
9. .01 by .01.
10. 1 by 1000.
11. 1 by .0001.
12. 1 by 10,000.
13. 1 by .00001.
14. .001 by .001.
15. 1 by 100,000.
16. 1 by .000001.
17. .0001 by .0001.
18. .00001 by .00001.
19. 33 by .11.
20. 33 by 110.
21. .33 by .11.
22. 3.3 by 1.1.
23. .0001 by 1.
24. 33 by .011.
25. 33 by 1100.
26. .0001 by .1.
27. 3300 by .11.
28. 330 by .011.
29. 33 by .0011.
30. 33 by 11000.
31. .0001 by .01.
32. .033 by .011.
33. .0001 by .001.
34. .0033 by .0011.
35. .0001 by .0001.
36. .0001 by .00001.

WRITTEN EXERCISE

Divide:

1. 5842 by .046.
2. 2.592 by .108.
3. 1.750 by 8750.
4. .00338 by .013.
5. 1.728 by .0024.
6. 2200 by .44.
7. 231.6 by 579.
8. 950 by 19,000.
9. 81.972 by .00009.
10. 115.814 by .00079.
11. 16 by .0064.
12. 1.86 by 31,000.
13. 1600 by 64,000.
14. .0004 by 20,000.
15. 100 by .000001.

Find the sum of the quotients:

16.	17.	18.
8.1 ÷ .9.	72 ÷ 8.	125 ÷ 250.
81 ÷ .09.	72 ÷ .8.	12.5 ÷ 2.5.
8.1 ÷ .09.	7.2 ÷ .8.	1.25 ÷ 2.5.
.81 ÷ 900.	72 ÷ .08.	12.5 ÷ 250.
.0081 ÷ 9.	.72 ÷ .08.	125 ÷ 2500.
8.1 ÷ 900.	72 ÷ .008.	.125 ÷ .025.
810 ÷ .009.	72 ÷ 8000.	12500 ÷ .25.
.0081 ÷ 9000.	72 ÷ .0008.	125 ÷ 25000.
81000 ÷ .009.	.072 ÷ .008.	12500 ÷ .025.
81 ÷ .000009.	72 ÷ .00008.	125 ÷ 250000.
8100 ÷ 90000.	.0072 ÷ .0008.	.125 ÷ .00025.
.00081 ÷ 90000.	.00072 ÷ .00008.	12500 ÷ .0025.

19.	20.	21.
8.8 ÷ 2.2.	17 ÷ 68.	36 ÷ .072.
.88 ÷ .22.	1.7 ÷ 6.8.	3.6 ÷ .072.
88 ÷ .0022.	.17 ÷ .68.	.36 ÷ .072.
8.8 ÷ 2200.	1.7 ÷ 680.	360 ÷ .072.
880 ÷ 2200.	170 ÷ 680.	.036 ÷ .072.
8.8 ÷ 2.200.	.017 ÷ .068.	3.6 ÷ 72000.
880 ÷ .2200.	1.7 ÷ 68000.	36 ÷ 720000.
8800 ÷ 2200.	1700 ÷ 6800.	360 ÷ .00072.
880 ÷ 22000.	1700 ÷ 68000.	3600 ÷ .0072.
880 ÷ .00022.	.0017 ÷ .0068.	.0036 ÷ .0072.
88000 ÷ .0022.	.00017 ÷ .00068.	3.6 ÷ .000072.
88000 ÷ .00022.	.000017 ÷ .000068.	.00036 ÷ .00072.

22. The product of two numbers is 0.00025. If one of the numbers is 0.0025, what is the other?

23. A retailer bought 450 yd. of cloth for $1237.50 and sold it at $3.25 per yard. How much did he gain per yard?

24. A drover bought a flock of sheep at the rate of $3.30 per head. He sold them at a profit of $0.20 per head and received $700. How many sheep were there in the flock and what was his gain?

25. Copy and complete the following table. Check the results.

LARGEST OAT-GROWING STATES IN A RECENT YEAR

STATE	YIELD IN BUSHELS	FARM PRICE PER BUSHEL	FARM VALUATION
Illinois	?	31¢	54,818,000
Iowa	?	31¢	58,811,000
Wisconsin	?	31¢	27,119,000
Minnesota	?	31¢	31,926,000
Total	?	31¢	?

26–28. Make and solve three self-checking problems in division of decimals.

DIVIDING BY POWERS AND MULTIPLES OF TEN

ORAL EXERCISE

1. 6.4 is what part of 64? \$0.17 is what part of \$1.70?

2. Compare (as in problem 1) \$240.60 with \$24,060; 17.75 ft. with 1775 ft.

3. Compare (as in problem 1) .1 with 1; .01 with 1; .001 with 1; .0001 with 1.

4. Read aloud the following, supplying the missing terms: Removing the decimal —— place to the —— divides the value of the decimal by 10; two places, —— the value of the decimal by ——; three places, —— the value of the decimal by ——.

5. Compare the quotient of $28 \div .7$ with the quotient of $\overline{28 \times 10} \div \overline{.7 \times 10}$; the quotient of $28 \div .7$ with the quotient of $280 \div 7$.

6. Compare the quotient of $16.4 \div 40$ with the quotient of $\overline{16.4 \div 10} \div \overline{40 \div 10}$; the quotient of $16.4 \div 40$ with the quotient of $1.64 \div 4$. What is the quotient of 56.77 divided by 7000?

SOLUTION. Removing the decimal point three places to the left and dropping the ciphers of the divisor is equivalent to dividing both dividend and divisor by 1000 and does not change the value of the quotient.

$$7\overline{).05677}$$
$$.00811$$

Buying and Selling by the Hundred

ORAL EXERCISE

1. Compare $460 \div 100 \times \$2$ with $4.60 \times \$2$.
2. Find the cost of 450 lb. of guano at \$4 per cwt.
3. Find the cost of 600 lb. of wire nails at 34¢ per cwt.
4. Find the cost of 4950 paving stones at \$8 per C.

Solution. C stands for 100. 4950 paving stones are 49.5 times 100 paving stones. Since 1 hundred paving stones cost \$8, 49.5 hundred paving stones will cost 49.5 times \$8, or \$396.

$$\begin{array}{r} 49.5 \\ 8 \\ \hline 396.0 \end{array}$$

WRITTEN EXERCISE

Find the cost:

	Quantity	Price per Hundredweight		Quantity	Price per Hundredweight
1.	450 lb.	55¢	5.	1600 lb.	71¢
2.	510 lb.	77¢	6.	2600 lb.	15¢
3.	640 lb.	60¢	7.	4900 lb.	70¢
4.	330 lb.	56¢	8.	3100 lb.	88¢

Buying and Selling by the Thousand

ORAL EXERCISE

1. Compare $\overline{3500 \div 1000} \times \$9$ with $3.500 \times \$9$.
2. Compare $\overline{12200 \div 1000} \times \$5$ with $12.2 \times \$5$.
3. Find the cost of 7150 feet of lumber at \$11 per M.

Solution. M stands for thousand. 7150 feet are 7.15 times 1000 feet. Since 1 thousand feet of lumber cost \$11, 7.15 thousand feet will cost 7.15 times \$11, or \$78.65.

$$\begin{array}{r} 7.15 \\ 11 \\ \hline 78.65 \end{array}$$

Find the cost of:

4. 8500 tiles at \$8 per M; at \$9 per M.
5. 4500 bricks at \$6 per M; at \$7 per M.
6. 7500 shingles at \$12 per M; at \$14 per M.
7. 3200 ft. lumber at \$14 per M; at \$12 per M.
8. 15,000 ft. lumber at \$11 per M; at \$12 per M.
9. 12,000 ft. lumber at \$16 per M; at \$15 per M.

WRITTEN EXERCISE

1. Find the cost of 17,500 shingles at $4 per M.

2. What is the cost of 2700 envelopes at $2.25 per M ?

3. Find the cost of 27,560 feet of oak lumber at $21 per M.

4. Find the total cost of :

125 bolts at $2.75 per C.
750 bolts at $3.50 per C.
450 fence posts at $6 per C.
275 lb. nails at $3.50 per cwt.
750 lb. wire at $3.75 per cwt.
750 lb. guano at $4.75 per cwt.

5. Find the total cost of :

7600 shingles at $4 per M.
14,400 ft. plank at $9 per M.
24,560 bricks at $3.50 per M.
9000 tiles at $9.375 per M.
2320 ft. lumber at $23 per M.
1,270,500 bricks at $6.75 per M.

6. Find the total cost of :

760 lb. bran at $.60 per cwt.
5875 lb. bran at $.70 per cwt.
5970 lb. meal at $1.12 per cwt.
4275 lb. meal at $1.10 per cwt.
5600 lb. feed at $1.10 per cwt.
500 lb. oatmeal at $2.50 per cwt.

7. Find the total freight on :

8000 lb. oil at 70¢ per cwt.
1500 lb. fish at 58¢ per cwt.
5100 lb. salt at 73¢ per cwt.
4950 lb. ale at 52¢ per cwt.
9900 lb. beef at 72¢ per cwt.
4950 lb. pork at 57¢ per cwt.

8. Find the amount of the following bill :

Bangor, Me., July 27, 19—

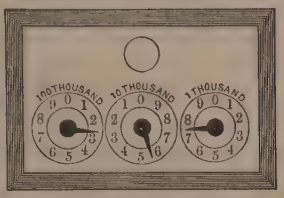
Messrs. Gray, Salisbury & Co.
Springfield, Mass.

Bought of McGraw, Eldridge & Co.

Terms 5% off if paid in 10 da.; 30 days net

		22000 ft. Clear Pine	23¢ per M			
		16000 Cedar Posts	6¢ per C			
		12500 ft. Pine Plank	20¢ per M			
		16500 Cedar R.R. Ties	12¢ per C			
		25000 Shingles	4¢ per M			
		92000 ft. Flooring	31¢ per M			

140. The accompanying illustration shows the three dials of a gas meter. Each division on the dial at the right denotes 100 cu. ft. of gas consumed; each division on the center dial 1000 cu. ft.; and each division on the dial at the left 10,000 cu. ft. The dials are read from left to right by simply taking the figures which the hands have just passed and adding two ciphers to them.



Thus, the above dial registers 20,000 cu. ft. + 5000 cu. ft. + 700 cu. ft. = 25,700 cu. ft.; but it is only necessary to write 257 (2, 5, 7) and add two ciphers to get this result.

WRITTEN EXERCISE

1. Read the accompanying meters and find the cost of the gas consumed during the period Jan. 1 to Feb. 1 at \$1.20 per 1000 cu. ft.

Jan. 1, 1906

Feb. 1, 1906

2. The following is the number of cubic feet of gas used in a residence for the six months ending July 1: January, 2900; February, 3200; March, 3700; April, 2900; May, 2700; June, 1200. Find the total gas bill for the six months at \$0.90 per 1000 cu. ft.

3. Assuming that gas is worth \$0.95 per 1000 cu. ft., find the amount of the following bill, less 10%.

GAS DEPARTMENT
87 West Street
Office Hours from 8 A.M. to 5 P.M.

Boston, Mass., Dec 1, 19—

Mr Fred M Allison
34 Atkinson St

To THE BOSTON GAS AND ELECTRIC LIGHT CO., Dr.

For Gas supplied by meter		
2210 0 cu. ft. as shown by Meter Dial Nov 29, 19—		
1860 0 cu. ft. as shown by Meter Dial Oct 30, 19—		
350 0 cu. ft. at \$1.00 per 1000 cu. ft.		350
Less 10%		35
		3 15

Discount of 10% allowed if paid on or before Dec 10, 19—

Received payment for the Company Dec 4, 19—
C. E. Bannigan

Buying and Selling by the Ton of 2000 Pounds

ORAL EXERCISE

1. Compare $\overline{8000 \div 2000} \times 8$ with $\overline{8000 \div 1000} \times 4$.
2. Compare $\overline{7000 \div 2000} \times 18$ with 7×9.
3. Find the cost of 4250 lb. coal at \$8 per ton.

Solution. 4250 lb. is 4.25 times 1000 lb. If the cost of 2 thousand pounds is \$8, the cost of 1 thousand pounds is \$4. Since 1 thousand pounds of coal cost \$4, 4.25 thousand pounds will cost 4.25 times \$4, or \$17.

$$\begin{array}{r} 4.25 \\ 4 \\ \hline 17.00 \end{array}$$

WRITTEN EXERCISE

1. At \$9 per ton, find the cost of the hay in the following weigh ticket. Also find the cost at \$8.75 per ton.

SCALES OF E. H. ROBINSON & CO.

No. 250 Clyde, N.Y., July 26, 19

Load of Hay

From C. A. Brown To W. D. Eaton

Gross weight 4810 lb.

Tare 1860 lb.

Net weight 2950 lb.

Geo. W. Ball
Weigher

2. At \$7.50 per ton find the cost of the coal in the following weigh ticket. Also find the cost at \$6.95 per ton.

WELLINGTON-WILD COAL CO.
126 Main Street, Rochester, N.Y.

No. 573 Nov. 16, 19

Mr. E. M. Robinson,
360 Monroe Ave., City

2930 lb. Stove Coal

Teamster Mann Received by E. M. Robinson

3. What will 8650 lb. of hay cost at $12 per ton?

4. Find the cost of 2150 lb. of coal at $6 per ton.

5. At $32 per ton, what is the cost of 26,480 lb. of phosphate?

6. Find the cost of 54,260 pounds of coal at $5.80 per ton.

7. Find the cost of 12 loads of coal weighing 4100, 3900, 4306, 4100, 4060, 4300, 3286, 3980, 3850, 4130, 3700, 3950 lb. net, at $5.20 per ton.

8. Find the total cost of: 5265 lb. hard coal at $8.40 per ton; 12,200 lb. soft coal at $3 per ton; 8275 lb. cannel coal at $11.75 per ton; 34,160 lb. egg coal at $6.20 per ton; 12,275 lb. nut coal at $5.75 per ton; 8753 lb. grate coal at $5.80 per ton; 24,160 lb. stove coal at $6.50 per ton.

9. During the month of January, in a recent year, there were consumed in a manufacturing plant 72 loads of coal weighing as follows: 6100, 6500, 6700, 6840, 7210, 6680, 7250, 8400, 6100, 6100, 6250, 6380, 6480, 6300, 6500, 6160, 6410, 6370, 6410, 6570, 6480, 6240, 6370, 6430, 6480, 6300, 7400, 7580, 7620, 7240, 7110, 7220, 7420, 7480, 6390, 6100, 6250, 6250, 6900, 6270, 6280, 6290, 6270, 6390, 6420, 6120, 6120, 6200, 6300, 6120, 6430, 6430, 8100, 6100, 6200, 6310, 6204, 6160, 6170, 6240, 6390, 6140, 6240, 7190, 7240, 7140, 7200, 6340, 8420, 6310, 7420, 6120 lb. net. Find the cost at 5.87\frac{1}{2}$ per ton.

WRITTEN REVIEW EXERCISE

1. Of what number is 25.56 both the divisor and quotient?

2. The sum of the divisor and quotient is 414.06. If the divisor is .6, what is the dividend?

3. In what time will 3 boys at $.75 per day earn as much as 2 men earn in 75 da. at $2.25 per day?

4. A merchant sold a quantity of flour for $370 and realized a gain of $34. If the selling price was $7.40 per barrel, what was the cost per barrel?

5. What number is that which is 165 times as great as 82.5?

6. If 450 bbl. of beef sold for $5872.50, what was the selling price per hundred barrels?

7. What will be the cost, at 15¢ per yard, of a paper border for a room 8 yd. wide and 12 yd. long?

8. An article was sold for $22.50; if $\frac{1}{6}$ of the cost was lost, what was the loss?

9. Wood costing $3.50 per cord is sold for $4.10 per cord. How many cords must be handled to gain $240?

10. Find the cost of 8 bbl. of pork weighing 280, 281, 286, 290, 285, 277, 285, and 290 lb. net, at $8.50 per hundred pounds.

11. A flock of 200 sheep was bought for $700. Ten of the sheep died, and the remainder of the flock was sold at $3.95 per head. What was the gain or loss?

12. If the actual cost of the necessities of life for one person, in a given year, amounted to $81.45, and 10 yr. later, owing to the advance in prices, the same necessities cost $108.60, what was the fractional increase in the cost of living?

13. A, B, and C bought a stock of goods for $7500, A contributing $2500, B $3000, and C the remainder. They sold the goods for $8400 and divided the profits equally. How much of the $8400 should A, B, and C, respectively, receive?

14. The following were the transactions of a merchant for 1 mo.: merchandise on hand July 1, $3378.50, sold for cash, $2374.20; bought for cash, $1945.35; sold on account, $2276.30; bought on account, $876.40; on hand July 31, $2056.35. Expenses for the month, $284.25. What was the net gain for the month?

15. The following were the transactions of a merchant for 1 mo.: merchandise on hand January 1, $8120.90; bought for cash, $3265.90; bought on account, $2845.10; sold for cash, $5157.65; sold on account, $4218.25; on hand January 31, $7253.25. The sale to Jas. S. Greet, on account, cannot be collected, $51.20. What was the net gain for the month?

16. What is the total freight on 12,250 lb. of hardware at \$.65 per hundredweight and 15,670 lb. of hardware at \$.60 per hundredweight?

17. A merchant bought 250 yd. of cloth at \$3.50 per yard, and 150 yd. at \$4.25. At what average price per yard should the whole be sold to realize an average profit of \$1 per yard?

18. What is the cost of 25 bbl. of sugar containing 312, 304, 309, 317, 330, 325, 315, 318, 317, 305, 319, 320, 325, 330, 335, 330, 325, 315, 315, 320, 320, 330, 330, 315, 315 lb. net, at $5\frac{3}{8}¢$ per pound?

19. A received \$1088 from the sale of his barley crop. If he received \$0.85 per bushel for the barley and his farm produced an average of 32 bu. to the acre, how many acres did it take to produce the barley?

20. A manufacturing pay roll shows that 40 hands are employed at \$1.45 per day, 50 hands at \$1.40 per day, 10 hands at \$3 per day, 40 hands at \$2.50 per day, and 5 hands at \$8 per day. Find the average daily wages.

21. A hardware merchant found that his stock of goods, Jan. 1, amounted to \$34,350.65. During the year he bought goods amounting to \$211,165.45, and sold goods amounting to \$220,540.45. Dec. 31, he took an account of stock and found that the goods on hand at cost prices were worth \$81,275.64. What was his gain or loss for the year?

22. Without copying the following figures, find (*a*) the sum of each line, and (*b*) the sum of each column. Prove the work by adding the line totals and comparing the sum with the sum of the column totals.

17.035	18.0135	186.02	126.42	6.009
8.005	5.07	142.004	.0634	3.14
32.972	18.0981	165.42	1.7538	9.314
126.83	4.931	.628	6.75	.048
95.16	6.815	.8467	8.41	.062
101.215	21.214	21.221	2.61	18.641

A REVIEW TEST

Without copying, find the quotients and the sum of the quotients, in each problem. Time, approximately, forty minutes.

In these problems the quotient in each division is apparent at a glance, hence the attention is fixed on placing the decimal point.

1.	2.	3.
12.5 ÷ 5	96 ÷ .4	3.9 ÷ .3
1.25 ÷ .5	.96 ÷ .004	39 ÷ .03
.125 ÷ .5	9.6 ÷ 4	3.9 ÷ .003
.125 ÷ .05	.96 ÷ .04	.0039 ÷ .003
.0125 ÷ 5	9.6 ÷ .04	.039 ÷ .0003
125 ÷ .5	960 ÷ .4	390 ÷ .3
12.5 ÷ .05	.096 ÷ .04	3.9 ÷ 3
.0125 ÷ .005	960 ÷ .004	390 ÷ .03
12.5 ÷ .005	9.6 ÷ .04	39 ÷ .003

4.	5.	6.
.0065 ÷ 1.3	.69 ÷ 23	.085 ÷ .17
6.5 ÷ 1.3	690 ÷ 2.3	.85 ÷ .017
.65 ÷ .13	6.9 ÷ 23	8.5 ÷ .017
65 ÷ .013	6.9 ÷ 2.3	.85 ÷ .0017
6.5 ÷ 130	.69 ÷ 230	85 ÷ .17
65 ÷ 130	.0069 ÷ .23	.085 ÷ .0017
6.5 ÷ 1300	6.9 ÷ 230	8.5 ÷ 1.7
.65 ÷ 13	.069 ÷ .0023	85 ÷ 1700
6500 ÷ 130	690 ÷ 2300	8.5 ÷ 170

7.	8.	9.
5.7 ÷ .19	75 ÷ .25	55 ÷ 1.1
57 ÷ .019	75 ÷ 2.5	550 ÷ 1.1
570 ÷ 1.9	.75 ÷ 2.5	.055 ÷ 110
57 ÷ 190	7.5 ÷ 250	5.5 ÷ 110
.0057 ÷ 1.9	.75 ÷ .25	55 ÷ 110
5.7 ÷ 190	75 ÷ .025	.055 ÷ .011
57 ÷ .19	.75 ÷ 250	550 ÷ .11
.57 ÷ 190	7.5 ÷ .025	550 ÷ 1100

CHAPTER XI

FACTORS, DIVISORS, AND MULTIPLES

FACTORS

ORAL EXERCISE

1. Name two factors of 63; of 88; of 144; of 128.
2. What are the factors of 49? of 77? of 35? of 21?
3. Name three factors of 45; of 66; of 24; of 60; of 80.
4. Name a factor that is common to 35 and 77; 36, 63, and 81.
5. Name three factors that are common to 30, 60, and 210.
6. Which of the following numbers have no factors except itself and one? 11, 27, 15, 37, 49, 62, 73, 81, 23.

141. An **even number** is an integer of which two is a factor. An **odd number** is an integer of which two is not a factor. A **prime number** is a number that has no integral factor except itself and one. A **composite number** is a number that has one or more integral factors besides itself and one.

Numbers are **mutually prime** when they have no common factor greater than one.

WRITTEN EXERCISE

1. Make a list of all the odd numbers from 1 to 100 inclusive; of all the prime numbers; of all the even numbers; of all the composite numbers.

ORAL EXERCISE

1. Is 2 a factor of 28? of 125? of 42? of 49? By what means do you readily determine this?
2. Is 5 a factor of 125? of 170? of 224? of 1255? of 1056? By what means do you readily determine this?
3. When is a number divisible by 10? by 3? by 9?

Tests of Divisibility of Numbers

142. A number is divisible by:

1. *Two*, when it is even, that is, when it ends with 0, 2, 4, 6, or 8.
2. *Three*, when the sum of its digits is divisible by 3.
3. *Four*, when the number expressed by its two right-hand figures is divisible by 4.
4. *Five*, when it ends with 0 or 5.
5. *Six*, when it is even and the sum of its digits is divisible by 3.
6. *Eight*, when the number expressed by the last three right-hand figures is divisible by 8.
7. *Nine*, when the sum of its digits is divisible by 9.
8. *Ten*, when its right-hand figure is a cipher.

ORAL EXERCISE

Name one or more factors of each of the following numbers:

1. 184.	**5.** 6984.	**9.** 51625.	**13.** 14128.
2. 2781.	**6.** 2750.	**10.** 83870.	**14.** 66438.
3. 1449.	**7.** 8975.	**11.** 13599.	**15.** 31284.
4. 638172.	**8.** 71168.	**12.** 123125.	**16.** 17375.

Factoring

143. Factoring is the process of separating a number into its factors.

144. Example. Find the prime factors of 780.

Solution. Since the number ends in a cipher, divide it by the prime factor 5; since the resulting quotient is an even number, divide it by 2. Since 78 is an even number, divide it by 2; since the sum of the digits in the resulting quotient is divisible by 3, divide by 3. The prime factors are then found to be 5, 2, 2, 3, and 13.

$$\begin{array}{r|l} 5 & 780 \\ 2 & 156 \\ 2 & 78 \\ 3 & 39 \\ & 13 \end{array}$$

WRITTEN EXERCISE

Find the prime factors of:

1. 112.	**4.** 786.	**7.** 968.	**10.** 408.	**13.** 2718.	**16.** 6900.
2. 126.	**5.** 392.	**8.** 689.	**11.** 650.	**14.** 3240.	**17.** 2064.
3. 288.	**6.** 315.	**9.** 1098.	**12.** 762.	**15.** 3205.	**18.** 7400.

Cancellation

ORAL EXERCISE

1. $(4 \times 15) \div (4 \times 3) = 15 \div 3$. Why?
2. Divide $\overline{2 \times 5 \times 7}$ by $\overline{5 \times 2}$; $\overline{8 \times 7 \times 5}$ by $\overline{5 \times 2 \times 7}$.
3. $\frac{3 \times 7 \times 8}{7 \times 3} = ?$ $\frac{5 \times 2 \times 8 \times 3}{2 \times 8 \times 3} = ?$ $\frac{2 \times 9 \times 7 \times 5}{5 \times 7 \times 2 \times 3} = ?$
4. What effect on the quotient has rejecting equal factors in both dividend and divisor?

145. Cancellation is the process of shortening computations by rejecting or canceling equal factors from both dividend and divisor.

146. Example. Divide the product of 6, 8, 12, 32, and 84 by the product of 3, 4, 6, and 24.

$$\frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{8}} \times \overset{2}{\cancel{12}} \times \overset{4}{\cancel{32}} \times \overset{28}{\cancel{84}}}{\underset{3}{\cancel{3}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{6}} \times \underset{8}{\cancel{24}}} = 2 \times 2 \times 2 \times 4 \times 28 = 896.$$

Solution. Do not form the products, but indicate the multiplication by the proper signs and write the divisor below the dividend as shown above. 3, 4, and 6 in the divisor are factors of 6, 8, and 12, respectively, in the dividend; hence, reject 3, 4, and 6 in the divisor and write 2, 2, and 2, respectively, in the dividend; then cancel the common factor 8 from 24 in the divisor and 32 in the dividend, retaining the factors 3 and 4, respectively; next cancel the common factor 3 in the divisor from 84 in the dividend and there remains the uncanceled factors 2, 2, 2, 4, and 28 in the dividend. Hence, the quotient is $2 \times 2 \times 2 \times 4 \times 28$, or 896.

WRITTEN EXERCISE

1. $\overline{14 \times 21 \times 48} \div \overline{7 \times 21 \times 6} = ?$
2. $\overline{128 \times 48 \times 88} \div \overline{64 \times 24 \times 4} = ?$
3. Divide $128 \times 18 \times 36$ by $64 \times 18 \times 12$.
4. $\frac{12 \times 16 \times 24 \times 8 \times 92 \times 28}{6 \times 8 \times 23 \times 7} = ?$

5. If 18 T. of hay cost $270, what will 25 T. cost at the same rate?

6. How many days' work at $2.75 will pay for 2 A. of land at $110 per acre?

7. If 75 bbl. of flour may be made from 375 bu. of wheat, how many bushels will be required to make 120 bbl. of flour?

8. If 45 men can complete a certain piece of work in 120 da., how many men can complete the same piece of work in 30 da.?

9. The freight on 350 lb. of evaporated apricots is $1.47. At that rate how much freight should be paid on 7350 lb. of evaporated apricots?

10. If 15 rm. of paper are required to print 400 copies of a book of 300 pp., how many reams will be required to print 32,000 copies of a book of 300 pp.?

DIVISORS AND MULTIPLES

Common Divisors

ORAL EXERCISE

1. Name a factor that is common to 35 and 49.
2. Name two factors that are common to both 48 and 64.
3. Name the greatest factor that is common to 75 and 100.

147. A **common divisor** is a factor that is common to two or more given numbers. The **greatest common divisor** (g. c. d.) is the greatest factor that is common to two or more given numbers.

148. Example. Find the g. c. d. of 24, 84, and 252.

Solutions. (*a*) Separate each of the numbers into its prime factors. The factor 2 occurs twice in all the numbers and the factor 3 once in all the numbers. None of the other factors occur in all the numbers; hence, $2 \times 2 \times 3$, or 12, is the greatest common divisor of 24, 84, and 252.

(*a*)

$24 = 2 \times 2 \times 2 \times 3$

$84 = 2 \times 2 \times 3 \times 7$

$252 = 2 \times 2 \times 3 \times 3 \times 7$

(*b*) The common prime factors of two or more given numbers may be found by dividing the numbers by their prime factors successively until the quotients contain no common factor, as shown in the margin.

(*b*)

2)24 — 84 — 252
2)12 — 42 — 126
3)6 — 21 — 63
2 — 7 — 21

Ever since decimal fractions came into quite general use the subject of greatest common divisor has been stripped of most of its practical value. When fractions like $\frac{1261}{1849}$ were quite generally used, it was necessary to reduce them to their lowest terms before they could be conveniently handled in an operation. For this purpose, the greatest common divisor (here 97) was found and canceled from each term, thus greatly simplifying the fraction (here $\frac{13}{19}$). Now, however, the greatest common divisor of the terms of the fractions used in business is easily found by inspection, and the need for finding the greatest common divisor is slight.

ORAL EXERCISE

1. What is the greatest common divisor of 65 and 75? of 12 and 32? of 75 and 125?

2. What is the greatest common divisor of 12, 30, and 96? of 8, 24, and 42? of 36, 90, and 96?

3. What divisor should be used in reducing $\frac{75}{125}$ to its lowest terms? $\frac{128}{640}$? $\frac{64}{320}$? $\frac{16}{160}$? $\frac{48}{240}$? $\frac{72}{240}$?

WRITTEN EXERCISE

Find the greatest common divisor of:

1. 48, 240. 2. 42, 28, 144. 3. 88, 144, 220.

4. A real estate dealer has four plots of land which he wishes to divide into the largest number of building lots of the same size. If the plots contain 168, 280, 182, and 252 square rods, respectively, how many square rods will there be in each building lot?

Common Multiples

ORAL EXERCISE

1. Name a multiple of 7; of 9; of 16; of 64.

2. Name two other multiples of each of the above numbers.

3. Name two multiples that are common to 3 and 4; to 5 and 9; to 8 and 12. Which of the multiples just named is the least common multiple?

149. A **common multiple** is any integral number of times two or more given numbers. The **least common multiple** (l. c. m.) of two or more numbers is the least number which is an integral number of times each of the given numbers.

150. Example. Find the l. c. m. of 28, 42, and 84.

SOLUTIONS. (*a*) Resolve each of the numbers into its prime factors. The factor 2 occurs twice in 28 and in 84, the factor 3 occurs once in 42 and 84, the factor 7 occurs once in each of the numbers. Therefore, the least common multiple is $2 \times 2 \times 3 \times 7$, or 84; or

(*a*)

$28 = 2 \times 2 \times 7$

$42 = 2 \times 3 \times 7$

$84 = 2 \times 2 \times 3 \times 7$

(*b*) Arrange the numbers in a horizontal line and divide by any prime factor that will exactly divide any two of them. Divide the numbers in the resulting quotient by any prime factor that will divide any two of them, and so continue the operation until quotients are found that are prime to each other. Find the product of the several divisors and the last quotients and the result is the l.c.m. $2 \times 2 \times 3 \times 7 = 84$, the l. c. m.

(*b*)

2)	28	42	84
2)	14	21	42
3)	7	21	21
7)	7	7	7
	1	1	1

All numbers that are factors of other given numbers may be disregarded in finding the l. c. m. Thus the common multiples of 4, 8, 16, 32, 64, and 80 are the same as the multiples of 64 and 80.

ORAL EXERCISE

State the least common multiple of:

1. 6, 5, and 3.
2. 6, 8, 12, and 24.
3. 4, 5, 15, and 30.
4. 2, 4, 7, 8, 48, 24.
5. 6, 42, 84, 168, 336.
6. 5, 15, 75, 150, 300.

WRITTEN EXERCISE

Find the least common multiple of:

1. 6, 7, 8, and 5.
2. 6, 18, 24, and 84.
3. 12, 24, 36, and 96.
4. 32, 46, 92, and 128.
5. 4, 20, 12, and 48.
6. 62, 78, 30, and 142.
7. 35, 105, 125, and 225.
8. 114, 240, 72, and 320.
9. What number is that of which 2, 3, 5, and 11 are the only prime factors?

CHAPTER XII

COMMON FRACTIONS

ORAL EXERCISE

1. When a quantity is divided into 3 equal parts, what is each part called? into 8 equal parts? into 12 equal parts?

2. The shaded part of A is what part of the whole hexagon? the shaded part of B? the shaded part of C?

3. In the shaded part of A how many sixths? in the shaded part of B?

4. One half of the hexagon is how many sixths of it? How many sixths in the whole hexagon?

5. In the unshaded part of B how many thirds? Two thirds are how many sixths?

6. In the unshaded part of C how many sixths?

7. Read the following fractions in the order of their size, the largest first: $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$.

8. Complete the following statement: Such parts of a unit as .5, .25, $\frac{1}{3}$, $\frac{5}{6}$, etc., are called ——.

151. Common fractions are expressed by two numbers, one written above and one below a short horizontal line.

152. The number written above the line is called the **numerator** of the fraction, and the number written below, the **denominator** of the fraction.

153. The *numerator* tells the *number of parts* expressed by the fraction; the *denominator names the parts* expressed by the fraction.

Thus, in the fraction $\frac{3}{4}$, 4 tells that a number has been divided into four equal parts and 3 shows that three of these parts have been taken.

154. It is clear that the *greater the number* of equal parts into which a unit is divided, the *smaller* is each part; and the *fewer* equal parts into which a unit is divided, the *greater* the size of each part. Hence,

Of two fractions having the same denominator, the one having the greater numerator expresses the greater value; and

Of two fractions having the same numerator, the one having the smaller denominator expresses the greater value.

155. The **terms of a fraction** are the numerator and denominator.

156. A **unit fraction** is a fraction whose numerator is one.

Thus $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ are unit fractions. $\frac{1}{3}$ in. is read *one third of an inch.*

157. An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator.

Thus, $\frac{6}{6}$, $\frac{5}{3}$, and $\frac{25}{5}$ are improper fractions. The value of an improper fraction is always equal to or greater than one.

158. A **mixed number** is the sum of a whole number and a fraction.

Thus, $2\frac{1}{7}$ and $4\frac{2}{3}$, read *two and one seventh* and *four and two fifths*, are mixed numbers.

ORAL EXERCISE

1. What takes the place of the denominator in .5? in .25?

2. Read aloud the following fractions in the order of their size, the largest first: $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{100}$.

3. Read aloud the following fractions in the order of their size, the smallest first: $\frac{2}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{2}{7}$, $\frac{1}{3}$, $\frac{2}{9}$, $\frac{1}{10}$, $\frac{1}{6}$, $\frac{4}{7}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{7}$.

4. Read aloud the following: $\frac{1}{2}$ mi.; $\frac{3}{4}$ T.; $27\frac{1}{2}$ yd.; $\frac{1}{1728}$ cu. ft.; $275\frac{3}{4}$ A.; $250\frac{5}{16}$ lb.; £$18\frac{9}{20}$; £$271\frac{1}{4}$; $\frac{1}{144}$ sq. ft.

5. Of all the cotton produced in the United States in a recent year the principal cotton-growing states contributed approximately as follows: North Carolina, $\frac{1}{14}$; South Carolina, $\frac{1}{8}$; Georgia, $\frac{1}{6}$; Oklahoma and Indian Territory, $\frac{1}{15}$; Alabama, $\frac{1}{9}$; Mississippi, $\frac{1}{13}$; Louisiana, $\frac{1}{40}$; Texas, $\frac{1}{4}$; Arkansas, $\frac{1}{17}$; Tennessee, $\frac{1}{36}$. Name the principal cotton-growing states, in the order of production, for that year.

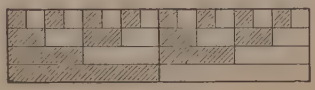
REDUCTION

To Higher Terms

ORAL EXERCISE

1. How many halves in 1? how many fourths? how many eighths? how many sixteenths?

2. How many fourths in $\frac{1}{2}$? how many eighths? how many sixteenths?



3. How many eighths in $\frac{1}{4}$? how many sixteenths?

4. How many fourths in $\frac{2}{16}$? how many eighths in $\frac{4}{16}$? how many halves in $\frac{8}{16}$?

5. What effect is produced upon the value of a fraction by multiplying or dividing both terms of a fraction by the same number?

6. Change 14 gal. to quarts. Compare the size of the units in 14 gal. with the size of the units in 56 qt.; the number of units; the value of the two numbers.

7. Change $\frac{1}{2}$ to twelfths; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$.

8. Name three fractions equal in value to $\frac{1}{8}$; to $\frac{2}{3}$; to $\frac{3}{4}$.

159. It has been seen that *multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.*

160. A fraction is reduced to **higher terms** when the given numerator and denominator are expressed in larger numbers.

ORAL EXERCISE

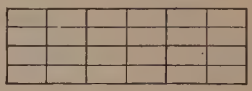
1. Reduce to twelfths: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$.
2. Reduce to sixteenths: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{5}{8}$.
3. Reduce to twentieths: $\frac{3}{4}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{3}{5}$, $\frac{4}{5}$.
4. Reduce to twenty-fourths: $\frac{3}{8}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{3}{4}$, $\frac{7}{8}$.
5. Reduce to thirty-seconds: $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{3}{16}$, $\frac{1}{16}$, $\frac{9}{16}$, $\frac{7}{16}$.
6. Reduce to one-hundredths: $\frac{3}{4}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{4}{25}$, $\frac{9}{20}$, $\frac{1}{4}$, $\frac{7}{25}$.
7. Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to fractions having the denominator 24.

To Lowest Terms

ORAL EXERCISE

1. $\frac{8}{24}$ equals how many thirds? $\frac{12}{24}$ equals how many halves?

2. Name the largest possible unit fraction. Why is this the largest possible unit fraction?



3. Change $\frac{6}{12}$ to the largest possible unit fraction; $\frac{8}{16}$; $\frac{25}{100}$; $\frac{50}{200}$; $\frac{125}{1000}$. Express $\frac{14}{16}$ in its simplest form. Reduce $\frac{25}{200}$ to its lowest terms.

161. A fraction is reduced **to its lowest terms** when the numerator and denominator are changed to numbers that are mutually prime.

162. Example. Reduce $\frac{96}{108}$ to its lowest terms.

Solution. 6 is a common factor of 96 and 108; dividing both terms by 6, the result is $\frac{16}{18}$. 2 is a common factor of 16 and 18; dividing both terms by 2, the result is $\frac{8}{9}$.

$$\frac{96}{108} = \frac{16}{18} = \frac{8}{9}$$

ORAL EXERCISE

1. Reduce to fifteenths: $\frac{1}{3}$, $\frac{2}{5}$, $\frac{2}{3}$, $\frac{4}{5}$.
2. Reduce to eighths: $\frac{9}{24}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{12}{32}$, $\frac{14}{16}$, $\frac{1}{4}$.
3. Reduce to fiftieths: $\frac{1}{2}$, $\frac{3}{5}$, $\frac{24}{100}$, $\frac{7}{10}$, $\frac{8}{25}$, $\frac{16}{200}$.
4. Change to twentieths: $\frac{1}{2}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{9}{10}$, $\frac{3}{5}$.
5. Reduce to lowest terms: $\frac{6}{16}$, $\frac{8}{10}$, $\frac{8}{12}$, $\frac{14}{16}$, $\frac{9}{12}$, $\frac{6}{8}$.

WRITTEN EXERCISE

1. Reduce to sixteenths: $\frac{110}{160}$, $\frac{180}{320}$, $\frac{7}{8}$, $\frac{25}{80}$, $\frac{3}{4}$, $\frac{160}{640}$.
2. Reduce to lowest terms: $\frac{720}{1728}$ cu. ft., $\frac{48}{160}$ A., $\frac{448}{2240}$ T.
3. Reduce to lowest terms: $\frac{100}{320}$ mi., £$\frac{165}{240}$, $\frac{48000}{57600}$ lb., $\frac{28}{80}$ mi.
4. Reduce to three-hundred-twentieths: $\frac{1}{8}$ mi., $\frac{1}{4}$ mi., $\frac{1}{16}$ mi.
5. Reduce to their simplest common fractional form: $\frac{1840}{2240}$ T., $\frac{1260}{2400}$ T., $\frac{92}{160}$ A., $\frac{124}{160}$ A., $\frac{360}{640}$ sq. mi., $\frac{492}{640}$ sq. mi., $\frac{248}{320}$ mi.

Integers and Mixed Numbers to Improper Fractions

ORAL EXERCISE

1. How many quarts in 1 gal.? in 3 gal.?
2. How many sixths in 1? in 3? in 5? in 7?
3. How many fifths in 1? in $1\frac{1}{5}$? in $1\frac{3}{5}$? in $3\frac{1}{5}$?
4. Express as fourths: $6\frac{1}{4}$, $12\frac{3}{4}$, 13, 87, $61\frac{1}{4}$, $28\frac{1}{2}$.
5. Express as eighths: 15, 12, $10\frac{1}{2}$, $1\frac{1}{4}$, $2\frac{5}{8}$, $1\frac{3}{8}$, $9\frac{7}{8}$.
6. Express as halves: 27, 14, $30\frac{1}{2}$, $171\frac{1}{2}$, $182\frac{1}{2}$, 249.

WRITTEN EXERCISE

Reduce to improper fractions:

1. $83\frac{1}{3}$.
2. $166\frac{2}{3}$.
3. $333\frac{1}{3}$.
4. $666\frac{2}{3}$.
5. $180\frac{5}{32}$.
6. $212\frac{1}{64}$.
7. $265\frac{1}{48}$.
8. $319\frac{5}{16}$.
9. $146\frac{15}{32}$.
10. $3150\frac{5}{9}$.
11. $1625\frac{1}{6}$.
12. $2150\frac{5}{12}$.

Improper Fractions to Integers or Mixed Numbers

ORAL EXERCISE

1. How many pecks in 240 qt.? $\frac{240}{8} = ?$ $\frac{129}{3} = ?$
2. Change to integers: $\frac{160}{4}$, $\frac{153}{9}$, $\frac{192}{16}$, $\frac{288}{12}$, $\frac{4860}{60}$, $\frac{1920}{32}$.
3. Express $28\frac{1}{4}$ as fourths; express $\frac{113}{4}$ as a mixed number.
4. Change to mixed numbers: $\frac{27}{4}$, $\frac{173}{3}$, $\frac{157}{8}$, $\frac{161}{4}$, $\frac{214}{3}$.
5. What is the value of: $\frac{288}{12}$ lb.? $\frac{128}{16}$ lb.? $\frac{168}{4}$ bu.? $\frac{244}{8}$ pk.? $\frac{366}{12}$ ft.? $\frac{4800}{160}$ A.? $\frac{960}{320}$ mi.? $\frac{250}{12}$ lb.? $\frac{720}{144}$ sq. ft.?

WRITTEN EXERCISE

Reduce to integers or mixed numbers:

1. $\frac{856}{320}$ mi.
2. $\frac{1930}{160}$ A.
3. $\frac{9655}{2240}$ T.
4. $\frac{1728}{160}$ A.
5. $\frac{82275}{2000}$ T.
6. $\frac{69223}{2000}$ T.
7. $\frac{2456}{16}$ lb.
8. $\frac{9504}{1728}$ cu. ft.
9. $\frac{9200}{640}$ sq. mi.

163. When expressing **final results** reduce all proper fractions to their lowest terms and all improper fractions to integers or mixed numbers.

To Least Common Denominator

ORAL EXERCISE

1. How many pounds in 1 T. 500 lb.? 5 T. + 1000 lb. = ? lb. 5 T. 1000 lb. = ? T.

2. How must numbers be expressed before they can be added or subtracted?

3. $\frac{1}{2} = \frac{?}{8}$; $\frac{1}{2} + \frac{3}{8} = ?$ $\frac{1}{4} = \frac{?}{16}$; $\frac{1}{4} - \frac{1}{16} = \frac{?}{16}$; $\frac{1}{3} = \frac{?}{6}$; $\frac{1}{3} - \frac{1}{6} = ?$

4. What kind of fractions can be added or subtracted?

5. Express $\frac{3}{4}$ as sixteenths. Add $\frac{3}{4}$ and $\frac{5}{16}$; $\frac{1}{4}$ and $\frac{9}{16}$; $\frac{3}{4}$ and $\frac{7}{8}$.

6. Express $\frac{1}{2}$ as eighths. Subtract $\frac{1}{2}$ and $\frac{3}{8}$; $\frac{1}{2}$ and $\frac{3}{16}$; $\frac{3}{4}$ and $\frac{1}{16}$.

164. Two or more fractions whose denominators are the same are said to have a **common denominator;** if this denominator is the smallest possible, the fractions are said to have a **least common denominator.** Two or more fractions having the same denominator are sometimes called **similar fractions.**

ORAL EXERCISE

Change to similar fractions:

1. $\frac{1}{2}, \frac{1}{4}$.
2. $\frac{1}{2}, \frac{1}{3}$.
3. $\frac{1}{3}, \frac{1}{5}$.
4. $\frac{2}{3}, \frac{1}{2}$.
5. $\frac{3}{5}, \frac{4}{10}$.
6. $\frac{1}{5}, \frac{1}{4}$.
7. $\frac{2}{3}, \frac{3}{4}$.
8. $\frac{1}{2}, \frac{4}{5}$.
9. $\frac{3}{5}, \frac{2}{3}$.
10. $\frac{2}{5}, \frac{7}{8}$.
11. $\frac{2}{3}, \frac{7}{9}$.
12. $\frac{1}{2}, \frac{3}{16}$.
13. $\frac{2}{5}, \frac{7}{10}$.
14. $\frac{3}{4}, \frac{9}{10}$.
15. $\frac{3}{8}, \frac{7}{10}$.
16. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$.
17. $\frac{1}{3}, \frac{1}{9}, \frac{4}{5}$.
18. $\frac{1}{4}, \frac{1}{8}, \frac{5}{32}$.
19. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$.
20. $\frac{1}{2}, \frac{3}{8}, \frac{5}{16}$.

WRITTEN EXERCISE

Change to fractions having the least common denominator:

1. $\frac{7}{8}, \frac{5}{32}, \frac{18}{64}$.
2. $\frac{3}{5}, \frac{9}{10}, \frac{1}{25}$.
3. $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{9}$.
4. $\frac{2}{5}, \frac{7}{9}, \frac{3}{10}, \frac{2}{3}$.
5. $\frac{5}{6}, \frac{7}{8}, \frac{5}{12}, \frac{7}{32}$.
6. $\frac{2}{3}, \frac{3}{5}, \frac{7}{36}, \frac{5}{48}$.
7. $\frac{1}{4}, \frac{7}{16}, \frac{9}{32}, \frac{17}{64}$.
8. $\frac{9}{10}, \frac{5}{24}, \frac{7}{12}, \frac{1}{8}$.
9. $\frac{1}{12}, \frac{5}{9}, \frac{2}{3}, \frac{1}{48}$.
10. $\frac{11}{16}, \frac{5}{32}, \frac{1}{4}, \frac{19}{64}$.
11. $\frac{15}{320}, \frac{3}{4}, \frac{17}{160}, \frac{5}{8}$.
12. $\frac{10}{640}, \frac{6}{16}, \frac{1}{32}, \frac{5}{32}$.

Change the fractions to form for addition or subtraction:

13. $31\frac{5}{16}, 7\frac{1}{10}$.
14. $134\frac{1}{64}, 112\frac{5}{20}$.
15. $6126\frac{5}{12}, 178\frac{5}{16}$.

ADDITION

165. It has been seen that *only like numbers and parts of like units can be added.*

ORAL EXERCISE

State the sum of:

1. $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$.
2. $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{4}$.
3. $\frac{1}{7}$, $\frac{3}{7}$, $\frac{4}{7}$.
4. $\frac{2}{11}$, $\frac{5}{11}$, $\frac{6}{11}$.
5. $\frac{1}{5}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{7}{5}$.
6. $\frac{1}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$.
7. $2\frac{1}{2}$, $3\frac{1}{2}$, $12\frac{1}{2}$, $19\frac{1}{2}$.
8. $5\frac{1}{4}$, $12\frac{1}{4}$, $7\frac{1}{4}$, $10\frac{1}{4}$.
9. $7\frac{7}{8}$, $2\frac{5}{8}$, $8\frac{1}{8}$, $1\frac{1}{8}$, $2\frac{1}{8}$.
10. $2\frac{1}{3}$, $5\frac{2}{3}$, $8\frac{1}{3}$, $12\frac{1}{3}$, $10\frac{2}{3}$.
11. $1\frac{1}{6}$, $10\frac{5}{6}$, $15\frac{1}{6}$, $18\frac{1}{6}$, $12\frac{1}{6}$.
12. $5\frac{7}{16}$, $2\frac{9}{16}$, $1\frac{5}{16}$, $8\frac{3}{16}$, $3\frac{1}{16}$.

By horizontal addition find the sum of:

13. 2 pieces of gingham containing 41^1 and 43^2 yd.

In the dry-goods business fourths (quarters) are very common fractions. They are usually written without denominators by placing the numerators a little above the integers. Thus, 51^1 equals $51\frac{1}{4}$, 54^2 equals $54\frac{2}{4}$ ($54\frac{1}{2}$), and 52^3 equals $52\frac{3}{4}$.

14. 4 pc. stripe containing 42^1, 38^1, 40^2, and 49 yd.
15. 3 pc. fancy plaid containing 42^1, 40^2, and 41 yd.
16. 4 pc. duck containing 48^1, 47^3, 46^2, and 40^2 yd.
17. 2 pc. monument cotton containing 54^2 and 55^2 yd.
18. 4 pc. dress silk containing 32^1, 34^2, 35^3, and 32^2 yd.

166. Examples. 1. Find the sum of $\frac{7}{8}$ and $\frac{2}{5}$.

SOLUTION. $\frac{7}{8}$ and $\frac{2}{5}$ are not similar fractions; hence, make them similar by reducing them to equivalent fractions having a least common denominator. $\frac{7}{8} = \frac{35}{40}$ and $\frac{2}{5} = \frac{16}{40}$. $\frac{35}{40} + \frac{16}{40} = \frac{51}{40} = 1\frac{11}{40}$.

l. c. m. of 8 and 5 = 40

$\frac{7}{8} = \frac{35}{40}$; $\frac{2}{5} = \frac{16}{40}$

$\frac{35}{40} + \frac{16}{40} = \frac{51}{40} = 1\frac{11}{40}$

2. Find the sum of $56\frac{1}{3}$, $34\frac{1}{8}$, $52\frac{3}{4}$.

SOLUTION. By inspection determine the least common denominator of the given fractions; then make the fractions similar and add them, as shown in the margin. The result is $1\frac{5}{24}$, which added to the sum of the integers equals $143\frac{5}{24}$, the required result.

$56\frac{1}{3} = 8$

$34\frac{1}{8} = 3$

$52\frac{3}{4} = \frac{18}{29}$

$143\frac{5}{24}$ $\quad \frac{29}{24} = 1\frac{5}{24}$.

WRITTEN EXERCISE

Find the sum of:

1. $\frac{7}{16}$, $\frac{5}{8}$.
2. $\frac{9}{16}$, $\frac{17}{64}$.
3. $2\frac{1}{8}$, $17\frac{1}{4}$.
4. $12\frac{1}{2}$, $19\frac{5}{16}$.
5. $\frac{2}{3}$, $4\frac{1}{5}$, $19\frac{1}{4}$.
6. $2\frac{1}{2}$, $4\frac{3}{4}$, $25\frac{9}{16}$.
7. $12\frac{3}{5}$, $172\frac{7}{10}$.
8. $8\frac{1}{4}$, $\frac{3}{5}$, $\frac{18}{10}$, $27\frac{1}{8}$.
9. $52\frac{5}{8}$, $59\frac{1}{2}$, $57\frac{3}{4}$, $52\frac{1}{16}$.
10. $60\frac{5}{8}$, $18\frac{3}{4}$, $21\frac{5}{16}$, $142\frac{1}{16}$.
11. $20\frac{1}{4}$, $12\frac{1}{5}$, $18\frac{1}{8}$, $92\frac{1}{5}$, $75\frac{3}{7}$.
12. $140\frac{3}{4}$, $260\frac{1}{8}$, $145\frac{1}{2}$, $216\frac{7}{8}$, $390\frac{1}{3}$.

13. A carpet dealer sold at different times $125\frac{5}{8}$ yd., $272\frac{1}{4}$ yd., $169\frac{1}{2}$ yd., $186\frac{3}{4}$ yd., $241\frac{1}{4}$ yd., $265\frac{1}{8}$ yd., $296\frac{7}{8}$ yd., and $314\frac{5}{8}$ yd. of Axminster carpet, at \$2.65 per yard. If it cost him \$2.45 per yard, what was his gain?

14. A dry-goods merchant bought 50 pc. of dress silk at \$1 per yard. If the pieces contained 42^1, 43^2, 44^2, 47^3, 44^1, 45^2, 40^3, 46^2, 45^1, 42, 47^1, 48^2, 40^3, 40^1, 40^2, 40^3, 50^2, 40^3, 47^2, 48^3, 40^3, 45^1, 40^2, 45^2, 44^2, 47^3, 46^2, 41^1, 51^3, 42^3, 53^2, 57^2, 53^1, 51^1, 48^3, 47^2, 40^1, 45^2, 45^2, 40^3, 40^1, 45^3, 47^2, 48^1, 51^1, 52^2, 57^2, 61^3, 60^2, 50^1 yd., respectively, and he sold the entire purchase at \$1.25 per yard, what was his gain?

Short Methods in Addition

ORAL EXERCISE

1. $\frac{1}{7}+\frac{1}{9}=\frac{16}{63}$. Observe that the numerator of the sum is equal to the sum of the denominators in the given fractions.

2. $\frac{1}{5}+\frac{1}{8}=?$ Give a short method for adding any two simple fractions whose numerators are 1.

3. $\frac{2}{7}+\frac{2}{5}=\frac{24}{35}$. Observe that the numerator of the sum is equal to the sum of the denominators multiplied by the numerator of either of the given fractions.

4. $\frac{2}{3}+\frac{2}{7}=?$ Give a short method for adding any two fractions whose numerators are alike.

5. Find the sum of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

SOLUTION. $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$; $\frac{7}{12}+\frac{1}{5}=\frac{47}{60}$, the required result.

ORAL EXERCISE

State the sum of:

1. $\frac{1}{2}, \frac{1}{3}$.
2. $\frac{1}{4}, \frac{1}{5}$.
3. $\frac{1}{5}, \frac{1}{6}$.
4. $\frac{1}{7}, \frac{1}{8}$.
5. $\frac{1}{9}, \frac{1}{6}$.
6. $\frac{1}{3}, \frac{1}{5}$.
7. $\frac{1}{2}, \frac{1}{8}$.
8. $\frac{1}{4}, \frac{1}{6}$.
9. $\frac{1}{9}, \frac{1}{7}$.
10. $\frac{2}{7}, \frac{2}{9}$.
11. $\frac{2}{5}, \frac{2}{7}$.
12. $\frac{2}{5}, \frac{2}{3}$.
13. $\frac{1}{4}, \frac{1}{3}$.
14. $\frac{3}{8}, \frac{3}{7}$.
15. $\frac{4}{5}, \frac{4}{9}$.
16. $\frac{5}{8}, \frac{5}{9}$.
17. $\frac{3}{4}, \frac{3}{5}$.
18. $\frac{4}{7}, \frac{4}{5}$.
19. $\frac{7}{9}, \frac{7}{8}$.
20. $\frac{6}{7}, \frac{6}{10}$.
21. $\frac{1}{5}, \frac{1}{8}, \frac{1}{3}$.
22. $\frac{1}{4}, \frac{1}{6}, \frac{1}{2}$.
23. $\frac{1}{9}, \frac{1}{8}, \frac{1}{7}$.
24. $\frac{2}{3}, \frac{2}{5}, \frac{2}{9}$.

167. The most common business fractions are usually small and of such a nature that they may be added with equally as much ease as integers. The following exercise will be found helpful to the student in learning to add these fractions in practically the same manner that he adds integers.

168. Example. Find the sum of $\frac{5}{16}$, $\frac{1}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$.

SOLUTION. By inspection determine that the least common denominator is 16. Then mentally reduce each fraction to 16ths and add as in whole numbers. Thus, 5, 7, 19, $\frac{27}{16}$, $1\frac{11}{16}$.

ORAL EXERCISE

Find the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{7}{9}$
$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{7}{9}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{2}{3}$
$\frac{1}{2}$	$\frac{1}{9}$	$\frac{3}{10}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{6}$

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{1}{3}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{3}{7}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{4}{7}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$
$\frac{4}{5}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{5}{4}$
$\frac{2}{3}$	$\frac{7}{10}$	$\frac{3}{8}$	$\frac{3}{7}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{12}$	$\frac{5}{7}$	$\frac{1}{6}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{1}{5}$	$\frac{7}{10}$
$\frac{1}{3}$	$\frac{9}{10}$	$\frac{5}{12}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{7}{8}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{3}{10}$
$\frac{4}{5}$	$\frac{7}{10}$	$\frac{7}{12}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{11}{16}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{10}$

Exercises similar to the foregoing should be continued until the student can name the successive results in the addition without hesitation.

169. The ordinary mixed numbers that come to an accountant should be arranged for addition practically the same as integers. In adding, the fractions should be combined first and then the integers.

170. Example. Find the sum of $2\frac{1}{2}$, $5\frac{1}{4}$, and $3\frac{7}{8}$.

SOLUTION. By inspection determine that the least common denominator of the fractions is 8. Mentally find the sum of the fractions and the result is $1\frac{5}{8}$. Add this result to the integers and the entire sum is $11\frac{5}{8}$.

$$\begin{array}{r} 2\frac{1}{2} \\ 5\frac{1}{4} \\ 3\frac{7}{8} \\ \hline 11\frac{5}{8} \end{array}$$

ORAL EXERCISE

State the sum of:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$2\frac{1}{2}$	$3\frac{1}{3}$	$3\frac{1}{4}$	$8\frac{1}{5}$	$14\frac{1}{2}$	$5\frac{3}{4}$	$4\frac{1}{5}$	$2\frac{7}{8}$	$3\frac{1}{3}$	$14\frac{1}{7}$
$3\frac{1}{4}$	$2\frac{1}{6}$	$5\frac{1}{3}$	$7\frac{1}{2}$	$17\frac{3}{4}$	$13\frac{1}{2}$	$7\frac{1}{3}$	$16\frac{2}{5}$	$17\frac{1}{8}$	$16\frac{2}{9}$

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$9\frac{3}{4}$	$5\frac{2}{3}$	$1\frac{1}{6}$	$1\frac{3}{7}$	$8\frac{1}{5}$	$4\frac{1}{4}$	$5\frac{1}{3}$	$4\frac{1}{9}$	$4\frac{1}{5}$	$4\frac{1}{2}$
$10\frac{1}{2}$	$4\frac{1}{4}$	$2\frac{5}{6}$	$6\frac{1}{2}$	$3\frac{3}{5}$	$2\frac{1}{3}$	$2\frac{2}{3}$	$5\frac{2}{3}$	$2\frac{2}{3}$	$1\frac{3}{8}$
$13\frac{1}{4}$	$11\frac{1}{2}$	$7\frac{7}{12}$	$5\frac{1}{2}$	$2\frac{1}{10}$	$4\frac{1}{5}$	$4\frac{1}{9}$	$6\frac{5}{6}$	$6\frac{4}{5}$	$7\frac{3}{4}$
$10\frac{1}{4}$	$12\frac{1}{12}$	$8\frac{5}{12}$	$13\frac{4}{7}$	$4\frac{9}{10}$	$3\frac{3}{4}$	$6\frac{7}{9}$	$2\frac{1}{3}$	$3\frac{3}{10}$	$12\frac{1}{4}$

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
$4\frac{1}{2}$	$1\frac{1}{2}$	$5\frac{1}{6}$	$3\frac{1}{4}$	$8\frac{1}{4}$	$4\frac{1}{2}$	$5\frac{2}{3}$	$4\frac{7}{9}$	$4\frac{1}{2}$	$4\frac{1}{8}$
$5\frac{1}{4}$	$5\frac{2}{3}$	$2\frac{2}{3}$	$1\frac{3}{7}$	$6\frac{3}{4}$	$2\frac{1}{8}$	$1\frac{1}{6}$	$1\frac{1}{9}$	$2\frac{1}{5}$	$6\frac{1}{2}$
$3\frac{3}{4}$	$5\frac{3}{4}$	$2\frac{5}{6}$	$2\frac{5}{7}$	$7\frac{1}{5}$	$4\frac{1}{5}$	$9\frac{5}{6}$	$7\frac{5}{6}$	$6\frac{2}{3}$	$9\frac{3}{8}$
$6\frac{7}{8}$	$5\frac{2}{7}$	$7\frac{3}{4}$	$6\frac{1}{2}$	$2\frac{3}{5}$	$3\frac{1}{4}$	$4\frac{1}{9}$	$5\frac{1}{3}$	$6\frac{4}{5}$	$7\frac{5}{8}$
$3\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$7\frac{1}{2}$	$3\frac{1}{5}$	$3\frac{1}{2}$	$7\frac{5}{9}$	$2\frac{2}{3}$	$9\frac{3}{5}$	$8\frac{3}{4}$
$4\frac{1}{4}$	$1\frac{1}{3}$	$2\frac{5}{6}$	$2\frac{5}{7}$	$5\frac{1}{5}$	$3\frac{1}{8}$	$3\frac{7}{9}$	$7\frac{1}{6}$	$4\frac{1}{5}$	$2\frac{1}{2}$
$3\frac{1}{8}$	$4\frac{1}{3}$	$8\frac{1}{12}$	$3\frac{4}{7}$	$2\frac{1}{4}$	$3\frac{1}{5}$	$2\frac{2}{3}$	$9\frac{5}{6}$	$3\frac{3}{10}$	$6\frac{1}{4}$
$5\frac{1}{8}$	$2\frac{7}{12}$	$8\frac{5}{12}$	$13\frac{3}{7}$	$5\frac{7}{10}$	$3\frac{1}{4}$	$12\frac{1}{3}$	$15\frac{2}{3}$	$9\frac{7}{10}$	$12\frac{1}{2}$

Exercises similar to the above should be continued until the student can add with great facility. If the principles of grouping have not been well mastered, simple addition should be carefully reviewed.

WRITTEN EXERCISE

Copy or write from dictation and find the sum of:

1.	2.	3.	4.	5.	6.
$1649\frac{1}{3}$	$1672\frac{3}{4}$	$1436\frac{1}{2}$	$2110\frac{1}{3}$	$6214\frac{1}{8}$	$1214\frac{1}{3}$
$4372\frac{1}{6}$	$1485\frac{7}{8}$	$1390\frac{3}{4}$	$1640\frac{1}{9}$	$1745\frac{1}{4}$	$2167\frac{1}{4}$
$8431\frac{2}{3}$	$1635\frac{1}{4}$	$2415\frac{1}{5}$	$3680\frac{1}{2}$	$3146\frac{3}{4}$	$3159\frac{1}{6}$
$5132\frac{7}{9}$	$1264\frac{1}{2}$	$1867\frac{1}{2}$	$4590\frac{7}{9}$	$1864\frac{1}{2}$	$9275\frac{1}{4}$
$1654\frac{1}{3}$	$1269\frac{3}{4}$	$1639\frac{1}{8}$	$2169\frac{2}{9}$	$2839\frac{5}{8}$	$7215\frac{3}{4}$
$1831\frac{5}{6}$	$1748\frac{1}{3}$	$4136\frac{3}{5}$	$8432\frac{2}{3}$	$6241\frac{7}{8}$	$5261\frac{1}{2}$
$1831\frac{3}{4}$	$1936\frac{5}{8}$	$1652\frac{1}{4}$	$4041\frac{1}{2}$	$4036\frac{3}{4}$	$7215\frac{2}{3}$
$1462\frac{1}{2}$	$5413\frac{1}{8}$	$3116\frac{1}{2}$	$6542\frac{5}{6}$	$8130\frac{5}{16}$	$5144\frac{3}{4}$
$1851\frac{1}{4}$	$2114\frac{7}{16}$	$1439\frac{1}{10}$	$1862\frac{2}{3}$	$2148\frac{7}{16}$	$6257\frac{2}{3}$
$1114\frac{7}{12}$	$1116\frac{9}{16}$	$2243\frac{7}{10}$	$3246\frac{8}{9}$	$1439\frac{9}{16}$	$2186\frac{5}{12}$

7.	8.	9.	10.	11.	12.
$9124\frac{1}{4}$	$7249\frac{1}{5}$	$1649\frac{1}{6}$	$7529\frac{1}{5}$	$7365\frac{1}{5}$	$2814\frac{1}{2}$
$2716\frac{1}{2}$	$2724\frac{4}{5}$	$2724\frac{1}{2}$	$6214\frac{1}{2}$	$2614\frac{1}{9}$	$2910\frac{1}{4}$
$2514\frac{1}{3}$	$8692\frac{1}{2}$	$8695\frac{1}{3}$	$1825\frac{4}{5}$	$1583\frac{1}{3}$	$2817\frac{1}{8}$
$2967\frac{1}{2}$	$2476\frac{1}{4}$	$1565\frac{1}{9}$	$8614\frac{3}{5}$	$1695\frac{1}{5}$	$2714\frac{1}{8}$
$2964\frac{1}{4}$	$8695\frac{1}{2}$	$2724\frac{7}{9}$	$9215\frac{3}{5}$	$1762\frac{1}{9}$	$2913\frac{7}{8}$
$6875\frac{1}{3}$	$6214\frac{1}{4}$	$8619\frac{1}{3}$	$6719\frac{3}{4}$	$1875\frac{5}{9}$	$2874\frac{5}{8}$
$8875\frac{3}{4}$	$7241\frac{1}{2}$	$2924\frac{2}{3}$	$8516\frac{1}{4}$	$1629\frac{7}{9}$	$2619\frac{3}{8}$
$2658\frac{1}{2}$	$8614\frac{1}{2}$	$6529\frac{1}{9}$	$7528\frac{1}{8}$	$7214\frac{2}{9}$	$1472\frac{1}{8}$
$8425\frac{2}{3}$	$4725\frac{1}{4}$	$8592\frac{1}{2}$	$7216\frac{5}{8}$	$2510\frac{1}{9}$	$2615\frac{3}{4}$
$8273\frac{3}{4}$	$1649\frac{1}{4}$	$2725\frac{1}{9}$	$6729\frac{1}{8}$	$2625\frac{2}{3}$	$1813\frac{1}{4}$
$1782\frac{3}{4}$	$1286\frac{1}{2}$	$8647\frac{2}{9}$	$3514\frac{1}{4}$	$8614\frac{1}{3}$	$1962\frac{1}{2}$
$8695\frac{1}{4}$	$6248\frac{1}{4}$	$8725\frac{5}{9}$	$1686\frac{3}{4}$	$2729\frac{1}{5}$	$1862\frac{1}{2}$
$2472\frac{1}{2}$	$1286\frac{1}{2}$	$6219\frac{7}{9}$	$1725\frac{1}{8}$	$2816\frac{1}{3}$	$1759\frac{1}{2}$
$6273\frac{1}{6}$	$8537\frac{3}{4}$	$8413\frac{1}{3}$	$2538\frac{3}{8}$	$2814\frac{1}{3}$	$2864\frac{1}{4}$
$9685\frac{5}{6}$	$6982\frac{1}{4}$	$7226\frac{5}{6}$	$1758\frac{5}{8}$	$2716\frac{2}{3}$	$1624\frac{1}{4}$
$9685\frac{11}{12}$	$3685\frac{1}{5}$	$1825\frac{1}{6}$	$2752\frac{7}{8}$	$1762\frac{1}{9}$	$1729\frac{1}{2}$
$1925\frac{5}{12}$	$2614\frac{3}{5}$	$4725\frac{1}{3}$	$2114\frac{1}{5}$	$1875\frac{1}{9}$	$1805\frac{3}{4}$
$4212\frac{1}{12}$	$8796\frac{4}{5}$	$2816\frac{2}{3}$	$2216\frac{1}{4}$	$2614\frac{2}{3}$	$1721\frac{7}{8}$
$2729\frac{1}{12}$	$1592\frac{3}{4}$	$2519\frac{1}{3}$	$1872\frac{1}{5}$	$2075\frac{1}{8}$	$1465\frac{3}{8}$

SUBTRACTION

ORAL EXERCISE

1. 172 A. − 154 A. = ? $\frac{5}{7} - \frac{1}{7} = ?$ 1 bu. − 3 pk. = ?

2. Find the difference between $\frac{1}{4}$ and $\frac{1}{8}$; $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{3}{7}$ and $\frac{2}{5}$.

171. It is clear that *only like numbers and parts of like units can be subtracted.*

172. Examples. **1.** Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

SOLUTION. The given fractions must be reduced to equivalent fractions having a least common denominator. The least common denominator is 24. $\frac{7}{8} = \frac{21}{24}$ and $\frac{5}{12} = \frac{10}{24}$. $\frac{21}{24} - \frac{10}{24} = \frac{11}{24}$, the required result.

2. From $21\frac{1}{3}$ take $17\frac{1}{2}$.

SOLUTION. Change the given fractions to similar fractions as in example 1. $\frac{3}{6}$ cannot be subtracted from $\frac{2}{6}$, hence 1 is taken from 21 and mentally united to $\frac{2}{6}$, making $\frac{8}{6}$. $\frac{3}{6}$ from $\frac{8}{6}$ leaves $\frac{5}{6}$, and 17 from 20 leaves 3. The required result is therefore $3\frac{5}{6}$.

ORAL EXERCISE

Find the value of:

1. $2\frac{3}{4} - \frac{1}{3}$.
2. $2\frac{5}{7} - \frac{1}{4}$.
3. $3\frac{7}{9} - \frac{4}{5}$.
4. $7\frac{5}{8} - 1\frac{3}{8}$.
5. $4\frac{5}{7} - 1\frac{2}{3}$.
6. $6\frac{7}{8} - 4\frac{5}{16}$.
7. $7\frac{1}{8} - 3\frac{5}{16}$.
8. $12\frac{1}{4} - 6\frac{1}{8}$.
9. $30 - 11\frac{7}{8}$.
10. $45 - 16\frac{3}{4}$.
11. $11\frac{1}{2} - 6\frac{2}{3}$.
12. $70\frac{3}{4} - 20\frac{1}{2}$.

The following is a recent clipping from a daily paper. It shows the prices of wheat on the Chicago market. The first line of prices is for wheat to be delivered in July, and the second line for wheat to be delivered in September.

CHICAGO WHEAT QUOTATIONS

DELIVERY	PREVIOUS CLOSING	OPENING	HIGHEST	LOWEST	CLOSING
July	$91\frac{1}{4}¢$	$90\frac{7}{8}¢$	$92\frac{7}{8}¢$	$90\frac{3}{4}¢$	$92\frac{1}{4}¢$
September	$87\frac{5}{8}¢$	$87\frac{1}{4}¢$	$88\frac{1}{4}¢$	$87¢$	$87\frac{1}{2}¢$

13. What was the difference between the highest and the lowest price of July wheat? of September wheat?

14. What was the difference between the opening and the closing price of September wheat? of July wheat?

15. What was the difference between the opening price and the previous closing (yesterday's closing) price of July wheat? of September wheat?

16. A bought 1000 bu. July wheat at the lowest price and sold the same at the closing price. What was his gain?

SUGGESTION. $1\frac{1}{4}¢ = \$0.015$; 1000 times $\$0.015 = \$$?

17. B bought 1000 bu. September wheat at the opening price and sold it at the highest price. What was his gain? Had he bought at the lowest price and sold at the closing price, what would have been his gain?

18. C bought 25,000 bu. July wheat at the opening price and sold it at the highest price. What was his gain?

WRITTEN EXERCISE

Find the value of:

1. $39 - 11\frac{5}{6}$.
2. $85 - 21\frac{3}{4}$.
3. $168 - 45\frac{5}{6}$.
4. $264\frac{9}{16} - 131\frac{1}{3}$.
5. $165\frac{1}{2} - 41\frac{1}{32}$.
6. $245\frac{3}{4} - 17\frac{3}{16}$.
7. $177\frac{2}{3} - 17\frac{3}{16}$.
8. $2150 - 121\frac{5}{16}$.
9. $\frac{5}{8} - \frac{1}{4} - \frac{1}{3}$.
10. $\frac{7}{8} - \frac{9}{16} - \frac{1}{8}$.
11. $2\frac{1}{2} + 1\frac{5}{8} - 1\frac{7}{12}$.
12. $25\frac{1}{2} - 8\frac{3}{4} - 15\frac{1}{2}$.

173. When the numerators of any two fractions are alike, the subtraction may be performed as in the following examples.

174. Examples. **1.** From $\frac{1}{7}$ take $\frac{1}{9}$. **2.** From $\frac{3}{5}$ take $\frac{3}{8}$.

SOLUTIONS. **1.** $9 - 7 = 2$, the new numerator. $9 \times 7 = 63$, the new denominator. Therefore, the required result is $\frac{2}{63}$. **2.** $8 - 5 \times 3 = 9$, the new numerator. $8 \times 5 = 40$, the new denominator. Therefore, $\frac{9}{40}$ is the required result.

ORAL EXERCISE

State the value of:

1. $\frac{1}{2} - \frac{1}{3}$.
2. $\frac{1}{2} - \frac{1}{4}$.
3. $\frac{1}{2} - \frac{1}{6}$.
4. $\frac{1}{3} - \frac{1}{7}$.
5. $\frac{1}{3} - \frac{1}{8}$.
6. $\frac{1}{3} - \frac{1}{9}$.
7. $\frac{1}{5} - \frac{1}{7}$.
8. $\frac{1}{5} - \frac{1}{8}$.
9. $\frac{1}{5} - \frac{1}{9}$.
10. $\frac{1}{4} - \frac{1}{5}$.
11. $\frac{1}{4} - \frac{1}{6}$.
12. $\frac{1}{4} - \frac{1}{7}$.
13. $\frac{1}{4} - \frac{1}{8}$.
14. $\frac{1}{6} - \frac{1}{7}$.
15. $\frac{1}{6} - \frac{1}{8}$.
16. $\frac{2}{3} - \frac{2}{7}$.
17. $\frac{3}{4} - \frac{3}{5}$.
18. $\frac{3}{4} - \frac{3}{7}$.
19. $\frac{4}{5} - \frac{4}{7}$.
20. $\frac{5}{6} - \frac{5}{7}$.
21. $\frac{5}{6} - \frac{5}{9}$.
22. $\frac{5}{6} - \frac{5}{8}$.
23. $\frac{7}{8} - \frac{7}{9}$.
24. $\frac{2}{7} - \frac{2}{9}$.
25. $12\frac{1}{2} - 6\frac{1}{3}$.
26. $13\frac{1}{4} - 2\frac{1}{5}$.
27. $14\frac{1}{5} - 7\frac{1}{6}$.
28. $16\frac{3}{5} - 12\frac{3}{8}$.

MULTIPLICATION

ORAL EXERCISE

1. 12 times 2 A. are how many acres? 12 times 2 fifths ($\frac{2}{5}$) are how many fifths? $\frac{24}{5} = ?$

2. 32 mi. divided by 4 equals how many miles? $\frac{1}{4}$ of 32 mi. equals how many miles? Multiplying by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, etc., is the same as dividing by what integer?

3. If 5 men can dig 125 bu. of potatoes in 1 da., how many bushels can 3 men dig in the same time? $\frac{3}{5}$ of 125 bu. equals how many bushels?

175. Example. Multiply $\frac{3}{8}$ by 248.

Solutions. (*a*) 248 times 3 eighths = 744 eighths $= \frac{744}{8} = 93$; but,

$$\frac{3}{8} \times 248 \overset{(a)}{=} \frac{744}{8} = 93$$

(*b*) If the multiplication is indicated as in the margin, the work may be shortened by cancellation.

$$\frac{\overset{31}{\cancel{248}} \text{ times } 3}{\cancel{8}} \overset{(b)}{=} 93$$

176. Therefore, to find the product of an integer and a fraction, *find the product of the integer and the numerator, and divide it by the denominator.*

Before actually multiplying, indicate the multiplication and cancel if possible.

ORAL EXERCISE

1. If 1 yd. of cloth costs \$0.87$\frac{1}{2}$ (\$$\frac{7}{8}$), what will 16 yd. cost? 48 yd.? 128 yd.? 72 yd.?

2. When oats cost \$0.33$\frac{1}{3}$ ¢ (\$$\frac{1}{3}$) a bushel, how much must be paid for 29 bu.? for 36 bu.? for 129 bu.?

3. A boy earns \$0.75 (\$$\frac{3}{4}$) a day. How much will he earn in 18 da.? in 40 da.? in 84 da.? in 128 da.? in 160 da.?

4. When property rents for \$720 a year, what is the rent for $\frac{1}{2}$ yr.? for $\frac{1}{4}$ yr.? for $\frac{1}{3}$ yr.? for $\frac{1}{12}$ yr.? for $\frac{1}{6}$ yr.?

5. A ship is worth \$48,000. What is $\frac{1}{8}$ of the ship worth? $\frac{7}{16}$ of the ship? $\frac{5}{6}$ of the ship? $\frac{7}{8}$ of the ship? $\frac{7}{12}$ of the ship?

WRITTEN EXERCISE

Find the product of:

1. $98 \times \frac{5}{8}$.
2. $80 \times \frac{2}{3}$.
3. $50 \times \frac{7}{24}$.
4. $97 \times \frac{7}{10}$.
5. $92 \times \frac{5}{16}$.
6. $188 \times \frac{1}{64}$.
7. $\frac{7}{8}$ of 95.
8. $\frac{5}{7}$ of 25.
9. $\frac{4}{5}$ of 88.
10. $\frac{9}{16}$ of 51.
11. $\frac{5}{24}$ of 99.
12. $\frac{7}{16}$ of 77.
13. $784 \times \frac{5}{7}$.
14. $459 \times \frac{1}{9}$.
15. $400 \times \frac{1}{16}$.
16. $510 \times \frac{7}{10}$.
17. $990 \times \frac{1}{64}$.
18. $800 \times \frac{15}{16}$.
19. $\frac{5}{6}$ of 2420.
20. $\frac{7}{8}$ of 2500.
21. $\frac{2}{3}$ of 3240.
22. $\frac{5}{8}$ of 5117.
23. $\frac{7}{16}$ of 7254.
24. $\frac{7}{16}$ of 1024.

177. Example. Multiply 25 by $4\frac{3}{8}$.

SOLUTION. $\frac{3}{8}$ of $25 = \frac{75}{8}$ or $9\frac{3}{8}$. Write $\frac{3}{8}$ as shown in the margin, and carry 9 to the product of the integers. $4 \times 25 + 9 = 109$. Therefore, 25 multiplied by $4\frac{3}{8} = 109\frac{3}{8}$.

$$\begin{array}{r} 25 \\ 4\frac{3}{8} \\ \hline 109\frac{3}{8} \end{array}$$

178. Therefore, to find the product of a mixed number and a whole number, *multiply the integer and the fraction separately and find the sum of the products.*

ORAL EXERCISE

Find the cost of:

1. $15\frac{3}{4}$ lb. of fish at 9¢.
2. $7\frac{3}{8}$ yd. of cloth at $3.
3. 16 lb. of beef at $12\frac{1}{2}$¢.
4. $16\frac{1}{2}$ lb. of sugar at 5¢.
5. 12 yd. of cloth at $11\frac{1}{4}$¢.
6. $6\frac{3}{4}$ bu. turnips at 32¢.
7. $12\frac{1}{2}$ bu. of oats at 39¢.
8. $10\frac{1}{2}$ yd. of calico at 4¢.
9. $16\frac{1}{2}$ yd. of ribbon at 20¢.
10. $8\frac{1}{2}$ gal. of molasses at 40¢.

WRITTEN EXERCISE

1. A merchant bought 24 pc. of English serge, containing 52, 47^2, 50^1, 48^3, 49, 51^3, 47, 48^2, 45^3, 49^1, 52^2, 50^2, 51^3, 50, 52, 53^1, 52^3, 47^3, 48^1, 51^2, 51^3, 48^2, 49, and 53 yd., at $\$1.12\frac{1}{2}$ per yard, and sold it all at $1.35 per yard. What did he gain?

2. I bought 25 pc. taffeta silk, containing 42^1, 40^2, 43, 44^3, 45, 41^2, 43, 46^1, 47^2, 44, 45^2, 49^1, 47^1, 45^1, 46, 44, 43^3, 40, 41^3, 46, 47, 40^2, 45^1, 42, and 47^3 yd., at $87\frac{1}{2}$¢ per yard, and sold the first 15 pc. at $1.05 and the remainder at $1.10. What did I gain?

3. A merchant bought 25 pc. of striped denim containing 41^1, 41^1, 42^2, 43^2, 42^1, 44^2, 43^1, 40^2, 42^1, 45^3, 42^1, 40^2, 41^2, 47^3, 45^1, 41^1, 43^2, 47^2, 44^3, 42^3, 43^2, 39^1, 42^1, 48^2, and 47 yd., at 11¢ per yard. If he sold the first 11 pc. at 15¢ per yard and the remainder at 17¢ per yard, what was his gain?

4. Copy and find the amount of the following bill:

Little Falls, N.Y., June 7, 19

Messrs. Sibley, Lindsay & Curr Co.
Rochester, N. Y.

Terms 60 days. *Bought of* **The Eureka Mills**

	5	pc. Bombay Gingham				
		$41'\ 42^3\ 45'\ 40\ 41$	210 yd.	8¾¢		
	5	pc. Pontiac Seersucker				
		$42^3\ 41\ 39^3\ 40^2\ 42$???		7¼¢		
	10	pc. York Denim				
		$40\ 42^2\ 41^3\ 40\ 41^2$ $41\ 39\ 42\ 41\ 40$???		15⅝¢		

179. The expressions $\frac{1}{2}$ of $\frac{3}{5}$ and $\frac{1}{2} \times \frac{3}{5}$ have the same meaning; hence, the sign of multiplication may be read *of*, or *multiplied by*, when it immediately follows a fraction.

180. Examples. 1. Multiply $\frac{3}{5}$ by $\frac{2}{3}$.

SOLUTION. To multiply $\frac{3}{5}$ by $\frac{2}{3}$ is to find $\frac{2}{3}$ of $\frac{3}{5}$.

Let the line AF in the accompanying diagram represent a unit divided into 5 equal parts.

Then AD will represent $\frac{3}{5}$. Subdivide each of the five equal parts into 3 equal parts and the line AF will represent a unit divided into 15 equal parts, each of which is $\frac{1}{15}$ of the whole. It is then clear that $\frac{1}{3}$ of $\frac{1}{5}$ equals $\frac{1}{15}$. Since $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{3}{5}$ is $\frac{3}{15}$. But $\frac{2}{3}$ of $\frac{3}{5}$ is 2 times $\frac{1}{3}$ of $\frac{3}{5}$; therefore, $\frac{2}{3}$ of $\frac{3}{5}$ equals $\frac{6}{15}$.

2. Find the product of $2\frac{1}{2}$, $\frac{4}{5}$, and $\frac{7}{15}$.

SOLUTION. Reduce the mixed number $2\frac{1}{2}$ to an improper fraction and obtain $\frac{5}{2}$. Cancel, and there remains in the numerators 2 times 7, and in the denominators 15, from which obtain the fraction $\frac{14}{15}$.

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} \times \frac{7}{15} = \frac{14}{15}$$

181. Hence, to multiply a fraction by a fraction:

Reduce the mixed numbers and integers to improper fractions and cancel all factors common to the numerators and denominators.

Find the product of the remaining numerators for the required numerator, and the product of the remaining denominators for the required denominator.

ORAL EXERCISE

1. How many yards in $\frac{2}{3}$ rd.? feet in $\frac{3}{5}$ rd.?

2. When barley is worth $25\frac{1}{2}¢$ per bushel, what is the value of $\frac{1}{2}$ bu.? of $\frac{3}{4}$ bu.?

3. A book, the retail price of which was \$5, was sold at wholesale for $\frac{4}{5}$ of the retail price, with $\frac{1}{10}$ off from that for cash. Find the selling price of 10 books.

WRITTEN EXERCISE

Reduce to their simplest form:

1. $\frac{3}{4}$ of $\frac{3}{5}$ of $\frac{3}{7}$.
2. $\frac{3}{4}$ of $\frac{8}{9}$ of $2\frac{1}{3}$.
3. $7\frac{1}{5} \times 25 \times \frac{5}{8}$.
4. $3\frac{5}{8} \times 4\frac{1}{2} \times 20$.
5. $50 \times \frac{7}{10} \times 7\frac{3}{5}$.
6. $1\frac{2}{3} \times 4\frac{1}{2} \times 8\frac{4}{5}$.

7. A saves \$9.75 per week and B $\frac{2}{3}$ as much. How much more will A have than B at the end of a year?

8. A merchant bought a piece of cloth containing $43\frac{1}{2}$ yd. at \$1.50 per yard. He sold $\frac{3}{4}$ of it at $\$1.62\frac{1}{2}$ a yard, and the remainder at $\$1.37\frac{1}{2}$ a yard. Did he gain or lose, and how much?

The following is a recent clipping from a daily paper. It shows the prices of corn on the New York market.

NEW YORK CORN QUOTATIONS

DELIVERY	PREVIOUS CLOSING	OPENING	HIGHEST	LOWEST	CLOSING
July	$66\frac{1}{2}¢$	$65\frac{7}{8}¢$	$66\frac{1}{2}¢$	$64\frac{7}{8}¢$	$65\frac{3}{8}¢$
September	$65\frac{1}{4}¢$	$64\frac{7}{8}¢$	$65\frac{1}{8}¢$	$64\frac{1}{4}¢$	$64\frac{5}{8}¢$

9. D bought 25,000 bu. September corn at the opening price and sold it at the highest price. What was his gain? Had he bought at the lowest price and sold at the highest price, what would he have gained?

10. E bought 12,500 bu. July corn at the lowest price and sold it at the closing price. What was his gain? Had he bought at the lowest price and sold at the highest price, what would he have gained?

11. A gold dollar weighs 25.8 troy grains. For every 90 parts of pure gold there are ten parts of alloy. How many grains of each kind in a gold dollar? in a 5-dollar gold piece?

12. A 5-cent piece weighs 77.16 troy grains. For every part of nickel there are three parts of copper. How many grains of each kind in a 5-cent piece?

13. The second general coinage act (1834) of the United States made one silver dollar weigh approximately as much as sixteen gold dollars, and this ratio of **sixteen to one** has been maintained up to the present time. What is the approximate weight of a silver dollar? If silver coins are $\frac{9}{10}$ pure, approximately how much pure silver in 10 silver dollars?

Short Methods in Multiplication

182. When mixed numbers are large, they may be multiplied as shown in the following example.

183. Example. Multiply $255\frac{1}{3}$ by $24\frac{2}{5}$.

Solution. Multiply the fractions together and obtain $\frac{2}{15}$, which write as shown in the margin. Multiply the integer in the multiplicand by the fraction in the multiplier and obtain 102. Multiply the fraction in the multiplicand by the integer in the multiplier and obtain 8. Multiply the integers together and add the partial products. The result is $6230\frac{2}{15}$.

$$
\begin{array}{rl}
255\frac{1}{3} & \\
24\frac{2}{5} & \\
\hline
\frac{2}{15} & = \frac{2}{5} \text{ of } \frac{1}{3} \\
102 & = \frac{2}{5} \text{ of } 255 \\
8 & = 24 \text{ times } \frac{1}{3} \\
\left.\begin{array}{r} 1020 \\ 510 \end{array}\right\} & = 24 \text{ times } 255 \\
\hline
6230\frac{2}{15} & = 24\frac{2}{5} \text{ times } 255\frac{1}{3}
\end{array}
$$

WRITTEN EXERCISE

Multiply:

1. $975\frac{1}{4}$ by $18\frac{1}{2}$.
2. $876\frac{3}{4}$ by $21\frac{7}{8}$.
3. $720\frac{7}{8}$ by $21\frac{3}{4}$.
4. $445\frac{1}{2}$ by $46\frac{3}{4}$.
5. $512\frac{7}{16}$ by $16\frac{1}{2}$.
6. $450\frac{9}{16}$ by $20\frac{3}{4}$.

SQUARING NUMBERS ENDING IN $\frac{1}{2}$ OR .5

184. Examples. 1. Multiply $9\frac{1}{2}$ by $9\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, which write as shown in the margin. $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to either the integer in the multiplicand or multiplier. Therefore, add 1 to the integer in the multiplicand and multiply by the multiplier. $9 \times 10 = 90$. Then, $9\frac{1}{2} \times 9\frac{1}{2} = 90\frac{1}{4}$.

$$\begin{array}{r} 9\frac{1}{2} \\ 9\frac{1}{2} \\ \hline 90\frac{1}{4} \end{array}$$

2. Find the cost of 8.5 T. of coal at \$8.50 per ton.

SOLUTION. The principles embodied in this example are practically the same as those in problem 1. $.5 \times .5 = .25$, $8 \times 9 = 72$. Therefore, 8.5 tons of coal at \$8.50 per ton will cost \$72.25.

$$\begin{array}{r} 8.5 \\ 8.5 \\ \hline 72.25 \end{array}$$

3. Find the cost of 75 A. of land at \$75 per acre.

SOLUTION. This problem is similar to example 2, the only difference being in the matter of the decimal point. Since the decimal point has no particular bearing upon the steps in the process of multiplying, proceed to find the product as in example 2. $5 \times 5 = 25$, which write as shown in the margin. $7 \times 8 = 56$, which write to complete the product. 75 acres of land at \$75 an acre will therefore cost \$5625.

$$\begin{array}{r} 75 \\ 75 \\ \hline 5625 \end{array}$$

ORAL EXERCISE

Multiply:

1. $1\frac{1}{2}$ by $1\frac{1}{2}$.	6. $6\frac{1}{2}$ by $6\frac{1}{2}$.	11. $13\frac{1}{2}$ by $13\frac{1}{2}$.	16. $16\frac{1}{2}$ by $16\frac{1}{2}$.
2. $2\frac{1}{2}$ by $2\frac{1}{2}$.	7. 7.5 by 7.5.	12. $14\frac{1}{2}$ by $14\frac{1}{2}$.	17. $17\frac{1}{2}$ by $17\frac{1}{2}$.
3. $3\frac{1}{2}$ by $3\frac{1}{2}$.	8. 8.5 by 8.5.	13. $15\frac{1}{2}$ by $15\frac{1}{2}$.	18. $18\frac{1}{2}$ by $18\frac{1}{2}$.
4. $4\frac{1}{2}$ by $4\frac{1}{2}$.	9. 9.5 by 9.5.	14. 11.5 by 11.5.	19. 195 by 195.
5. $5\frac{1}{2}$ by $5\frac{1}{2}$.	10. 10.5 by 10.5.	15. 12.5 by 12.5.	20. 205 by 205.

WRITTEN EXERCISE

In the following problems make all the extensions mentally.

1. Find the total cost of:

85 lb. of tea at 85¢.	55 lb. tea at 55¢.
75 gal. sirup at 75¢.	75 bbl. flour at \$7.50.
45 gal. sirup at 45¢.	650 bbl. oatmeal at \$6.50.
$2\frac{1}{2}$ bu. beans at \$2.50.	25 doz. cans olives at \$2.50.
35 gal. molasses at 35¢.	95 cs. salad dressing at 95¢.
65 cs. horseradish at 65¢.	750 lb. cream codfish at $7\frac{1}{2}$¢.
$4\frac{1}{2}$ cs. baking powder at \$4.50.	$3\frac{1}{2}$ cs. baking powder at \$3.50.

MULTIPLICATION OF ANY NUMBERS ENDING IN $\frac{1}{2}$ OR .5

185. Examples. 1. Multiply $7\frac{1}{2}$ by $6\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to $\frac{1}{2}$ of $\overline{6+7}$, or $6\frac{1}{2}$, which added to $\frac{1}{2}$ of $\frac{1}{2}$ equals $6\frac{3}{4}$. Write $\frac{3}{4}$ as shown in the margin, and carry 6. $\overline{6\times 7}+6 = 48$. Therefore, $7\frac{1}{2}\times 6\frac{1}{2} = 48\frac{3}{4}$.

$$\begin{array}{r} 6\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 48\frac{3}{4} \end{array}$$

2. Multiply $7\frac{1}{2}$ by $9\frac{1}{2}$.

SOLUTION. $\frac{1}{2}$ of $\overline{7+9} = 8$, with no remainder. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$, which write as shown in the margin, and carry 8. $\overline{7\times 9}+8 = 71$. Therefore, $7\frac{1}{2}\times 9\frac{1}{2} = 71\frac{1}{4}$.

$$\begin{array}{r} 7\frac{1}{2} \\ 9\frac{1}{2} \\ \hline 71\frac{1}{4} \end{array}$$

Observe that : (1) in finding $\frac{1}{2}$ of any number (dividing a number by 2) there is either nothing remaining or 1 remaining; (2) in finding $\frac{1}{2}$ of an even number there can be no remainder, and in finding $\frac{1}{2}$ of an odd number there is always a remainder 1. Hence, to multiply numbers ending in $\frac{1}{2}$ or .5:

Mentally determine the sum of the integers in the multiplicand and multiplier. If it is an even number, write $\frac{1}{4}$ (.25 or 25) in the product. If it is an odd number, write $\frac{3}{4}$ (.75 or 75) in the product. Multiply the integers and to the product add $\frac{1}{2}$ of their sum.

ORAL EXERCISE

Multiply :

1. $3\frac{1}{2}$ by $7\frac{1}{2}$.	**4.** $17\frac{1}{2}$ by $2\frac{1}{2}$.	**7.** 3.5 by 8.5.
2. $4\frac{1}{4}$ by $5\frac{1}{2}$.	**5.** $14\frac{1}{2}$ by $6\frac{1}{2}$.	**8.** 7.5 by 6.5.
3. $16\frac{1}{2}$ by $4\frac{1}{2}$.	**6.** $21\frac{1}{2}$ by $9\frac{1}{2}$.	**9.** 5.5 by 8.5.

WRITTEN EXERCISE

Make the extensions in each of the following problems mentally.

1. Find the total cost of :

6.5 T. coal at \$8.50.	8.5 T. coal at \$9.50.
2.5 T. hay at \$17.50.	16.5 T. hay at \$11.50.
15.5 cd. wood at \$3.50.	14.5 cd. wood at \$5.50.

2. Find the total cost of :

45 bu. beans at \$2.50.	350 bu. wheat at \$1.05.
35 bbl. flour at \$6.50.	350 bu. beans at \$2.50.
45 bbl. flour at \$8.50.	85 bbl. oatmeal at \$7.50.

DIVISION

ORAL EXERCISE

1. 8 A. $\div 4 = ?$ 8 ninths $(\frac{8}{9}) \div 4$?

2. If 2 lb. of coffee costs \$0.66$\frac{2}{3}$ (\$$\frac{2}{3}$), what will 1 lb. cost? Divide $\frac{2}{3}$ by 2. What is the effect of dividing the numerator of a fraction?

3. $\frac{4}{5} \div 2 = ?$ $\frac{1}{2}$ of $\frac{4}{5} = ?$

4. Because $\frac{4}{5} \div 2 = \frac{1}{2}$ of $\frac{4}{5}$, therefore, $\frac{1}{4} \div 5 = \frac{1}{5}$ of $\frac{1}{4}$, or $\frac{1}{4} \times \frac{1}{5}$. $\frac{1}{4} \times \frac{1}{5} = ?$

5. What is the quotient of $\frac{1}{3} \div 5$? of $\frac{1}{2} \div 8$? of $\frac{1}{5} \div 2$?

6. Because $\frac{1}{3} \div 5 = \frac{1}{5}$ of $\frac{1}{3}$, therefore $\frac{2}{3} \div 5 = 2$ times $\frac{1}{3}$ of $\frac{1}{5}$. That is, $\frac{2}{3} \div 5 = \frac{1}{5}$ of $\frac{2}{3}$, or $\frac{2}{3} \times \frac{1}{5}$. $\frac{2}{3} \times \frac{1}{5} = ?$

7. How much is $\frac{3}{4} \div 5$? $\frac{5}{6} \div 3$? $7\frac{1}{2}$ $(7\frac{5}{2}) \div 8$? $3\frac{1}{2} \div 6$?

8. What is the effect of multiplying the denominator of a fraction?

186. In the above exercise it is clear that

Dividing the numerator of a fraction by an integer divides the whole fraction; and,

Multiplying the denominator of a fraction by an integer divides the whole fraction.

ORAL EXERCISE

Find the quotient of:

1. $\frac{8}{9} \div 4$.	**4.** $\frac{3}{4} \div 12$.	**7.** $\frac{7}{12} \div 4$.	**10.** $\frac{3}{5} \div 9$.	**13.** $\frac{1}{5} \div 19$.
2. $\frac{8}{13} \div 2$.	**5.** $\frac{5}{8} \div 12$.	**8.** $\frac{7}{16} \div 9$.	**11.** $\frac{1}{5} \div 6$.	**14.** $\frac{3}{16} \div 5$.
3. $\frac{15}{9} \div 5$.	**6.** $\frac{9}{10} \div 3$.	**9.** $\frac{5}{16} \div 7$.	**12.** $\frac{1}{8} \div 5$.	**15.** $\frac{7}{16} \div 5$.

187. Examples. **1.** Divide $28\frac{7}{8}$ by 7.

SOLUTION. First divide the integers and the result is 4; then divide the fraction by 7 and the result is $\frac{1}{8}$. Therefore, $28\frac{7}{8} \div 7 = 4\frac{1}{8}$.

$$7\overline{)28\frac{7}{8}} \quad 4\frac{1}{8}$$

2. Divide $26\frac{1}{2}$ by 8.

SOLUTION. Divide 26 by 8 and the result is 3 with a remainder 2. Join the remainder, 2, with the fraction, $\frac{1}{2}$, making $2\frac{1}{2}$. Reduce $2\frac{1}{2}$ to an improper fraction and the result is $\frac{5}{2}$. $\frac{5}{2} \div 8 = \frac{5}{16}$. Therefore, $26\frac{1}{2} \div 8 = 3\frac{5}{16}$.

$$8\overline{)26\frac{1}{2}} \quad 3\frac{5}{16}$$

ORAL EXERCISE

Divide:

1. $16\frac{1}{2}$ by 4.
2. $18\frac{1}{5}$ by 9.
3. $25\frac{3}{4}$ by 2.
4. $17\frac{1}{2}$ by 8.
5. $32\frac{3}{5}$ by 4.
6. $27\frac{1}{2}$ by 7.
7. $19\frac{7}{8}$ by 9.
8. $20\frac{2}{3}$ by 10.
9. $21\frac{1}{2}$ by 8.
10. $24\frac{6}{7}$ by 6.
11. $45\frac{5}{9}$ by 5.
12. $40\frac{2}{3}$ by 10.
13. $8\frac{1}{2}$ by 5.
14. $14\frac{3}{4}$ by 7.
15. $11\frac{1}{2}$ by 9.
16. $26\frac{1}{2}$ by 10.

ORAL EXERCISE

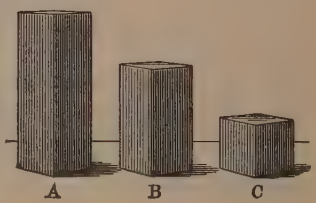
1. How many eighths in one? $1 \div \frac{1}{8} = ?$

2. What is the value of: $1 \div \frac{1}{10}$? $3 \div \frac{1}{5}$? $17 \div \frac{1}{8}$? $125 \div \frac{1}{12}$? $250 \div \frac{1}{4}$?

3. Read aloud the following, supplying the missing word: *To divide an integer by a unit fraction, multiply the integer by the —— of the fraction.*

4. What is the value of $25 \div \frac{1}{3}$? $2.5 \div \frac{1}{2}$? $7.5 \div \frac{1}{4}$? $25.5 \div \frac{1}{12}$? $54 \div \frac{1}{4}$? $48 \div \frac{1}{5}$? $29 \div \frac{1}{3}$? $2\frac{1}{2} \div \frac{1}{6}$?

5. If B, in the accompanying diagram, is 1, what is C? How many blocks like C in B? $1 \div \frac{1}{2} = ?$



6. If A is 1, what is B? A is how many times B? That is, $A \div B = ?$ $1 \div \frac{2}{3} = ?$

7. If $1 \div \frac{2}{3} = \frac{3}{2}$ $(1\frac{1}{2})$, then $2 \div \frac{2}{3} = ?$

8. What is the value of $4 \div \frac{2}{3}$? $5 \div \frac{3}{4}$? $12 \div \frac{2}{5}$? $15 \div \frac{2}{3}$?

9. Read aloud the following, supplying the missing words: If A is 1, B is ——, and C is ——. If B is contained in A $\frac{3}{2}$ $(1\frac{1}{2})$ times, it is contained in C $\frac{1}{3}$ of $\frac{3}{2}$ times or —— times. That is, $\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} =$ ——.

10. What is the value of $\frac{1}{2} \div \frac{1}{4}$? $\frac{2}{3} \div \frac{2}{5}$? $\frac{2}{7} \div \frac{2}{3}$? $\frac{3}{4} \div \frac{5}{6}$?

188. The **reciprocal of a fraction** is 1 divided by that fraction.

Thus, the reciprocal of $\frac{2}{3}$ is $1 \div \frac{2}{3}$, or $\frac{3}{2}$. That is, *the reciprocal of a fraction is the fraction inverted.*

189. **Reciprocal numbers**, as we use the terms in arithmetic, are numbers whose product is 1.

Thus, 4 and $\frac{1}{4}$, $\frac{4}{5}$ and $\frac{5}{4}$, $\frac{1}{6}$ and 6, $\frac{2}{3}$ and $\frac{3}{2}$, are reciprocal numbers, because their product is equal to 1.

190. It has been seen that the brief method for dividing a fraction or an integer by a fraction is to *multiply the dividend by the reciprocal of the divisor.*

The principles of cancellation should be used whenever possible. Integers and mixed numbers should be reduced to improper fractions before applying the rule.

WRITTEN EXERCISE

Divide:

1. $\frac{7}{8}$ by $\frac{2}{3}$.
2. $7\frac{1}{2}$ by $\frac{1}{8}$.
3. 95 by $\frac{5}{8}$.
4. 88 by $\frac{3}{4}$.
5. 16 by $\frac{3}{5}$.
6. $15\frac{1}{2}$ by $\frac{1}{5}$.
7. $\frac{3}{4}$ by $\frac{2}{3}$.
8. $4\frac{5}{7}$ by $\frac{2}{3}$.
9. $\frac{9}{10}$ by $\frac{7}{8}$.
10. $6\frac{2}{3}$ by $1\frac{1}{2}$.
11. 160 by $4\frac{1}{2}$.
12. 250 by $3\frac{2}{3}$.
13. $\frac{7}{8}$ by $\frac{4}{5}$.
14. $\frac{3}{5}$ by $\frac{2}{7}$.
15. 169 by $4\frac{7}{8}$.
16. 640 by $5\frac{3}{8}$.
17. 625 by $83\frac{1}{3}$.
18. $920\frac{3}{5}$ by 73.

191. Examples. **1.** Divide 2190 by $48\frac{2}{3}$.

SOLUTION. Multiplying both dividend and divisor by the same number does not affect the quotient; hence, multiply the dividend and divisor by 3 and obtain for the new dividend and divisor 6570 and 146, respectively. Divide the same as in simple numbers and obtain the result 45. Or,

Reduce both the dividend and divisor to thirds, obtaining $\frac{6570}{3}$ and $\frac{146}{3}$. Reject the common denominators and divide as in whole numbers.

$$\begin{array}{r} 48\frac{2}{3})2190 \\ 3 \quad 3 \\ \hline 146) \quad 6570 (45 \\ \quad\quad 584 \\ \hline \quad\quad 730 \\ \quad\quad 730 \\ \hline \end{array}$$

2. Divide $65\frac{1}{2}$ by $12\frac{1}{3}$.

SOLUTION. Multiply both dividend and divisor by 6, the least common denominator of the fractions, and divide as in simple numbers. The result is $5\frac{23}{74}$. Or,

Reduce both the dividend and divisor to sixths, obtaining as a result $\frac{74}{6}$ and $\frac{393}{6}$. Reject the common denominator and divide as in simple numbers.

$$\begin{array}{r} 12\frac{1}{3})65\frac{1}{2} \\ 6 \quad 6 \\ \hline 74) 393 (5\frac{23}{74} \\ \quad 370 \\ \hline \quad 23 \end{array}$$

WRITTEN EXERCISE

Divide:

1. $270\frac{1}{2}$ by $12\frac{1}{2}$.
2. $508\frac{1}{4}$ by $30\frac{2}{3}$.
3. $1431\frac{1}{5}$ by $20\frac{1}{2}$.
4. $962\frac{1}{2}$ by $31\frac{1}{4}$.
5. $650\frac{3}{4}$ by $26\frac{1}{2}$.
6. $1680\frac{1}{8}$ by $45\frac{1}{2}$.
7. 7552 by $78\frac{2}{3}$.
8. $470\frac{3}{4}$ by $17\frac{1}{8}$.
9. $1054\frac{1}{2}$ by $168\frac{1}{3}$.

FRACTIONAL RELATIONS

ORAL EXERCISE

1. If f in the accompanying diagram is 1, what is e? d? c? b? a?

2. What part of e is f? of d? of c? of b? of a? What part of 6 is 1? of 5? of 4? of 3? of 2?

3. What part of a is e? d? c? b? What part of 6 is 2? 3? 4? 5?

4. What part of d is f? What part of b is e? What part of $\frac{1}{2}$ ($\frac{3}{6}$) is $\frac{1}{6}$? What part of $\frac{5}{6}$ is $\frac{1}{3}$ ($\frac{2}{6}$)?

5. What part of 7 bu. is 1 bu.? What part of 7 eighths ($\frac{7}{8}$) is 1 eighth ($\frac{1}{8}$)?

6. What part of $\frac{3}{5}$ is $\frac{2}{5}$?

SOLUTION. $\frac{3}{5}$ and $\frac{2}{5}$ are similar fractions; hence they may be compared in the same manner as concrete integral numbers. 2 is $\frac{2}{3}$ of 3; therefore, $\frac{2}{5}$ is $\frac{2}{3}$ of $\frac{3}{5}$; or,

$$\tfrac{2}{5} \text{ is } \frac{\frac{2}{5}}{\frac{3}{5}} \text{ of } \tfrac{3}{5}. \quad \frac{\frac{2}{5}}{\frac{3}{5}} = \tfrac{2}{5} \times \tfrac{5}{3} = \tfrac{2}{3}.$$

7. $\frac{3}{4}$ is what part of $1\frac{3}{4}$ ($\frac{7}{4}$)? of $2\frac{3}{4}$? of $5\frac{1}{4}$?

8. $\frac{1}{8}$ is what part of $\frac{1}{2}$?

SOLUTION. $\frac{1}{2} = \frac{4}{8}$. $\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{4}{8}$, therefore, $\frac{1}{8} = \frac{1}{4}$ of $\frac{1}{2}$; or,

$$\tfrac{1}{8} \text{ is } \frac{\frac{1}{8}}{\frac{1}{2}} \text{ of } \tfrac{1}{2}. \quad \frac{\frac{1}{8}}{\frac{1}{2}} = \tfrac{1}{8} \times \tfrac{2}{1} = \tfrac{1}{4}.$$

192. To find what fraction one number is of another, *take the number denoting a part for the numerator of the fraction, and the number denoting the whole for the denominator.*

ORAL EXERCISE

1. If a piece of work can be performed in 12 da., what part of it can be performed in 5 da.? in 7 da.?

2. If A can do a piece of work in 15 da., what part of it can he do in 1 da.? in 2 da.? in 5 da.? in $7\frac{1}{2}$ da.?

3. If B can do a piece of work in $7\frac{1}{2}$ da., what part of it can he do in 1 da.? in 2 da.? in 5 da.? in $5\frac{1}{2}$ da.? in $6\frac{1}{2}$ da.?

4. What part of 100 is $33\frac{1}{3}$? $12\frac{1}{2}$? $66\frac{2}{3}$? $8\frac{1}{3}$? 25? 75? 125? $16\frac{2}{3}$? $83\frac{1}{3}$? $62\frac{1}{2}$? $22\frac{2}{9}$? $9\frac{1}{11}$? $56\frac{1}{4}$? $6\frac{2}{3}$?

5. What part of \$1 is $33\frac{1}{3}$¢? $66\frac{2}{3}$¢? 25¢? 75¢? $16\frac{2}{3}$¢? $8\frac{1}{3}$¢? $6\frac{2}{3}$¢? $3\frac{1}{3}$¢? $6\frac{1}{4}$¢? $62\frac{1}{2}$¢? $87\frac{1}{2}$¢? $37\frac{1}{2}$¢? $14\frac{2}{7}$¢?

6. What part of 1000 is 125? $166\frac{2}{3}$? $666\frac{2}{3}$? 625? $333\frac{1}{3}$?

7. What part of \$10 is \$$3.33\frac{1}{3}$? \$1.25? \$$1.66\frac{2}{3}$? \$$8.33\frac{1}{3}$? \$2.50? \$6.25? \$$6.66\frac{2}{3}$?

WRITTEN EXERCISE

1. A man asked for a horse $\frac{2}{5}$ more than it cost, but finally reduced the price $\frac{1}{10}$. He gained \$26. What was the cost of the horse? the price asked? the selling price?

2. A had $\frac{1}{3}$ of his money invested in bonds, $\frac{5}{24}$ in bank stock, and the remainder, \$1980, on deposit in the First National Bank. How much was invested in bonds? in bank stock?

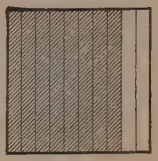
3. A man left his estate to his four sons. To the first son he gave $\frac{1}{3}$ of the estate; to the second, $\frac{1}{2}$ of the remainder; to the third, $\frac{1}{4}$ of the estate; to the fourth son, \$1556. What was the value of the estate?

4. A merchant reduced the marked price of a machine $\frac{1}{6}$, and then sold it so that he gained $\frac{1}{4}$ of the first cost. If he gained \$8, what was the first cost of the machine, and the marked price before any reduction was made?

5. A man placed a house and lot in the hands of a real estate agent to be sold at such a price that he, the owner, might realize \$5985, after paying the agent $\frac{1}{20}$ of the selling price of the property. For how much was the property sold?

6. A farmer had three bins containing wheat, rye, and oats respectively. The quantity of oats was three times that of the wheat, and the rye was one half of the quantity of the oats. If the value of the oats at 35¢ per bushel was \$1155, how many bushels of each kind of grain did the farmer have? If the wheat was worth 95¢ per bushel, and the rye $67\frac{1}{2}$¢ per bushel, what was the value of the entire lot of grain?

WRITTEN EXERCISE



1. The square in the margin represents the total population of the state of New York (state census of 1910), and the shaded area represents the urban (city) population. If the rural (country) population is 1,800,000, what is the entire population of the state? the urban population?

2. In a recent year the population of Massachusetts was in round numbers 3,360,000, and there were fourteen persons living in the cities of the state to each person living in the country. Represent this graphically as in problem 1, and find the city population and the country population for the state.

3. Suppose that *C* in the diagram represents the population of the United States in 1870, *A* the population in 1830, and *F* the population in 1900. If the population in 1870 was 38,400,000 (round numbers), what was the population (round numbers) in 1900? In 1830?

4. Suppose that *F* in the diagram represents the population of the United States in 1900, and *C* the proportion of this population living in cities in 1900. What proportion of the population lived in cities in 1900? Suppose that *F* represents the population in 1860 and *A* the proportion of this population living in cities. Assuming that the city population in 1860 was 5,240,554, find the total population for the same year.

5. The total population of New Jersey (state census of 1910) is 2,537,167, and the rural population, 629,957. Represent this graphically and find the urban population.

CONVERSION OF FRACTIONS

ORAL EXERCISE

1. What is the denominator of the decimal .6? of .75?
2. What is the numerator of .4? of .04? of .004? of .0004?
3. Write as a common fraction .7; .23; .079; .0013; .00123.

193. A decimal may be written as a common fraction.

194. Examples. 1. Reduce .0625 to a common fraction.

SOLUTION. .0625 means $\frac{625}{10000}$; but $\frac{625}{10000}$ may be expressed in simpler form. Dividing both terms of the fraction by 625, the result is $\frac{1}{16}$.

$$\frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

WRITTEN EXERCISE

Reduce to a common fraction or to a mixed number:

1. 0.375.	5. 0.9375.	9. 0.0335.	13. 260.675.
2. 0.0625.	6. $1.66\frac{2}{3}$.	10. $0.0056\frac{1}{4}$.	14. 126.1875.
3. 0.0016.	7. 0.4375.	11. 181.875.	15. 175.0625.
4. 0.5625.	8. 0.125.	12. 171.245.	16. 172.0075.

195. A common fraction may be written as a decimal.

196. Example. Reduce $\frac{3}{8}$ to a decimal.

SOLUTION. $\frac{3}{8}$ equals $\frac{1}{8}$ of 3 units. 3 units equals 3000 thousandths. $\frac{1}{8}$ of 3000 thousands equals 375 thousandths (.375).

$$8\overline{)3.000} \quad .375$$

ORAL EXERCISE

1. Reduce to equivalent decimals: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{3}{16}$, $\frac{1}{9}$, $\frac{1}{11}$.
2. Reduce to common fractions: .5, .25, .50, .75, $.33\frac{1}{3}$, $.66\frac{2}{3}$, $.16\frac{2}{3}$, $.12\frac{1}{2}$, .6, .4, .60, .40, .2, $.83\frac{1}{3}$, .20, $.08\frac{1}{3}$, .375, .125, $.37\frac{1}{2}$, $.87\frac{1}{7}$, .875, .0625, $.11\frac{1}{9}$, $.09\frac{1}{11}$.

WRITTEN EXERCISE

Reduce to equivalent decimals:

1. $\frac{7}{8}$.	3. $\frac{9}{16}$.	5. $\frac{9}{320}$.	7. $\frac{17}{480}$.	9. $\frac{5}{6400}$.	11. $21\frac{5}{8}$.
2. $\frac{5}{16}$.	4. $\frac{7}{16}$.	6. $\frac{11}{16}$.	8. $\frac{7}{25}$.	10. $5\frac{7}{12}$.	12. $165\frac{17}{20}$.

THE SOLUTION OF PROBLEMS

197. The steps in the solution of a problem are: (1) reading the problem to find what is given and what is required; (2) determining from what is given how to find what is required; (3) outlining a process of computation and then performing it; (4) checking results.

198. A problem should be thoroughly understood before any attempt is made to solve it; and when the relation of what is given to what is required has been discovered, the process of computation should be briefly indicated and then performed as briefly and rapidly as possible.

199. To insure accuracy the work should always be checked in some manner. If the answer to the problem is estimated in advance, it will prove an excellent check against absurd results.

Thus, 42 doz. boys' hose at \$48 a dozen is equal to approximately $40 \times \$50$; $9\frac{1}{3}\%$ of 1290 bu. is equal to approximately $\frac{1}{10}$ of 1290 bu.; etc.

200. Example. A tailor used 30 yd. of flannel in making 18 waistcoats; at that rate how many yards will he require in making 45 waistcoats?

SOLUTION

1. The quantity needed in making 18 waistcoats is given and the quantity needed in making 45 waistcoats is required.

2. One waistcoat requires $\frac{30}{18}$ yd.; 45 waistcoats will require 45 times $\frac{30}{18}$ yd.

3. $\frac{\overset{15}{\cancel{45}} \text{ times } \overset{5}{\cancel{30}}}{\underset{3}{\cancel{18}}} = 75$; that is 75 yd. of flannel are required in making 45 waistcoats.

4. $\frac{30}{18}$ yd. $= \frac{5}{3}$ yd.; $\frac{75}{45}$ yd. $= \frac{5}{3}$ yd.; therefore the work is probably correct.

201. If reasons for conclusions, processes, and results are given, they should be brief and accurate. It is also a mistake to try to use the language of the book or the instructor. Such artificial work stifles thought and conceals the condition of the learner.

The subject of analysis should not be unduly emphasized. A correct solution may generally be accepted as evidence that the correct analysis has been made.

ORAL EXERCISE

In the following problems first find each result as required, and then give a brief, accurate explanation of the steps taken in the solution. Do not use pen or pencil.

1. If 2 T. cost \$8, what will 5 T. cost?

SUGGESTION. \$20; since 2 T. cost \$8, 5 T., which are $2\frac{1}{2}$ times 2 T., will cost $2\frac{1}{2}$ times \$8, or \$20.

2. 24 is $\frac{6}{7}$ of what number? $\frac{3}{8}$ of what number? $\frac{2}{13}$ of what number?

3. 220 is $\frac{1}{2}$ less than what number? 450 is $\frac{1}{3}$ less than what number?

4. A, having spent $\frac{1}{5}$ of his money, finds he has \$84 left. How much had he at first?

5. \$124 is $\frac{1}{3}$ more than what sum of money? \$300 is $\frac{1}{4}$ more than what sum of money?

6. A man sold $\frac{5}{16}$ of an acre of land for \$35. At that rate what is his entire farm of 100 acres worth?

7. A man bought a stock of goods and sold it at $\frac{1}{4}$ above cost. If he received \$275, what was the cost of the goods?

8. B bought a stock of goods which he sold at $\frac{1}{5}$ below cost. If he received for the sale of the goods \$240, what was the cost and what was his loss?

9. $\frac{2}{16}$ of the students in a high school are girls and the remainder are boys. If the number of boys is 350, how many scholars in the school?

10. A bought a quantity of wheat which he sold at $\frac{1}{2}$ above cost. If he received \$300 for the wheat, what did it cost him and what was his gain?

11. A bought a quantity of dry goods and sold them so as to realize $\frac{1}{3}$ more than the cost. If the selling price was \$720, what was the cost and what was the gain?

12. D bought a stock of carpeting which he was obliged to sell at $\frac{1}{6}$ below cost. If he received \$750 for the sale of the carpeting, what was the cost of same, and what was his loss?

WRITTEN EXERCISE

In the following problems give both analysis and computation.

1. If $\frac{1}{2}$ lb. of tea cost 21¢, what will $9\frac{1}{2}$ lb. cost?

COMPUTATION	ANALYSIS
$9\frac{1}{2} = \frac{19}{2}$	$9\frac{1}{2} = \frac{19}{2}$; $9\frac{1}{2}$ is therefore 19 times $\frac{1}{2}$. If $\frac{1}{2}$ lb. cost
$19 \times 21¢ = \$3.99$	21¢, $9\frac{1}{2}$ lb. will cost 19 times 21¢, or \$3.99.

2. If $\frac{1}{2}$ of a pound of tea cost 42¢, what will $35\frac{1}{2}$ lb. cost?

3. If a drain can be dug in 17 da. by 45 men, how many men will it take to dig $\frac{1}{5}$ of it in 3 da.?

4. In what time will 3 boys at $\$0.62\frac{1}{2}$ per day earn as much as 4 men at \$2.25 each per day will earn in $45\frac{3}{4}$ da.?

5. A spends \$72 per week or $\frac{3}{4}$ of his income; B saves \$48 per week or $\frac{3}{8}$ of his income. How long will it take A to save as much as B saves in five weeks?

6. If 115 bu. of wheat are required to make 23 bbl. of flour, how many bushels will be required to make 50 bbl. of flour? 117 bbl. of flour? 259 bbl. of flour?

ORAL REVIEW EXERCISE

1. $.05 \times 6 \times 0 \times 2\frac{1}{2} = ?$

2. \$0.75 is what part of \$3?

3. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{10}$?

4. Find the value of $.45 + (.25 \times 5) - .04$.

5. 60 is $\frac{3}{5}$ of what number? $\frac{3}{4}$? $\frac{3}{7}$? $\frac{4}{5}$? $\frac{5}{6}$?

6. At 25¢ a yard, what will $2\frac{1}{2}$ yd. of cloth cost?

7. $\frac{6}{7}$ is $\frac{1}{2}$ of what number? $\frac{2}{5}$ is $\frac{3}{4}$ of what number?

8. If $\frac{3}{4}$ of an acre of land costs \$75, what will 50 A. cost?

9. If $\frac{2}{5}$ of a number is 84, what is 5 times the same number?

10. The dividend is $4\frac{1}{2}$ and the quotient is $6\frac{3}{4}$; what is the divisor?

11. If 6 bu. of apples cost \$15, what will 80 bu. cost at the same rate?

12. At \$460 per half mile, what will be the cost of grading 6 mi. of road?

13. How much will 4 carpenters earn in 10 da. at the rate of \$2.25 each per day?

14. At \$4.50 per cord, what will be the cost of $4\frac{1}{2}$ cd. of wood? of $6\frac{1}{2}$ cd.? of $12\frac{1}{2}$ cd.? of $7\frac{1}{2}$ cd.?

15. A bought a horse for \$96 and sold it for $\frac{7}{8}$ of its cost. What part of the cost was the loss sustained?

16. A bought $4\frac{1}{4}$ yd. of velvet at \$5.20 per yard and gave in payment a \$50 bill. How much change should he receive?

17. I sold 5 A. of land for \$375 and sustained a loss equal to $\frac{1}{6}$ of the original cost of the land. What did the land cost per acre?

18. D and E agree to mow a field for \$36. If D can do as much in 2 da. as E can do in 3, how should the money be divided?

19. N sold a watch to O and received $\frac{1}{3}$ more than it cost him. If O paid \$64 for the watch, what did it cost N? What per cent did N gain?

20. A earns \$125 per month. Of this sum he spends \$75 and saves the remainder. What part of his monthly earnings does he save? What per cent?

WRITTEN REVIEW EXERCISE

1. Find the cost of 1100 eggs at $23\frac{3}{4}¢$ per dozen.

2. Counting 2000 lb. to a ton, find the cost of $5\frac{7}{8}$ T. of steel at $1\frac{5}{16}¢$ per pound.

3. When flour is sold at \$6.02 per barrel of 196 lb., what should be paid for $55\frac{1}{2}$ lb.?

4. I bought 300 bbl. of flour at \$5.75 per barrel. At what price must I sell it per barrel in order to gain \$150?

5. The cost of 200 bu. of wheat was \$204.50 and the selling price \$212.35. What was the gain per bushel?

6. A can do a piece of work in $5\frac{1}{2}$ da. and B in $7\frac{1}{2}$ da. If they join in the completion of the work, how long will it take them?

7. How much will 7 men earn in 6 da., working 10 hr. per day, at 25 ¢ per hour?

8. At \$2.50 per day of 8 hr., how much should a man receive for $11\frac{1}{2}$ hours' work?

9. A boy works $4\frac{1}{2}$ da. at the rate of \$5.75 per week of 6 da. How much does he earn?

10. W, in $\frac{1}{3}$ of a day, earns \$1.25, and Y, in $\frac{1}{2}$ of a day, earns $\$0.87\frac{1}{2}$. How much will the two together earn in $40\frac{1}{2}$ da.?

11. A and B together can do a piece of work in 10 da. If A can complete the work alone in 16 da., how long will it take B to do it?

12. Nov. 1, in a recent year, was on Tuesday. How much did B earn during November if he was employed every working day at the rate of \$3.75 per day?

13. A farm is divided into 6 fields containing, respectively, $25\frac{5}{8}$, $26\frac{7}{16}$, $32\frac{3}{4}$, $56\frac{7}{8}$, $35\frac{9}{16}$, and $52\frac{1}{16}$ A. How much is the farm worth at \$37.50 per acre?

14. Find the total cost of: 630 lb. sugar at $4\frac{3}{4}$ ¢; 375 lb. tea at $38\frac{1}{2}$ ¢; 240 lb. crackers at $5\frac{7}{8}$ ¢; 65 lb. rice at $7\frac{9}{16}$ ¢; $52\frac{1}{2}$ lb. raisins at $7\frac{1}{2}$ ¢; and 250 lb. coffee at $24\frac{3}{4}$ ¢.

15. A retailer bought 5 bbl. of flour at \$6.50 per barrel, 12 bu. potatoes at 75 ¢ per bushel, and gave in payment a fifty-dollar bill. How much change should he receive?

16. Five garden lots measuring $2\frac{7}{8}$, $10\frac{1}{2}$, $12\frac{5}{8}$, $6\frac{7}{16}$, and $8\frac{9}{16}$ A. respectively, were bought at $\$212.87\frac{1}{2}$ per acre and sold at \$250.50 per acre. Find the gain resulting from the transaction.

17. I bought 4120^2 yd. of silk at \$1.02 per yard and sold $\frac{3}{4}$ of it at \$1.50 per yard, and the remainder for \$1600. What was the average price received per yard, and how much did I gain?

18. A, B, C, and D hire a pasture for \$419.50. A put in 25 head of cattle for 4 wk.; B, 31 head for 5 wk.; C, 44 head for 6 wk.; and D, 40 head for 8 wk. How much should each be required to pay?

19. A grain dealer bought $6750\frac{1}{2}$ bu. of corn at $60\frac{1}{2}¢$ per bushel, and $2130\frac{1}{4}$ bu. of oats at $32\frac{3}{4}¢$ per bushel. He sold the corn at $69\frac{7}{8}¢$ per bushel, and the oats at $39\frac{5}{8}¢$ per bushel. Did he gain or lose, and how much?

20. A grocer bought 15 bbl. of molasses, each containing 50 gal., at $25\frac{1}{2}¢$ per gallon. He retailed $150\frac{1}{2}$ gal. of it at $30¢$ per gallon, $170\frac{1}{4}$ gal. at $28¢$ per gallon, and the remainder at $35¢$ per gallon. Did he gain or lose, and how much?

21. Find the cost of 25 bx. of cheese weighing: $67-4$, $62-4$, $61-3$, $72-4$, $81-5$, $64-4$, $66-3$, $65-5$, $61-4$, $62-3$, $64-4$, $66-3$, $65-5$, $61-4$, $62-3$, $64-4$, $67-3$, $65-5$, $60-3$, $62-4$, $67-4$, $65-4$, $60-4$, $68-3$, $65-4$ lb., respectively, at $11\frac{3}{4}¢$ per pound.

22. A dry-goods merchant bought 25 pc. of Scotch cheviot containing 42^1, 40^2, 45^3, 41^1, 40^1, 45^2, 42^1, 43^3, 38^1, 35^1, 36^2, 41^2, 44^1, 45^2, 39^1, 37^1, 42^2, 47, 41, 42^1, 43^3, 40^1, 47^1, 38, 31 yd., respectively, at $39\frac{1}{2}¢$ per yard. If he sold the entire purchase at $43\frac{3}{4}¢$ per yard, did he gain or lose, and how much?

23. C. W. Bender failed in business. He owes A \$712.25; B, \$1421.25; C, \$625.25; D, \$1460.75; his entire resources amount to \$2109.75. What fractional part of his indebtedness can he pay? what per cent? How many cents on \$1? If his creditors accept payment on this basis, how much will each receive?

24. A dry-goods merchant bought 12 pc. of striped denim containing 40^1, 45^1, 40^1, 48^2, 41^2, 40^3, 45^2, 41^1, 44^2, 39^2, 51^1, 38 yd., respectively, at $14\frac{3}{4}¢$ per yard; 15 pieces of cashmere containing 39^1, 41^2, 42^1, 45^2, 39, 52, 40, 45, 46, 51, 47^2, 42^1, 41^1, 47^1, 48 yd., respectively, at \$1.12 per yard; 10 pc. wash silk containing 35^1, 30, 31^2, 30, 30, 30, 32^3, 32, 31^1, 32 yd., respectively, at $31¢$ per yard. He gave in payment, cash, \$300, and a 60-da. note for the balance. What was the face of the note?

25. Find the amount of the following bill:

Boston, Mass., Apr. 15, 19

MESSRS. CHARLES H. PALMER & CO.
Springfield, Mass.

Bought of EDGAR W. TOWNSEND & CO.

Terms: cash

250	lb. Rio Coffee	$0.24⅞
450	" Mocha Coffee	.20⅛
172	doz. Eggs	.24⅞
990	lb. White Sugar	.04¼
900	" Brown Sugar	.03⅛
975	" Granulated Sugar	.04⅝
172	" Butter	.16½
3021	" Ham	.13⅛
280	" Cream Codfish	.07⅛
11	pails Mackerel	1.87½
120	lb. Raisins	.07⅝
480	" Starch	.03⅛
225	" Japan Tea	.26⅛
210	" Young Hyson Tea	.24⅞
420	" Oolong Tea	.27⅞
157	" Pearl Tapioca	.03½
17	pkg. Yeast Cakes	.37½
375	lb. Java Coffee	.23⅝

26. C's salary is $17.50 per week of 48 hr. How much should he be paid for 11 da., working 9 hr. per day?

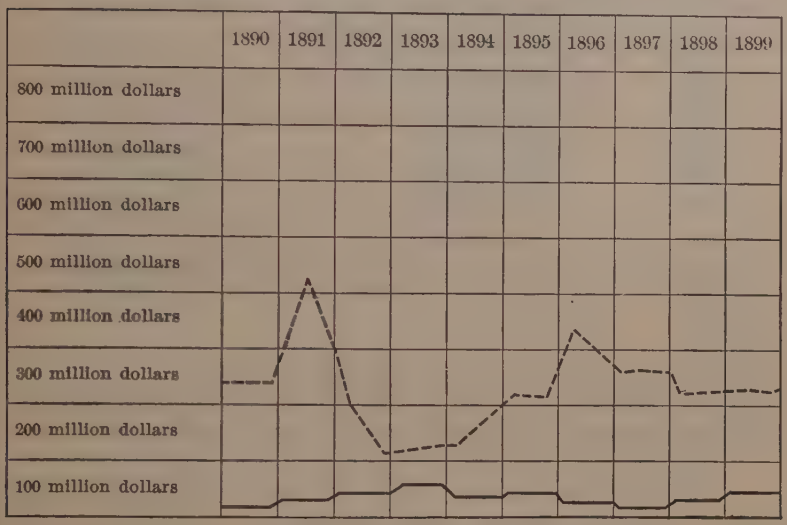
27. A man earning $2.75 per day of 10 hr. lost 7½ hr. during one week of 6 da. How much should he receive for the week's work?

28. E begins work at 7:30 A.M. and quits work at 6:30 P.M. If he is paid at the rate of $3.75 per day of 8 hr. and he takes the noon hour off for lunch, how much should he receive for his day's labor?

29. A factory foreman is paid $3.75 per day of 8 hr. and $0.50 an hour for overtime. How much should he be paid for a week in which he begins work at 7 o'clock A.M., quits work at 7:30 o'clock P.M., and takes 1½ hr. off each day for lunch?

GRAPHIC OUTLINE

A comparison of the money value of the wheat crop, and the fire losses paid by insurance companies, in the United States, 1890 to 1899 inclusive. - - - - - - - - - - - -value of wheat crop. ————————value of fire losses.



WRITTEN EXERCISE

1. The figures below give the value of the wheat crop, and the fire losses paid by fire insurance companies, in the United States, for the years 1890 to 1899 inclusive. (See Graphic Outline.)

	Farm Val. of Wheat	Fire Losses
1890	$334,773,678	$108,993,792
1891	513,473,711	143,764,967
1892	322,111,881	151,516,098
1893	213,171,381	167,544,370
1894	225,902,025	140,006,484
1895	237,938,998	142,110,233
1896	310,602,593	118,737,420
1897	428,547,121	116,254,575
1898	392,770,320	130,593,505
1899	319,545,259	153,597,830

2. Make a graphic outline comparing the wheat crop, and the fire losses paid by the fire insurance companies, in the United States, for the years 1880 to 1889 inclusive.

	Farm Val. of Wheat	Fire Losses
1880	$474,201,850	$74,643,400
1881	456,880,427	81,280,900
1882	445,602,123	84,505,024
1883	383,649,272	100,149,228
1884	330,862,260	110,008,611
1885	275,320,390	102,813,796
1886	314,226,020	104,924,750
1887	310,612,960	120,283,055
1888	385,248,030	110,885,665
1889	342,491,707	123,046,833

3. Make a graphic outline comparing the wheat crop, the cotton crop, and the fire losses paid by the fire insurance companies, in the United States, for the years 1900 to 1909 inclusive.

	Farm Val. of Wheat	Farm Val. of Cotton	Fire Losses
1900	$323,515,177	$515,828,431	$160,929,805
1901	467,350,156	439,166,710	165,817,810
1902	422,224,117	501,897,135	161,087,040
1903	443,024,826	660,549,230	145,302,155
1904	510,489,874	652,031,626	229,198,050
1905	518,372,727	632,298,332	165,221,650
1906	490,332,760	721,647,237	518,611,800
1907	554,437,000	700,956,011	215,084,709
1908	616,826,000	681,230,956	217,885,850
1909	730,046,000	812,089,833	188,705,150

Use a dotted line to represent the cotton crop.

The figures representing the fire losses do not include the cost of maintaining fire departments, nor the losses sustained by the interruption of business.

The United States exceeds all other countries in losses by fire. A large per cent of these losses are due to carelessness.

ORAL REVIEW EXERCISE

1. $\frac{3}{4}$ of 36 is what part of 81?
2. Multiply 126 by 101; 92 by 102.
3. Divide $4\frac{1}{2}$ by $2\frac{3}{4}$; $3\frac{2}{3}$ by $2\frac{1}{2}$.
4. Find the cost of each of the following:
   a. 35 bu. of seed at 35¢ per bushel.
   b. 65 A. of land at $65 per acre.
   c. 45 yd. of cloth at 45¢ per yard.
5. Divide $\frac{2}{3}$ by $\frac{4}{5}$; $\frac{8}{9}$ by $\frac{4}{7}$; $\frac{5}{6}$ by $\frac{2}{5}$.
6. Multiply 36 by $\frac{3}{4}$; 49 by $\frac{6}{7}$; 55 by $\frac{5}{11}$.
7. What is the square of 15? of 1.5? of $1\frac{1}{2}$?
8. $64 \times \frac{1}{2} = ?$ $64 \div \frac{1}{2} = ?$ $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = ?$
9. How many yards of cloth can be bought for $25 at $12\frac{1}{2}$¢ per yard?
10. If it costs $7.50 to harvest $6\frac{1}{2}$ A. of corn, what will it cost to harvest 65 A.?
11. An agent received $7.20 for collecting a debt, and the merchant received $232.80. What was the total debt?
12. C and D received $21.75 for work done jointly. If C does only half as much work as D, how should the money be divided?
13. If a lot of articles were bought at the rate of 3 for 2¢ and sold at the rate of 2 for 3¢, how many must be sold to gain $5?
14. A and B received $34 for work done jointly. If A can do as much work in 8 da. as B can do in 9 da., how should the money be divided?
15. Name the results quickly:

a. A can do a piece of work in 4 da., and B in 5 da. If they work together, in how many days will they finish the task?

b. F can do a piece of work in $3\frac{1}{3}$ da., and G in 5 da. If they work together, in how many days will they finish the task?

c. C can do a piece of work in 2 da., D in 3 da., and E in 4 da. If they work together, in how many days will they finish the task?

d. H and J together can do a piece of work in 20 da. If H alone can do the work in 30 da., in how many days can J alone do the work?

WRITTEN EXERCISE

Problems of the Farm

1. If 15 sheep consume 5785 lb. of dry fodder in a year, what is the cost per sheep if the fodder is worth $8.50 per ton?

2. If eggs are worth 24¢ per dozen, what is the difference in the value of two hens, in a year, if one lays 180 eggs and the other lays 96 eggs?

3. An apple tree produced $9\frac{1}{4}$ bu. of apples, $6\frac{1}{2}$ bu. of which graded as "firsts" and the remainder as "seconds." What fractional part of the yield were firsts, and what fractional part were seconds?

4. The apple tree referred to in Ex. 3 was sprayed the year following, and that year it produced $10\frac{1}{2}$ bu. of which $9\frac{1}{4}$ bu. were firsts and the remainder seconds. What fractional part of the yield were firsts? What part were seconds?

5. If the apples referred to in Exs. 3 and 4 were sold at $1.20 per bushel for firsts and 70¢ a bushel for seconds, what was the value of the spraying?

6. It is estimated that a quail in one year eats 28¢ worth of grain and saves $1.68 worth of grain by destroying insects and weeds. What is the value of a pair of quails to the farmer annually, not counting the value of the brood?

7. An undrained field produced 24 bu. of grain per acre, and after being drained it produced 33 bu. per acre. What was the fractional increase? What was the value of the increase if the grain sold for $55\frac{1}{2}$¢ per bushel?

8. A flock of hens averaged 78 eggs each per year. What would be the value to the farmer of introducing a better breed of hens that would produce 120 eggs each per year, if he kept a flock of 40 hens, and received 24¢ per dozen for the eggs?

9. If 6 A. of unfertilized land produced 275 bu. of corn, and if fertilized, it would have produced 350 bu., what would the farmer have gained by fertilizing the land if the corn was sold for 68¢ per bushel, and the fertilizer cost $24 per ton, and 400 lb. were used on each acre?

WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

1. A, B, and C hire a pasture for $81. A puts in 6 cows for 4 mo.; B, 6 cows for 6 mo.; and C, 6 cows for 5 mo. What sum should each pay?

2. A man owned $\frac{6}{7}$ of a tract of land; he sold $\frac{2}{3}$ of his share for $14,504.46. At that rate, what was the value of his original share? What was the whole tract worth?

3. The owner of a house received a net yearly income from rental of $408.90, after paying the following: insurance, $64.20; taxes, $74.50; repairs, $28.40. What was the monthly rental?

4. A man drew $\frac{1}{3}$ of his money from the bank and then paid bills of the following amounts: $12.50, $18.25, and $7.50; he then had left in cash $11.75. What sum had he in the bank before drawing the check?

5. A man placed a mortgage on his house and lot for $2967. The lot cost $2720; the improvements, $260.50; and the dwelling, $5920.50. The mortgage was what fractional part of the total value of the property?

6. At the end of a season a dealer sold a machine for $64, after reducing the marked price $\frac{1}{5}$. If he still gained $\frac{1}{7}$ of the cost, what was the first cost? The marked price was what fraction above the cost price?

7. From dictation, write results for the following: $\frac{1}{4}$ of $25\frac{1}{2}$; $\frac{1}{6}$ of $26\frac{1}{2}$; $\frac{1}{5}$ of $36\frac{1}{4}$; $\frac{1}{7}$ of $17\frac{1}{2}$; $\frac{1}{3}$ of $42\frac{2}{3}$; $\frac{1}{9}$ of $28\frac{1}{2}$; $\frac{1}{11}$ of $22\frac{1}{3}$; $\frac{1}{9}$ of $64\frac{1}{2}$; $\frac{1}{8}$ of $50\frac{1}{4}$; $\frac{1}{10}$ of $35\frac{1}{5}$; $\frac{1}{12}$ of $50\frac{1}{2}$.

8. A merchant closed his business under the following conditions: resources, $22,455.20; liabilities, $33,682.80. What fractional part of his debts can he pay? If he owes James S. Brown $202.50, how much will Brown receive in settlement?

9. A, B, and C are partners in a mercantile business in which A has invested $9180; B, $6120; and C, $3060. At the end of 1 yr. they divided a gain of $3060.90. If each partner received of the gain according to his fractional part of the investment, how much did each receive?

CHAPTER XIII

ALIQUOT PARTS

202. An **aliquot part** of a number is a part that is contained in the number an integral number of times.

Thus, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10.

ORAL EXERCISE

1. How many cents in $\$\frac{1}{2}$? in $\$\frac{1}{4}$? in $\$\frac{1}{8}$? in $\$\frac{1}{6}$?

2. What aliquot part of \$1 is $25¢$? $50¢$? $6\frac{1}{4}¢$? $12\frac{1}{2}¢$?

3. Read aloud the following, supplying the missing terms: $16 \times 50¢ = 16 \times \$\frac{1}{2} = \frac{1}{2}$ of \$16; $16 \times 25¢ = 16 \times \$\frac{1}{4} = \frac{1}{4}$ of \$16; $16 \times 12\frac{1}{2}¢ = 16 \times \$$ —— = —— of \$16; $16 \times 6\frac{1}{4}¢ = 16 \times \$$ —— = —— of \$16.

4. Give a short method for finding the cost when the quantity is given and the price is $50¢$; $25¢$; $12\frac{1}{2}¢$; $6\frac{1}{4}¢$.

5. What is the cost of 160 yd. of dress goods at \$1? at $50¢$? at $25¢$? at $12\frac{1}{2}¢$? at $6\frac{1}{4}¢$?

6. How many cents in $\$\frac{1}{3}$? in $\$\frac{1}{6}$? in $\$\frac{1}{12}$? in $\$\frac{1}{15}$? in $\$\frac{1}{7}$? in $\$\frac{1}{5}$? in $\$\frac{1}{10}$?

7. What aliquot part of \$1 is $33\frac{1}{3}¢$? $16\frac{2}{3}¢$? $8\frac{1}{3}¢$? $6\frac{2}{3}¢$? $14\frac{2}{7}¢$? $20¢$? $10¢$?

8. Read aloud the following, supplying the missing terms: $140 \times 14\frac{2}{7}¢ = 140 \times \$\frac{1}{7} = \frac{1}{7}$ of \$140; $90 \times 6\frac{2}{3}¢ = 90 \times \$$ —— = —— of \$90; $90 \times 20¢ = 90 \times \$$ —— = —— of \$90.

9. Read aloud the following, supplying the missing terms: $240 \times 33\frac{1}{3}¢ = 240 \times \$$ —— $= \frac{1}{3}$ of \$240; $240 \times 16\frac{2}{3} = 240 \times \$\frac{1}{6}$ = —— of \$240; $240 \times 12\frac{1}{2}¢ = 240 \times \$$ —— = —— of \$240.

10. Give a short method for finding the cost when the quantity is given and the price is $33\frac{1}{3}¢$; $16\frac{2}{3}¢$; $8\frac{1}{3}¢$; $6\frac{2}{3}¢$; $14\frac{2}{7}¢$.

11. Find the cost of 960 yd. of cloth at $33\frac{1}{3}¢$; at $16\frac{2}{3}¢$; at $8\frac{1}{3}¢$.

ORAL EXERCISE

State the cost of:

1. 240 lb. tea at 50¢; at $33\frac{1}{3}$¢; at 25¢.
2. 360 lb. coffee at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $12\frac{1}{2}$¢.
3. 720 gal. cider at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
4. 2400 doz. eggs at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢.
5. 2400 yd. prints at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢.
6. 960 yd. cotton at $6\frac{1}{4}$¢; at $8\frac{1}{3}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
7. 2040 yd. plaids at 50¢; at $33\frac{1}{3}$¢; at 25¢; at 20¢; at $16\frac{2}{3}$¢.
8. 480 lb. lard at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 10¢.
9. 3600 lb. raisins at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 20¢; at 25¢; at $33\frac{1}{3}$¢.
10. 480 yd. lining at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at 10¢; at $12\frac{1}{2}$¢; at $6\frac{2}{3}$¢.
11. 4200 yd. silesia at 10¢; at 20¢; at $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at $14\frac{2}{7}$¢.
12. 1500 yd. plaids at $1; at 50¢; at $33\frac{1}{3}$¢; at 25¢; at 20¢.
13. 420 yd. stripe at 10¢; at $12\frac{1}{2}$¢; at $14\frac{2}{7}$¢; at $16\frac{2}{3}$¢; at 25¢.
14. 120 yd. gingham at $8\frac{1}{3}$¢; at $6\frac{1}{4}$¢; at $6\frac{2}{3}$¢; at 10¢; at $12\frac{1}{2}$¢.
15. 1240 yd. wash silk at 25¢; at 50¢; at $33\frac{1}{3}$¢; at 20¢.
16. At the rate of 3 for 50¢, what will 27 handkerchiefs cost?
17. At $33\frac{1}{3}$¢ per half dozen, what will 12 doz. handkerchiefs cost? 17 doz.? 25 doz.? $7\frac{1}{2}$ doz.? $4\frac{1}{2}$ doz.?
18. A merchant bought cloth at $33\frac{1}{3}$¢ per yard and sold it at 50¢ per yard. What was his gain on 1680 yd.?

ORAL EXERCISE

1. What is the cost of $12\frac{1}{2}$ yd. of silk at 96¢ per yard?

SUGGESTION. The cost of $12\frac{1}{2}$ yd. at 96¢ = the cost of 96 yd. at $12\frac{1}{2}$¢. *Interchanging the multiplicand and multiplier considered as abstract numbers does not affect the product.*

2. Find the cost of 25 yd. of silk at $1.72 per yard.

SUGGESTION. The cost of 25 yd. at $1.72 (172¢) = the cost of 172 yd. at 25¢.

3. Find the cost of:

a. 25 yd. at 16¢. *c.* $6\frac{1}{4}$ lb. at 32¢. *e.* 25 yd. at 84¢.

b. $12\frac{1}{2}$ yd. at 48¢. *d.* $12\frac{1}{2}$ lb. at 80¢. *f.* $12\frac{1}{2}$ yd. at $1.75.

TABLE OF ALIQUOT PARTS

Nos.	$\frac{1}{2}$'s	$\frac{1}{4}$'s	$\frac{1}{8}$'s	$\frac{1}{16}$'s	$\frac{1}{3}$'s	$\frac{1}{6}$'s	$\frac{1}{12}$'s	$\frac{1}{15}$'s	$\frac{1}{5}$'s	$\frac{1}{10}$'s
1	.50	.25	.12$\frac{1}{2}$	.06$\frac{1}{4}$	.33$\frac{1}{3}$	.16$\frac{2}{3}$	.08$\frac{1}{3}$	.06$\frac{2}{3}$	.20	.10
10	5.	2$\frac{1}{2}$	1$\frac{1}{4}$	.62$\frac{1}{2}$	3$\frac{1}{3}$	1$\frac{2}{3}$	.83$\frac{1}{3}$	.66$\frac{2}{3}$	2.	1.
100	50.	25.	12$\frac{1}{2}$	6$\frac{1}{4}$	33$\frac{1}{3}$	16$\frac{2}{3}$	8$\frac{1}{3}$	6$\frac{2}{3}$	20.	10.
1000	500.	250.	125.	62$\frac{1}{2}$	333$\frac{1}{3}$	166$\frac{2}{3}$	83$\frac{1}{3}$	66$\frac{2}{3}$	200.	100.

WRITTEN EXERCISE

In the three problems following make all the extensions mentally.

1. Without copying, find quickly the total cost of:

84 lb. tea at 50¢.
75 lb. tea at 33$\frac{1}{3}$¢.
72 lb. coffee at 25¢.
84 lb. coffee at 33$\frac{1}{3}$¢.
25 lb. coffee at 28¢.
88 lb. candy at 12$\frac{1}{2}$¢.
24 lb. tapioca at 6$\frac{1}{4}$¢.
6$\frac{1}{4}$ lb. tea at 64¢.
25 lb. cocoa at 52¢.
12$\frac{1}{2}$ lb. cocoa at 48¢.
360 lb. codfish at 6$\frac{2}{3}$¢.
66 lb. crackers at 8$\frac{1}{3}$¢.
25 lb. chocolate at 36¢.
25 cs. horseradish at 64¢.

2. Without copying, find quickly the total cost of:

25 yd. silk at 84¢.
12$\frac{1}{2}$ yd. silk at 96¢.
750 pc. lace at 6$\frac{2}{3}$¢.
112 yd. ticking at 6$\frac{1}{4}$¢.
210 yd. plaids at 33$\frac{1}{3}$¢.
128 gro. buttons at 12$\frac{1}{2}$¢.
68 yd. lansdowne at 50¢.
77 yd. duck at 14$\frac{2}{7}$¢.
6$\frac{1}{4}$ gro. buttons at 32¢.
155 yd. cheviot at 20¢.
96 yd. gingham at 8$\frac{1}{3}$¢.
84 yd. shirting at 12$\frac{1}{2}$¢.
25 doz. spools thread at 25¢.
168 yd. striped denim at 8$\frac{1}{3}$ ¢.

3. Without copying, find quickly the total cost of:

25 bu. corn at 64¢.
25 bu. corn at $0.72.
12$\frac{1}{2}$ bu. oats at $0.36.
25 bu. beans at $2.80.
12$\frac{1}{2}$ bu. wheat at $1.04.
12$\frac{1}{2}$ bu. millet at $1.24.
25 bu. clover seed at $3.60.
50 bu. clover seed at $3.75.
25 bu. corn at $0.84.
25 bu. corn at $0.44.
25 bu. oats at $0.35.
12$\frac{1}{2}$ bu. rye at $1.04.
6$\frac{1}{4}$ bu. wheat at $1.20.
6$\frac{1}{4}$ bu. wheat at $1.12.
25 bu. timothy seed at $2.40.
50 bu. timothy seed at $2.75.

ORAL EXERCISE

1. Multiply by 10: 4; 15; .07; 8¢; $1.12; $24.60; $12.125.

2. Multiply by 100: 3; 17; .09; 12¢; $1.64; $21.17.

3. Multiply by 1000: 7; 29; .19; 15¢; $1.75; $23.72.

4. What aliquot part of $10 is $2.50? Find the cost of 16 articles at $10 each; at $2.50 each.

5. Find the cost of 84 bu. of wheat at $1.25.

SOLUTION. $1.25 is $\frac{1}{8}$ of $10. 84 bu. at $10 = $840; $\frac{1}{8}$ of $840 = $105.

6. Formulate a short method for finding the cost when the quantity is given and the price is $1.25.

SOLUTION. $1.25 is $\frac{1}{8}$ of $10; hence, *multiply the quantity by 10 and take $\frac{1}{8}$ of the product.*

7. Formulate a short method for finding the cost when the quantity is given and the price is $2.50; 3.33\frac{1}{3}$; 1.66\frac{2}{3}$.

8. Find the cost of 168 yd. of cloth at $1.25; at $2.50; at 3.33\frac{1}{3}$; at 1.66\frac{2}{3}$.

9. What aliquot part of $100 is $25? $12.50? $6.25?

10. Find the cost of 72 chairs at $25 each.

SOLUTION. 72 chairs at $100 = $7200; but the price is $25, which is $\frac{1}{4}$ of $100; therefore, $\frac{1}{4}$ of $7200, or $1800, is the required cost.

11. Give a short method for multiplying any number by 25; by 12$\frac{1}{2}$; by 6$\frac{1}{4}$; by 33$\frac{1}{3}$; by 8$\frac{1}{3}$.

12. Find the cost of 25 T. coal at $7.20; of 6$\frac{1}{4}$ T.; of 12$\frac{1}{2}$ T.

13. What aliquot part of 1000 is 250? 500? 125? 62$\frac{1}{2}$? 333$\frac{1}{3}$? 166$\frac{2}{3}$? 200? 100? 83$\frac{1}{3}$? 66$\frac{2}{3}$?

14. Formulate a short method for multiplying a number by 250.

SOLUTION. Since $250 = \frac{1000}{4}$, *multiply by 1000 and take $\frac{1}{4}$ of the product.*

15. Give a short method for finding the cost when the quantity is given and the price is $125; 166\frac{2}{3}$.

16. Multiply 84 by 50; by 25; by 12$\frac{1}{2}$; by 16$\frac{2}{3}$; by 33$\frac{1}{3}$.

17. Multiply 160 by 2$\frac{1}{2}$; by 1$\frac{1}{4}$; by 12$\frac{1}{2}$; by 125; by 62$\frac{1}{2}$.

18. Multiply 240 by 3$\frac{1}{3}$; by 1$\frac{2}{3}$; by 33$\frac{1}{3}$; by 16$\frac{2}{3}$; by 333$\frac{1}{3}$.

19. Find the cost of 250 sofa beds at $32 each.

SOLUTION. The cost of 250 beds at $32 = the cost of 32 beds at $250. The cost of 32 beds at $1000 = $32,000; but the price is $250, which is $\frac{1}{4}$ of $1000; therefore, $\frac{1}{4}$ of $32,000, or $8000, is the required cost.

20. Find the cost of 720 couches at $12.50 each.

21. Find the cost of 440 lb. sugar at $2\frac{1}{2}¢$.

SOLUTION. $2\frac{1}{2}¢$ is $\frac{1}{4}$ of $10¢$. The cost of 440 lb. at $10¢ = $44; but the price is $2\frac{1}{2}¢$, therefore, $\frac{1}{4}$ of $44, or $11 = the required cost.

22. Formulate a short method for finding the cost when the quantity is given and the price is $1\frac{1}{4}¢$.

SOLUTION. $1\frac{1}{4}¢ = \frac{1}{8}$ of $10¢$; hence, *point off one place in the quantity and take $\frac{1}{8}$ of the result.*

23. Give a short method for finding the cost when the quantity is given and the price is $2\frac{1}{2}¢$; $3\frac{1}{3}¢$; $1\frac{2}{3}¢$.

24. Find the cost of 160 lb. at $2\frac{1}{2}¢$; at $1\frac{1}{4}¢$; at $3\frac{1}{3}¢$; at $1\frac{2}{3}¢$. Also of 240 lb. at each of these prices.

25. Find the cost of 2400 lb. at $2\frac{1}{2}¢$; at $1\frac{1}{4}¢$; at $3\frac{1}{3}¢$; at $1\frac{2}{3}¢$. Also of 360 lb. at each of these prices.

ORAL EXERCISE

By inspection find the cost of:

1. 25 lb. tea at $54¢$.
2. 25 lb. tea at $33\frac{1}{3}¢$.
3. 125 lb. tea at $64¢$.
4. $6\frac{1}{4}$ A. land at $112.
5. 25 T. coal at $8.40.
6. 25 T. coal at $5.20.
7. 18 T. coal at $6.25.
8. 164 A. land at $25.
9. 72 T. coal at $6.25.
10. 250 yd. silk at $88¢$.
11. 250 yd. silk at $96¢$.
12. 25 pc. lace at $6.60.
13. 250 yd. silk at $1.12.
14. 192 A. land at $12.50.
15. 165 gro. buttons at $33\frac{1}{3}¢$.
16. $1\frac{1}{4}$ yd. silk at $88¢$.
17. 64 pc. lace at $1.25.
18. 125 yd. silk at $1.12.
19. 1250 bbl. beef at $24.
20. 78 yd. velvet at $2.50.
21. $2\frac{1}{2}$ bu. potatoes at $96¢$.
22. 640 bu. apples at $87\frac{1}{2}¢$.
23. 840 yd. prints at $16\frac{2}{3}¢$.
24. $12\frac{1}{2}$ bu. potatoes at $64¢$.
25. 84 bookcases at $12.50.
26. 810 bbl. pork at $12.50.
27. 125 yd. crepon at $3.60.
28. $12\frac{1}{2}$ yd. cheviot at $1.04.
29. 24 oak sideboards at $125.
30. $12\frac{1}{2}$ yd. gunner's duck at $48¢$.

WRITTEN EXERCISE

In the following problems make all the extensions mentally. See how many of the problems can be done in 10 minutes.

1. Without copying, find the total cost of:

425 lb. at 10 ¢.	2500 lb. at 64 ¢.	24 lb. at $1\frac{1}{4}$ ¢.
310 lb. at 20 ¢.	1600 lb. at 25 ¢.	48 lb. at $2\frac{1}{2}$ ¢.
100 lb. at 14 ¢.	1893 lb. at $3\frac{1}{3}$ ¢.	$2\frac{1}{2}$ lb. at 96 ¢.
1000 lb. at 27 ¢.	2500 lb. at 14 ¢.	125 lb. at 24 ¢.
1000 lb. at $4\frac{1}{2}$ ¢.	1400 lb. at 25 ¢.	192 lb. at $3\frac{1}{3}$ ¢.
1250 lb. at 44 ¢.	1250 lb. at 88 ¢.	88 lb. at $12\frac{1}{2}$ ¢.

2. Without copying, find the total cost of:

88 yd. at $1\frac{1}{4}$ ¢.	174 yd. at 10 ¢.	24 yd. at 12 ¢.
72 yd. at $3\frac{1}{3}$ ¢.	123 yd. at 11 ¢.	78 yd. at $3\frac{1}{3}$ ¢.
104 yd. at $2\frac{1}{2}$ ¢.	127 yd. at 11 ¢.	165 yd. at 20 ¢.
480 yd. at $6\frac{1}{4}$ ¢.	246 yd. at 25 ¢.	114 yd. at $6\frac{2}{3}$ ¢.
360 yd. at $8\frac{1}{3}$ ¢.	171^2 yd. at 10 ¢.	1280 yd. at $6\frac{1}{4}$ ¢.
121 yd. at 11 ¢.	178^3 yd. at 10 ¢.	192 yd. at $33\frac{1}{3}$ ¢.

3. Copy and find the total cost of:

450 lb. at $1\frac{1}{2}$ ¢.	249 lb. at 25 ¢.	$6\frac{1}{4}$ lb. at 88 ¢.
820 lb. at $1\frac{1}{2}$ ¢.	240 lb. at $3\frac{1}{3}$ ¢.	92 lb. at $2\frac{1}{2}$ ¢.
1200 lb. at $4\frac{1}{4}$ ¢.	200 lb. at $3\frac{1}{2}$ ¢.	$12\frac{1}{2}$ lb. at 24 ¢.
1400 lb. at $6\frac{1}{2}$ ¢.	450 lb. at $6\frac{2}{3}$ ¢.	18 lb. at $4\frac{1}{2}$ ¢.
7961 lb. at 50 ¢.	$79\frac{1}{2}$ lb. at 40 ¢.	125 lb. at 18 ¢.
1293 lb. at 30 ¢.	$78\frac{1}{2}$ lb. at 50 ¢.	648 lb. at $6\frac{1}{4}$ ¢.
1480 lb. at 40 ¢.	750 lb. at $33\frac{1}{3}$ ¢.	1900 lb. at $4\frac{1}{4}$ ¢.

4. Copy and find the total cost of:

750 gal. at $8\frac{1}{3}$ ¢.	99 gal. at 30 ¢.	360 gal. at 5 ¢.
488 gal. at $6\frac{2}{3}$ ¢.	60 gal. at $6\frac{2}{3}$ ¢.	625 gal. at 64 ¢.
640 gal. at $6\frac{1}{4}$ ¢.	50 gal. at 76 ¢.	810 gal. at $1\frac{1}{2}$ ¢.
194 gal. at 50 ¢.	25 gal. at 74 ¢.	920 gal. at $2\frac{1}{2}$ ¢.
176 gal. at 25 ¢.	$12\frac{1}{2}$ gal. at 88 ¢.	165 gal. at $6\frac{2}{3}$ ¢.
280 gal. at $12\frac{1}{2}$ ¢.	79 gal. at $33\frac{1}{3}$ ¢.	240 gal. at $62\frac{1}{2}$ ¢.
720 gal. at $33\frac{1}{3}$ ¢.	20 gal. at $1.79.	666 gal. at $66\frac{2}{3}$ ¢.
366 gal. at $16\frac{2}{3}$ ¢.	$6\frac{1}{4}$ gal. at $1.96.	1680 gal. at $16\frac{2}{3}$ ¢.

ORAL EXERCISE

1. How much less than \$1 is 75¢? what fractional part of \$1 less?

2. Find the cost of 144 pc. of lace at 75¢ per piece.

SOLUTION. At \$1 per piece the cost would be \$144; but the cost is not \$1 but $\frac{1}{4}$ less than \$1. Deducting $\frac{1}{4}$ of \$144, the result is \$108, the required cost.

3. Find the cost of 124 bookcases at \$7.50.

SOLUTION. \$7.50 is $\frac{1}{4}$ less than \$10. \$1240 less $\frac{1}{4}$ of itself = \$930, the required result.

4. Formulate a rule for multiplying a number by .75; by $7\frac{1}{2}$; by 75; by 750.

5. How much more than \$1 is $\$1.12\frac{1}{2}$? What fractional part of \$1 more?

6. Find the cost of 84 yd. of silk at $\$1.16\frac{2}{3}$ per yard.

SOLUTION. At \$1 per yard, the cost would be \$84; but $\$1.16\frac{2}{3}$ is $\frac{1}{6}$ more than \$1. Adding $\frac{1}{6}$ of \$84 to itself, the result is \$98, the required cost.

7. Formulate a short method for finding the cost when the quantity is given and the price is $\$1.12\frac{1}{2}$; $\$1.16\frac{2}{3}$; $\$1.33\frac{1}{3}$; \$11.25; \$112.50.

8. How much less than \$1 is $87\frac{1}{2}$¢? what fractional part of \$1 less? Formulate a short method for multiplying a number by $87\frac{1}{2}$.

9. Formulate a short method for multiplying a number by $.83\frac{1}{3}$; by 1.25.

10. Compare the cost of $87\frac{1}{2}$ yd. at 64¢ with the cost of 64 yd. at $87\frac{1}{2}$¢.

ORAL EXERCISE

State the cost of:

1. 24 yd. at 75¢.
2. 75 yd. at 24¢.
3. 192 yd. at $87\frac{1}{2}$¢.
4. 240 yd. at $83\frac{1}{3}$¢.
5. $87\frac{1}{2}$ yd. at \$2.48.
6. 176 yd. at $\$1.12\frac{1}{2}$.
7. $87\frac{1}{2}$ yd. at \$2.88.
8. 25 yd. at 4¢.
9. 28 yd. at 75¢.
10. 27 yd. at 75¢.
11. 75 yd. at 84¢.
12. 75 yd. at 16¢.
13. 270 yd. at $11\frac{1}{9}$¢.
14. 144 yd. at $11\frac{1}{9}$¢.
15. $11\frac{1}{9}$ yd. at 18¢.
16. 1125 yd. at 64¢.
17. 1125 yd. at 32¢.
18. 1125 yd. at 48¢.

WRITTEN REVIEW EXERCISE

1. Find the total of the costs called for in problems 1–15 in the oral exercise at the top of page 159.

2. Find the total cost of the items in the oral exercise at the bottom of page 162; of the items in the oral exercise at the bottom of page 164.

3. Find the total cost of:

84 yd. at 7¢.	98 yd. at 9¢.	72 yd. at 75¢.
112½ yd. at 5¢.	79 yd. at 11¢.	87½ yd. at 88¢.
112½ yd. at 6¢.	17 yd. at 16¢.	320 yd. at 11¢.

4. Find the total cost of:

71 yd. at 22¢.	85 yd. at 30¢.	30 yd. at 7½¢.
31 yd. at 44¢.	17 yd. at 25¢.	24 yd. at 8⅓¢.
82 yd. at 88¢.	12½ yd. at 39¢.	56 yd. at 83⅓¢.
71 yd. at 72¢.	250 yd. at 64¢.	124 yd. at $1.12½.

5. Find the total cost of:

192 lb. at 3⅓¢.	167 lb. at 12½¢.	1151¾ lb. at 10¢.
384 lb. at 6¼¢.	184 lb. at 37½¢.	1721½ lb. at 15¢.
378 lb. at 6¼¢.	2164 lb. at 2½¢.	2911½ lb. at 33⅓¢.
149 lb. at 6¼¢.	1369 lb. at 2½¢.	2706 lb. at 33⅓¢.

6. Copy and find the amount of the following bills, less 3%:

a.

Rochester, N.Y., Aug. 2, 19

Mr. C. G. Garlic
North Rose, N.Y.

To Smith, Perkins & Co., *Dr.*

Terms: cash, less 3%.

	330 lb.	Granulated Sugar	6¼¢			
	32 "	Butter	22¢			
	64 "	Cheese	16⅔¢			
	75 "	Young Hyson Tea	24¢			
	155 "	Dried Apples	8¢			
	300 "	Brown Sugar	3½¢			
	60 "	Oolong Tea	51¢			
	125 "	Rio Coffee	28¢			
	250 "	Mocha Coffee	24¢			

b.

Buffalo, N.Y., Aug. 5, 19

MR. GEORGE A. COLLIER
Savannah, N.Y.

Bought of GEORGE H. BUELL & CO.

Terms: cash, less 3%.

	72 pr. Boys' Hose	$12\frac{1}{2}¢$			
	18 doz. Linen Handkerchiefs	2.50			
	18 " Lace Handkerchiefs	$3.33\frac{1}{3}$			
	78 yd. Silk Velvet	$3.33\frac{1}{3}$			
	75 pc. Black Ribbon	$28¢$			
	347 yd. Pontiac Seersucker	$6\frac{1}{4}¢$			
	186 " Washington Cambric	$12\frac{1}{2}¢$			

ORAL EXERCISE

1. At $33\frac{1}{3}¢$ per pound, how many pounds of coffee can be bought for $12?

SOLUTION. $.33\frac{1}{3} = \$\frac{1}{3}$; 3 pounds can be bought for $1; then, 12×3 lb. = 36 lb., the required result.

2. When the cost is given and the price is $25¢$, how may the quantity be found?

SOLUTION. When the price is $25¢$, the quantity is 4 times the cost; hence, *multiply the cost by 4.*

3. Give a short method for finding the quantity when the cost is given and the price is $20¢$; $33\frac{1}{3}¢$; $12\frac{1}{2}¢$; $6\frac{1}{4}¢$; $6\frac{2}{3}¢$; $16\frac{2}{3}¢$.

4. Formulate a short method for dividing any number by 125.

SOLUTION. 125 is $\frac{1}{8}$ of 1000; then the quotient by 125 will be 8 times the quotient by 1000. Therefore, *divide by 1000 and multiply the result by 8.* Or, $\frac{1}{125} = \frac{8}{1000}$. Therefore, *multiply by 8 and move the decimal point three places to the left.*

5. Give a short method for dividing by $6\frac{1}{4}$.

SOLUTION. $6\frac{1}{4} = \frac{1}{16}$ of 100; then the quotient by $6\frac{1}{4}$ will be 16 times the quotient by 100. Therefore, *move the decimal point two places to the left and multiply the result by 16.* Or, $\frac{1}{6} = \frac{16}{100}$. Therefore, *multiply by 16 and move the decimal point two places to the left.*

6. Give a short method for dividing a number by $12\frac{1}{2}$; by $16\frac{2}{3}$; by $33\frac{1}{3}$; by $6\frac{1}{4}$; by $66\frac{2}{3}$; by $333\frac{1}{3}$; by $166\frac{2}{3}$.

7. Formulate a short method for dividing a number by .75.

SOLUTION. .75 increased by $\frac{1}{3}$ of itself = 1. When the divisor is 1 the quotient is the same as the dividend. Hence, to divide a number by .75 *increase the number by $\frac{1}{3}$ of itself.*

8. At 75¢ per bushel, how many bushels of wheat can be bought for \$144? for \$192? for \$240? for \$780? for \$1260? for \$360? for \$1350? for \$810?

9. At \$7.50 per dozen, how many dozen men's gloves can be bought for \$1440?

SOLUTION. \$7.50 + $\frac{1}{3}$ of itself = \$10. To divide by 10 is to point off one place to the left. \$1440 + $\frac{1}{3}$ of itself = \$1920; \$1920 ÷ \$10 = 192, the number of pairs of gloves.

10. State a short method for dividing a number by $7\frac{1}{2}$; by 75; by 750.

ORAL EXERCISE

Find the quantity:

	Cost	Price per Yard		Cost	Price per Pound
1.	\$65	$33\frac{1}{3}$¢	7.	\$75	$6\frac{2}{3}$¢
2.	\$250	25¢	8.	\$12	$1\frac{2}{3}$¢
3.	\$120	$6\frac{1}{4}$¢	9.	\$25	$1\frac{1}{4}$¢
4.	\$215	$2\frac{1}{2}$¢	10.	\$38	$1\frac{1}{2}$¢
5.	\$126	$12\frac{1}{2}$¢	11.	\$125	\$1.25
6.	\$125	20¢	12.	\$420	$12\frac{1}{2}$¢

WRITTEN EXERCISE

Find the quantity:

	Cost	Price per Yard		Cost	Price per Bushel
1.	\$570.00	75¢	6.	\$1721.00	$33\frac{1}{3}$¢
2.	\$612.00	75¢	7.	\$1842.50	25¢
3.	\$274.50	$7\frac{1}{2}$¢	8.	\$1785.00	$87\frac{1}{2}$¢
4.	\$281.50	$12\frac{1}{2}$¢	9.	\$2142.00	$33\frac{1}{3}$¢
5.	\$864.50	$12\frac{1}{2}$¢	10.	\$2720.50	$16\frac{2}{3}$¢

REVIEW EXERCISE

This exercise may be used in a number of different ways, some of which are suggested below.

1. One student may make the oral extension, using the first quantity, 60 yd., by each price in column 1; a second student may use the same quantity and make the extension by each price in column 2, and so on for the ten lists.

2. Each student in the class may take the same quantity and make the extension by each price in column 1, and foot the extensions. Compare results. Such an exercise should occupy one minute. This work may be continued for ten or fifteen minutes daily, as the instructor desires, a different quantity being used for each minute.

1	2	3	4	5	6	7	8	9	10
50¢	25¢	$66\frac{2}{3}$¢	60¢	25¢	$6\frac{1}{4}$¢	50¢	75¢	$1.50	1.33\frac{1}{3}$
20¢	$62\frac{1}{2}$¢	$6\frac{2}{3}$¢	$87\frac{1}{2}$¢	20¢	$16\frac{2}{3}$¢	90¢	$8\frac{1}{3}$¢	$1.25	1.66\frac{2}{3}$
$12\frac{1}{2}$¢	$33\frac{1}{3}$¢	75¢	$37\frac{1}{2}$¢	60¢	30¢	10¢	$37\frac{1}{2}$¢	1.12\frac{1}{2}$	$1.20
$16\frac{2}{3}$¢	30¢	$8\frac{1}{3}$¢	$6\frac{1}{4}$¢	$12\frac{1}{2}$¢	$66\frac{2}{3}$¢	$62\frac{1}{2}$¢	80¢	$1.10	1.16\frac{2}{3}$
90¢	10¢	40¢	80¢	$33\frac{1}{3}$¢	$6\frac{2}{3}$¢	40¢	$87\frac{1}{2}$¢	$1.75	$1.30

1. Find the cost of:

a.	*b.*	*c.*	*d.*	*e.*
60 yd.	72 yd.	90 yd.	75 yd.	48 yd.
80 yd.	50 yd.	20 yd.	84 yd.	96 yd.
40 yd.	44 yd.	25 yd.	54 yd.	64 yd.

2. Find the cost of:

a.	*b.*	*c.*	*d.*	*e.*
24 yd.	78 yd.	12 yd.	92 yd.	15 yd.
16 yd.	69 yd.	36 yd.	21 yd.	18 yd.
32 yd.	81 yd.	42 yd.	46 yd.	10 yd.

3. Find the cost of:

a.	*b.*	*c.*	*d.*	*e.*
100 yd.	108 yd.	160 yd.	200 yd.	320 yd.
120 yd.	135 yd.	180 yd.	240 yd.	400 yd.
150 yd.	144 yd.	128 yd.	300 yd.	360 yd.

WRITTEN REVIEW EXERCISE

Name	Quantity	Prices					
		1	2	3	4	5	6
Boucle Stripe	yd.	$0.10½	$0.11	$0.10	$0.11½	$0.12	$0.12½
Dress Silks	yd.	1.20	1.25	1.33⅓	1.37½	1.40	1.50
English Serge	yd.	1.33⅓	1.30	1.35	1.25	1.37½	1.45
Fancy Gingham	yd.	.06¼	.06	.06½	.07	.07½	.07¼
Fancy Plaids	yd.	.31½	.32	.33⅓	.35	.34	.37½
Gunner's Duck	yd.	.14	.15	.14½	.16	.17	.17½
Percale Shirting	yd.	.07	.07½	.08	.08½	.09	.09½
Scotch Cheviot	yd.	.39	.40	.37½	.45	.44	.48
Taffeta Silk	yd.	.87½	.85	.90	.88	.92½	.95
Wash Silk	yd.	.30	.37½	.40	.35	.42	.41½

Prepare each of the following invoices in correct form, omit the heading, and find the value by each price list.

1. 22 yd. Boucle Stripe
   27 yd. English Serge
   56 yd. Percale Shirting
   48 yd. Wash Silk

2. 72 yd. Dress Silk
   104 yd. Scotch Cheviot
   64 yd. Taffeta Silk
   60 yd. Gunner's Duck

3. 36 yd. Fancy Gingham
   88 yd. Scotch Cheviot
   92 yd. Wash Silk
   120 yd. Boucle Stripe
   80 yd. Gunner's Duck

4. 96 yd. Fancy Plaids
   36 yd. English Serge
   100 yd. Taffeta Silk
   70 yd. Percale Shirting
   90 yd. Wash Silk

5. 72 yd. Boucle Stripe
   56 yd. Wash Silk
   50 yd. Fancy Plaids
   80 yd. Dress Silk
   70 yd. Gunner's Duck

6. 84 yd. Taffeta Silk
   66 yd. Fancy Gingham
   84 yd. Scotch Cheviot
   66 yd. English Serge
   45 yd. Percale Shirting

CHAPTER XIV

BILLS AND ACCOUNTS

BILLS

203. A detailed statement of goods sold, or of goods bought to be sold, is called either a **bill** or an **invoice.** A detailed statement of goods bought to be used or consumed, such as office furniture, stationery, and fuel, or a statement of services rendered, or of a work performed, is called a **bill.**

Thus, a physician's statement of services rendered, or a transportation company's bill for work performed, and the charges for the same, is called a *bill;* but a statement of a quantity of silk bought or sold by a dry-goods merchant in the course of trade is called either a *bill* or an *invoice.*

204. The models following show a variety of current practices in billing. They will therefore be found helpful as studies.

1. Groceries

Boston, Mass., Oct. 15, **19**

Messrs. SMITH, PERKINS & CO.
Rochester, N.Y.

Bought of E. E. GRAY COMPANY

Terms 30 da. **Telephone,** Main 167

3	bbl. Rolled Oats	$6.25	18	75		
10	" Gold Medal Flour	6.50	65	00		
5	bx. Wool Soap	3.10	15	50	99	25

This is one of the simplest bill forms; it is the form that is common in a great many lines of business.

2. Groceries

Boston, Mass., Nov. 12, 19

Messrs. E. O. Sherman & Co.
Charlestown, Mass.

Bought of S. S. PIERCE COMPANY

Terms 30 da.; 3% 10 da.

10	Red Label Hams	146 lb.	$0.23	$33.58
20	mats Java Coffee	1500 "	.25	375.00
12	6-lb. tins Mustard	72 "	.36	25.92
15	6-lb. tins Cocoa	90 "	.34	30.60
				$465.10

Goods bought by the mat, chest, case, etc., are frequently billed by the pound. The above bill shows the form in such cases.

3. Hardware

The following bill is sometimes used in the hardware business. The first number after the name of the article is the quantity; the number above the horizontal line following, the price; and the number below the line, the grade. Thus, the first item in the bill shows that 12 doz. porcelain knobs in all were sold, of which 6 doz. were No. 8 at $1.25 and 6 doz. No. 16 at $1.33⅓.

New York, Apr. 6, 19

Messrs. L. M. Shipman & Son
Springfield, Mass.

Bought of The Eureka Hardware Company

Terms 3% 10 da.; net 30 da.

12	doz. Porcelain Knobs	6 1.25/8 6 1.33⅓/16	15	50	
15	" Steelyards	3 5.00/17 12 3.75/14	142	50	
25	" Jack Planes	5 6.25/16 5 6.00/20 15 6.66⅔/20	161	25	319 25

4. Wholesale Dry Goods

CHICAGO, Mar. 15, 19

Messrs. C. E. Boynton & Son,
Buffalo, N. Y.

Bought of **MARSHALL FIELD & CO.**
Franklin Street and Fifth Avenue

TERMS 30 da.

124	12	pc. Humboldt Jeans 42 40 42¹ 40 40³ 43² 41 43 41¹ 43² 40¹ 42²	500	33⅓¢	166	67		
125	12	pc. Washington Cambric 40 37³ 40³ 46² 37³ 43³ 39 32¹ 40¹ 41² 39¹ 40¹	479	6¼¢	29	94		
129	12	pc. Cotton Flannel 40 39 42 40 42 40 42 41 42 41 43 43	495	8⅓¢	41	25		
132	12	pc. Kellogg Opera Flannel 41 43 42 41 43 39 42 40 40 42 44 41	498	37½¢	186	75		
127	12	pc. Monument Cotton 42 40 40 40 43 39¹ 41 43 46 39 42 43³	499	6¾¢	33	27		
135	6	pc. Creton Plaids 37 41 39² 41² 37 42	238	11¢	26	18		
141	6	pc. Whipcord 39 41¹ 42³ 46 41 43	253	99¢	250	47		
	25	doz. Whalebones		90¢	22	50		
167	75	yd. Black Ribbon		75¢	56	25		
	100	doz. spools Thompson Thread		37½¢	37	50	850	78

In the wholesale dry-goods business the price is generally for a yard, and the number of yards to the piece varies in some kinds of cloth. The first item in the above bill is followed by a series of numbers, 41, 42, etc.; these represent the number of yards in each of the 12 pc. Immediately following these numbers is recorded the total number of yards in the 12 pc. The total number of yards should be found by horizontal addition.

5. Manufacturer's

The following is a bill for neckwear. The different styles are distinguished by the marks at the left of the quantity. This form is common among manufacturers, jobbers, and wholesalers. Bills on which trade discounts (see page 246) are allowed are arranged as shown in this bill.

New York, Oct. 10, 19

Messrs. J. E. Whiting & Co.
Boston, Mass.

Bought of Johnson Bros., Sons & Co.

Terms Net 30 da.

721	$1\frac{1}{2}$	doz. Neckwear	$4.50	6	75		
1026	$\frac{1}{2}$		27.00	13	50		
1025	$1\frac{1}{2}$		27.50	41	25		
1020	$\frac{3}{4}$		9.00	6	75		
923	$2\frac{1}{2}$		18.00	45	00		
1015	$1\frac{3}{4}$		24.00	42	00		
				155	25		
		Less 2%		3	11	152	14

6. Furniture

In the following bill the goods were sold delivered on the cars (f. o. b.) Boston, but the shippers prepaid the freight to Bangor. The freight is a part of the selling price and is added to the amount of the bill, as shown in the model.

BOSTON, May 23, 19

Messrs. J. D. Lyman & Bro.
Bangor, Me.

Bought of E. M. PRAY, SONS & CO.

Manufacturers of Fine Furniture

TERMS Net 30 da.

65	5	Turkish Rockers	24.25	123	75		
396	12	Mahogany Hall Stands	67.50	810	—		
626	10	Mahogany Office Chairs	17.25	172	50		
178	12	Roman Chairs	19.25	231	—		
				1337	25		
		Freight		12	10	1349	35

7. Wholesale Coal

F. H. OSBORN & CO.

SHIPPERS OF

Anthracite, Bituminous, and Gas Coal

Boston, Nov. 29, 19—

Sold to C. W. Graham & Co.,

162 Causeway St., City

Terms Cash

		27800# Grate	5.00	69	50			
		26900# Egg	5.00	67	25			
		29600# Stove	5.00	74	—			
		29700# Lump	2.75	40	84	251	59	
		Received payment,						
		F. H. Osborn & Co.,						
		per O.						

The above is a form of bill that is generally used for wholesale transactions in coal. It is called a **receipted bill,** and shows that the coal has been paid for.

8. Retail Coal

Boston, Jan. 31, 19—

Mr. Ira B. Chapin,

212 Huntington Ave., City

Terms Cash

Bought of **F. M. Everett & Co.**

Jan.	3	2 loads Grate Coal						
		6340-2140 6380-2180 8400#	6.00	25	20			
	18	2 loads Grate Coal						
		6380-2140 6400-2140 8500#	6.00	25	50	50	70	
		Paid						
		F. M. Everett & Co.,						
		per W. E. S.						

The foregoing bill shows a form sometimes used by retailers. The numbers at the left of the hyphen are the gross weights, and the numbers at the right the tares of the different loads.

9. China and Glassware

Boston, Nov. 6, *19*

M THE WENTWORTH-STRATTON CO.
Rochester, N.Y.

Bought of Osgood, Traver & Son

Terms 60 da. net; 2% 10 da.

1	Dinner Set, 130 pieces; viz.:							
	1 doz. Plates, 8 in.			1	88			
	1 " " 7 "			1	63			
	1 " " 6 "			1	38			
	1 " " 7 " (deep)			1	63			
	1 " Fruit Saucers, 4 in.				75			
	1 " Individual Butters				50			
	1/12 doz. Covered Dishes, 8 in.	$12.00		1	00			
	1/12 " Casseroles, 8 in.	13.50		1	13			
	1/4 " Dishes, 8 in.	2.50			63			
	1/12 " " 10 "	4.50			38			
	1/12 " " 12 "	7.50			63			
	1/12 " " 14 "	10.50			88			
	1/6 " Bakers, 8 in.	4.50			75			
	1/12 " Sauce Boats	4.00			33			
	1/12 " Pickles	3.00			25			
	1/12 " Bowls	2.00			17			
	1/12 " Sugars	6.00			50			
	1/12 " Creams	2.79			23			
	1 " Handled Teas			2	00			
	1/2 " " Coffees	2.33		1	17			
	1/12 " Pitchers	6.00			50			
	1/12 " Covered Butters and Drainers	9.00			75	19	07	
25	more Dinner Sets as above	19.07				476	75	
						495	82	
	Crates					7	50	
	Carting					2	10	505 42

The above form is common in the china and glassware business. In this case a charge is made for the crates used in packing and the prices do not include delivery. The cost of the crate and the cost for carting are therefore made a part of the bill.

10. Lumber

The H. M. Bickford Co.

Boston, Mass., Oct. 8, *19*

Sold to L. A. Hammond & Co.
Paterson, N.J.

Terms Fgt. net cash; bal. in 5 da. less $1\frac{1}{2}\%$

23,289 ft.	$\frac{1}{2} \times 2\frac{1}{2}$	#1	N. C. Ceiling	$18.50	$430.85
3,520 "	"	2	" " "	17.00	59.84
10,307 "	$\frac{3}{8} \times 2\frac{1}{2}$	1	" " "	13.50	139.14
1,690 "	"	2	" " "	12.50	21.13
					$650.96
			Less freight (45,200 lb. at 24¢)		108.48
					$542.48

Lumber is generally sold by the thousand feet. In the above bill the goods were sold free on board cars (f. o. b.) Paterson, N.J., but the shippers have not prepaid the freight. They find that these charges are $108.48 and deduct this amount from the total of the bill. In the wholesale lumber business the prices quoted usually include the cost of delivery, and when the freight charges are not known at the time of making the shipment, they are paid by the consignees and deducted from the amount of the bill on the arrival of the goods. The freight bill is then sent to the shippers for credit.

WRITTEN EXERCISE

1. Study the model bill, page 170. Increase the price of each article 25¢ and then copy and find the amount of the bill.

2. Study the first model bill, page 171, and then copy and find the amount of it at the following prices: hams, 27¢; coffee, 23¢; mustard, 31¢; cocoa, 39¢.

3. Study the second model bill, page 171, and then copy and find the amount of it at the following prices: porcelain knobs #8, $1.12\frac{1}{2}$; #16, $1.25; steelyards #64, $11; #17, $8.33\frac{1}{3}$; jack-planes #14, $6; #21, $6.25; #48, $6.75.

4. Apr. 15, you bought of S. S. Pierce Co., Boston, Mass., for cash: 25 gal. finest New Orleans molasses at 48¢; 15 gal. fancy sugar-house sirup at 49¢; 75 lb. raw mixed coffee at 29¢; 25 lb. raw Pan-American coffee at 19¢; 5 cartons Fowle's entire-wheat flour at 39½¢; ½ bbl. Franklin Mills flour at $6.75; ⅛ bbl. pastry flour at $5.25. Write the bill.

5. Mar. 19, Frank M. Richmond & Co., New York City, sold to Charles M. Thompson, Poughkeepsie, N.Y., 12 doz. porcelain knobs: 3 doz. #71 at $6.35, 9 doz. #74 at $6.75; 12 doz. shingle hatchets: 6 doz. #16 at $9.75, 6 doz. #34 at $12.50; 7 doz. steel squares: 3 doz. #91 at $35, 4 doz. #73 at $33. Terms: 30 da. Write the bill.

6. Study the model bill on page 172. Increase the prices of the articles marked 124 and 132 five cents each and the remainder of the articles one cent each; then copy and find the amount of the bill.

7. Nov. 15, J. B. Ford & Co., Albany, N.Y., bought of the Clinton Mills, Little Falls, N.Y., 10 pc. percale shirting containing 42, 48, 52^{1}, 58, 62, 38, 49, 51, 54, and 46^{1} yd., at 7½¢; 10 pc. fine wool cheviot containing 58^{1}, 42, 49, 51, 44^{2}, 46, 48, 41^{2}, 39, and 42 yd., at $1.12½; 5 pc. cashmere containing 49^{3}, 40^{1}, 48^{3}, 49^{1}, and 49 yd. at $1.37½. Terms: 60 da., or 3% discount for cash within 10 da. Write the bill.

8. Study the first model bill on page 173. Increase the prices of styles 1026, 1025, 1020, and 923, 25¢ each and diminish the prices of the other styles 25¢ each; then copy and find the amount of the bill. Omit the discount.

9. Sept. 24, Geo. W. Fairchild, Buffalo, N.Y., bought of E. M. Lawrence & Co., New York City, silk ribbon as follows: 12 pc. #1142 at $2.25; 5 pc. #1321 at $1.25; 25 pc. #171 at $4.37½; 8 pc. #1927 at $1.75; 36 pc. #2114 at $1.66⅔; 15 pc. #1371 at $1.33½; 15 pc. #624 at $4.37½; 12 pc. #909 at $1.87½; 25 pc. #1008 at $3.33⅓; 25 pc. #1246 at $4.75; 18 pc. #2119 at $1.12½. Terms: 30 da., or 2% discount for cash in 10 da. Write the bill.

10. Study the second model bill on page 173. Increase the price of the articles marked 65 and 396, 25¢ each, and diminish the price of the other articles $12\frac{1}{2}$¢ each; then copy and find the amount of the bill. Freight added, $14.70.

11. July 20, The Hayden Furniture Co., Rochester, N.Y., bought of John H. Pray & Son, Boston, Mass., 25 #31 card tables at $11; 24 #94 china closets at $27.50; 15 #16 dining sets at $85; 25 #3060 fancy rockers at $9.25; 15 #35 music cabinets at $2.75; 25 #26 mahogany office chairs at $12.50; 12 #89 oak sideboards at $125. Terms: 30 da. The prices are free on board Boston, and the shipper prepaid the freight, $34.50. Write the bill.

12. Study the first model bill on page 174. Increase the price of the stove coal 25¢ per ton and the price of each of the other kinds $12\frac{1}{2}$¢ per ton; then copy and find the amount of the bill. Receipt the bill for F. H. Osborn & Co.

13. May 19, C. E. Williams & Co., Cleveland, O., bought of Fairbanks & Co., Scranton, Pa.: 3 car loads of stove coal weighing 20,500, 26,400, and 25,600 lb., respectively, at $4.75 per ton (2000 lb.); 1 car load grate coal weighing 21,900 lb. at $4.25 per ton; 1 car load cannel coal weighing 22,500 lb. at $7.75 per ton. Terms: 30 da., or 3% discount for cash in 10 da. Write the bill.

14. Study the second model bill, page 174, then copy and find the amount of it at $6.25 per ton for each sale.

15. Copy the bill in problem 14 in accordance with the model shown on page 174. Make the price of the coal $6.66\frac{2}{3}$.

16. Study the model bill on page 175. Increase each price given five cents and then copy and find the amount of the bill. Cost of crates used in packing, $6.40; carting, $2.80.

17. July 15, Henry Nelson & Co., Portland, Me., bought of Jones, Stratton & Co., New York City, 5 doz. plates, 8 in., at $1.50; 35 doz. plates, 7 in., at $1.35; 15 doz. plates, 6 in., at $1.10; 10 doz. plates, 5 in., at 90¢; 65 doz. handled teas at $1.85. Terms: 30 da. Cost of crate used in packing, $2; cartage, 75¢. Write the bill.

STATEMENTS

FOLIO 72

Albany, N.Y., Feb. 1, 19

Mr. C. B. McMenimen
St. Louis, Mo.

In account with Baker, Taylor & Co.

Jan.	5	To mdse.	300	10		
	12	" "	599	90		
	25	" "	97	10	997	10
	16	By cash	300	10		
	30	" "	371	30	671	40
					325	70

205. A **statement** is an abstract of a customer's account, showing under proper dates the details and totals of debits and credits and the balance remaining unpaid.

FOLIO 72

Albany, N.Y., Mar. 1, 19

Mr. C. B. McMenimen
St. Louis, Mo.

In account with Baker, Taylor & Co.

Feb.	1	To account rendered	325	70		
	8	" mdse.	300	60		
	12	" "	460	20	1086	50
	9	By mdse. returned	12	40		
	15	" cash	300	—		
	27	" "	361	90	674	30
					412	20

The first model on the preceding page is a statement of C. B. McMenimen's account for January. It shows that the charges aggregate $997.10, the credits $671.40, and that the balance remaining unpaid is $325.70.

The second model on the preceding page is a statement of C. B. McMenimen's account for January and February. The items on the January statement are summarized in the record "To account rendered, $325.70." The first item on the March statement will be "To account rendered, $412.20."

WRITTEN EXERCISE

1. During March, F. E. Smith, Buffalo, N.Y., bought merchandise of The Hayden Furniture Co., Rochester, N.Y., as per bills rendered: namely, Mar. 3, $400.80; Mar. 15, $360.90; Mar. 20, $200.70; Mar. 26, $260.90; Mar. 28, $130.50. During the same time he made cash payments on account as follows: Mar. 15, $400.80; Mar. 23, $360.90. On Mar. 27 he also returned goods for credit amounting to $18.60. Render a statement of F. E. Smith's account.

2. During April the above account was charged for merchandise as follows: Apr. 15, $720.50; Apr. 27, $260.90. The account was also credited for cash as follows: Apr. 16, $200.70; Apr. 28, $100.00. Render the April statement.

3. Copy and find the balance of the following statement:

Boston, Mass., Feb. 1, 19

MRS. C. M. SHERMAN
931 BEACON ST., City

In account with SPENCER, MEAD & CO.

Jan.	1	Account rendered		13	64	
	3	2 pr. Gloves	2.50			
		3 yd. Velvet	3.75			
		12 " Black Silk	2.10			
	12	6 pr. Hose	35¢			
		2 Hats	9.00			
		30½ yd. Muslin	12½¢			
		Cr.				
	5	2 pr. Gloves	2.50			
	15	1 Hat	9.00			

PAY ROLLS

PAY ROLL For the week ending Sept 18, 19

No.	NAME	Number of Hours' Work Each Day M	T	W	T	F	S	Total No. of Hours	Wages per Hour	Total Wages		REMARKS
1	John W Alden	9	9	9	9	9	9	54	25¢	13	50	
2	C E Brown	8½	9½	8½	8½	8	8	51	25¢	12	75	
3	Fred W Dunn	9½	10	9½	10	10	10	59	35¢	20	65	
4	N F Good	7½	9½	9½	9½	10	9½	54½	33⅓¢	18	08	
5	Frank McInnes	8	8	8	8	8	8	48	35¢	16	80	
6	J P Kaplan	9	9	9	9	9	9	54	37½¢	20	25	
7	D K Martin	7	8	9	9½	9½	9½	52	35¢	18	20	
8	Fred Reed	8	9	7	8	7½	8½	48	27½	13	20	
9	C L Spaulding	10	8	9	9	9	9	54	18½	9	84	
10	James Spaulding	10	7½	9½	7	9	9	52	21¢	10	92	
		90	85½	85½	87	89	89	526½		154	19	

This form is most common among manufacturing establishments, but it is also used by printers, contractors, and builders.

Checks are sometimes used in paying off employees, but most large concerns find the envelope system the most convenient and satisfactory. To pay off employees by the envelope system it is necessary for the bookkeeper to find first the amount of money required and then the bills and fractional currency that are necessary to pay each employee. The amount required is the total of the pay roll, and the bills and fractional currency desired may be found as shown in the following illustration. This illustration, called a **change memorandum**, shows the method of finding just the denominations wanted for the pay roll at the top of the page. A change memorandum may be proved correct as shown in the **pay-roll memorandum** at the top of page 182.

No.	Bills					Coins				
	$20	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1		1		1	1	1				
2		1		1		1	1			
3	1					1		1	1	
4		1	1	1	1				1	3
5		1	1		1	1	1		1	
6	1						1			
7		1	1	1	1			2		
8		1		1	1			2		
9			1	2		1	1		1	4
10		1				1	1	1	1	2
	2	7	4	7	5	6	5	6	5	9

When the amount of the pay roll and the necessary bills and fractional currency have been determined, a check payable to the order of *Pay Roll* is written. A pay-roll memorandum similar to the accompanying form is then attached to the check and both are sent to the bank. The pay-roll memorandum should foot the same as the pay-roll book, and is therefore a check upon the correctness of the change memorandum.

FIRST NATIONAL BANK
Westfield, Mass.
PAY-ROLL MEMORANDUM
NELSON & CO.
require the following:

Pennies	9	09
Nickels	5	25
Dimes	6	60
Quarters	5	1 25
Halves	6	3 00
Dollars	5	5 00
2's	7	14 00
5's	4	20 00
10's	7	70 00
20's	2	40 00
		154 19

In a large pay roll the adept bookkeeper frequently *estimates* the kind of change required. This is done by scanning the pay roll first to find the number of pennies required, then the number of nickels, etc. The experieneed book-keeper can make a very accurate estimate.

PAY ROLL For the week ending Nov. 6, 19—

No.	NAME	Time in hours M	T	W	T	F	S	Total time	Rate per hour	Amount advanced	Amount due	Bills and silver necessary $20	$10	$5	$2	$1	50c	25c	10c	5c	1c	REMARKS
1	Ralph Roe	9	9	9	9	9	9	54	44¢		24 —		1		2							
2	Emma Tyng	9	9	9	9	9	9	54	44¢		24 —		1		2							
3	John Walsh	8	9	8	9	9	9	52	35¢		18 20			1	1	1			2			
4	Edward Laine	8½	8½	9	9	9	9	53	35¢		18 55			1	1	1	1			1		
5	Dora Small	9½	9	8½	9	8½	9½	54	22½¢		12 —			1		1						
6	Ellen Dale	9	8½	8	8½	8½	9½	53	33⅓¢	5 —	12 67			1		1		1		1	1	2
7	J. Bloom	9	9	9	9	9	9	54	35¢		18 90			1	1	1	1	1	1	1		
8	Charles Frey	8	9	9	9	9	9	53	25¢		13 25		1			1	1		1			
9	Benj. A. Green	9	9	9	9	9	9	54	18¢		9 84				1	2		1	1		1	4
10	F. M. Spessy	8	7½	12	9	9	9	57½	22½¢		12 78		1			1		1	1			3
11	Ada Neal	10	10	10	9	8	8	55	14¢		8 13				1	1	1			1		3
12	Ora Snell	9	9	9	8	8	8	51	35¢		17 85		1	1	1		1	1	1			
13	C. H. Spring	9	9	8	9	9	9	53	30¢		15 90		1	1			1	1	1	1		
14	Sam'l Dobson	9	9	9	9½	9½	8	57½	22½¢		12 —			1		1						
		124	124½	127	128	124	124	753½		5 —	218 07	2	10	7	16	5	7	6	7	5	12	

40 199 35 32 8 3 2 22 7 4 2 1 = 218 22

WRITTEN EXERCISE

1. Study the model pay roll, page 181, and find the amount of it at the following wages per hour: #1, 18¢; #2, $21\frac{2}{9}$¢; #3, 25¢; #4, 35¢; #5, $33\frac{1}{3}$¢; #6, 35¢; #7, $37\frac{1}{2}$¢; #8, 35¢; #9, $27\frac{1}{2}$¢; #10, $18\frac{2}{9}$¢. Make a change memorandum.

2. Study the model pay roll on page 182, and then find the amount of it at the following wages per hour: #1, 50¢; #2, 45¢; #3, $33\frac{1}{3}$¢; #4, 35¢; #5, $27\frac{1}{2}$¢; #6, $37\frac{1}{2}$¢; #7, 25¢; #8, $33\frac{1}{3}$¢; #9, $44\frac{4}{9}$¢; #10, $22\frac{2}{9}$¢; #11, $22\frac{2}{9}$¢; #12, $14\frac{3}{7}$¢; #13, $12\frac{1}{2}$¢; #14, 30¢.

3. Make a pay roll memorandum from problem 2.

WRITTEN REVIEW EXERCISE

1. Find the amount of each of the following bills:

New York, May 31, *19*

MESSRS. GRAY, SALISBURY & CO.

Rochester, N.Y.

Bought of J. E. PAGE, SONS & CO.

Terms: net, 60 da.; 2% 10 da.

CASE	PIECES	DESCRIPTION OF ARTICLES	YDS.	PRICE	ITEMS	AMOUNT
#364	10	Velveteen				
		42^1 40 40 46 38^1				
		40 42 42 41 39		25¢		
#359	12	Corduroy				
		36 38^1 39^2 42 41^2 39^2				
		37 37 41 45 41 40^1		$66\frac{2}{3}$¢		
#371	15	Gray Homespun				
		39 38 35 42 41				
		45 39 41 34 37				
		41 40 41 38 42^8		$83\frac{1}{3}$¢		
#360	6	Storm Serge				
		40 42^1 43 42 39 42^1		44¢		
#373	24	Fine English Serge				
		42 38 42 42 40^2 42^1				
		40 39 40 41 40^1 43				
		42 42 38^2 38 41 42				
		43 44 41 40 37^1 37		1.37½		
#381	24	Groveland Flannel				
		32 40 39 42 41 45				
		45 46 35 41 38 41				
		37 42 43 40 37 42				
		37 40 42 41 44 41		$33\frac{1}{3}$		

2. Make out a bill for the following order. Bill the English breakfast tea at 41¢; Finest oolong tea at 65¢; Young Hyson tea at 97½¢; Choice Japan tea at 59¢; Orinda kaughphy at $1.90; raw Java coffee at 30½¢; gluten flour at 30¢ a carton and $7.75 per barrel. Assume that half a chest of tea weighs 75 lb., and a mat of coffee 70 lb.

E. M. BARBER & SON
RETAIL GROCERS

Springfield, Mass., Aug. 13, 19

S. S. Pierce Company
Boston, Mass.

Gentlemen:

Please ship us via B. & A. R.R., the following goods:

3 hf. cht. English Breakfast Tea
3 " " Finest Oolong Tea
5 " " Young Hyson Tea
25 lb. Choice Japan Tea
5 5-lb. cans Orinda Kaughphy
7 mats Raw Java Coffee
5 hf. bbl. Gluten Flour
25 5-lb. cartons Gluten Flour

Respectfully yours

E. M. Barber & Son

3. Boston, Mass., Apr. 16, E. O. Burrill, Philadelphia, Pa., bought of Jones, Talcott & Co., on account, 30 da., 25 Turkish rugs $4\frac{1}{2} \times 7$ at $10.25; 750 yd. matting at 55¢; 225 yd. linoleum at 27½¢; 25 Turkish rugs $8\frac{1}{2} \times 12$ at $21.75; 25 Persian rugs 6×9 at $12.25; 12 Persian rugs 7×11 at $16.25; 10 rolls, each containing 150 yards, Brussels carpeting at $2.25; 275 yd. Moquette carpeting at $1.75. Find the amount of the bill.

TIME SLIP

Friday, 4/26, 19—

	IN	OUT	IN	OUT
1	751	1201	1256	502
2	747	1202	1247	503
3	744	1204	1254	504
4	751	1205	1255	501
5	753	1206	1259	505
6	751	1202	1259	503
7	859	1201	1254	502
8	756	1202	1250	501
9	757	1201	1259	502

TIME SLIP

Saturday, 4/27, 19—

	IN	OUT	IN	OUT
1	753	1200	1258	502
2	757	1204	1259	501
3	753	1200	1256	504
4	755	1202	1254	502
5	749	1201	1257	504
6	828	1203	1255	503
7	755	1202	1253	501
8	857	1201	1254	502
9	758	1202	1258	504

The above slips show a record of time for 9 employees for 2 da. in a large printing establishment. The records are made by a large mechanical timekeeper, and at convenient periods are copied in the pay-roll book. Fractions are recorded to the nearest $\frac{1}{6}$ of an hour. The above record is based on an eight-hour day, from 8 to 12 A.M., and 1 to 5 P.M. If a record is made before the hour for beginning work, either in the morning or at noon, it is not counted in the regular time, but if an employee is late, his record for time begins at the nearest multiple of ten after the record is made. For instance, if an employee rang in at 8:42 A.M., his time for the morning would begin at 8:50. In the above slip for Friday, No. 1 rang in at 7:51 and left at 12:01; he rang in at 12:56 and rang out at 5:02; time, 8 hr.

4. Copy the following pay roll, enter the time for Friday and Saturday (from the above slips), find the amount of the pay roll; make a change memorandum and a pay-roll memorandum.

PAY ROLL FOR THE WEEK ENDING APRIL 27, 19—

No.	Name	Number of Hours' Work Each Day						Total No. of Hours	Wages per Hour	Total Wages		Remarks
		M.	T.	W.	T.	F.	S.					
1	A. B. Comer	8	8	8	8				55⅝¢			
2	W. D. Ball	8	8	8	8				44⅘¢			
3	A. M. Snow	8	8	8	8				44⅘¢			
4	R. O. Mark	8	8	8	8				33⅓¢			
5	Miss Mary Cane	8	7½	8	8				33⅓¢			
6	Miss Ellen Kyle	8	7½	8	8				35¢			
7	D. M. Garson	8	7½	8	8				35¢			
8	S. D. Lane	8	7½	7½	8				25¢			
9	Miss Cora Knapp	8	8	7½	8				22⅔¢			

EXPRESSAGE AND FREIGHTAGE

These express rates went into effect February 1, 1914, in conformity with the order of the Interstate Commerce Commission:

Between New York and Chicago		Between New York and New Orleans		Between New York and Denver	
1st class	*2d class*	*1st class*	*2d class*	*1st class*	*2d class*
5 lb. 31¢	31¢	41¢	41¢	47¢	47¢
6 33	32	46	46	53	53
7 35	32	50	48	58	57
8 38	32	54	48	64	57
9 40	32	59	48	69	57
10 42	32	63	48	75	57
15 53	40	84	63	1.02	77
20 64	48	1.06	80	1.30	98
25 75	57	1.27	96	1.57	1.18

Between Chicago and Boston		Between Chicago and Galveston		Between Chicago and Portland, Ore.	
1st class	*2d class*	*1st class*	*2d class*	*1st class*	*2d class*
5 lb. 31¢	31¢	39¢	39¢	63¢	63¢
6 34	33	43	43	72	72
7 36	33	47	45	81	80
8 38	33	51	45	89	80
9 41	33	55	45	98	80
10 43	33	59	45	1.06	80
15 54	41	78	59	1.50	1.13
20 66	50	98	74	1.93	1.45
25 77	58	1.17	88	2.36	1.77

Under the new system the country is divided into blocks, each block covering a certain designated territory, and fixed rates are made for certain weights and distances, not as under the old system, at so much per pound.

The tables given herewith suggest the manner in which charges are made for given amounts to certain points.

First-class matter includes all merchandise; *second-class* matter applies to *food* and *drink*, and these terms are elaborated and explained in the instructions to express agents. On small packages the difference in charges for first-class matter or second-class matter is very slight (sometimes the same), but for large packages and long distances the difference is marked.

WRITTEN EXERCISE

1. Find the total cost of sending the following packages:

10 lb. first class from New York to Chicago.
5 lb. second class from Chicago to Boston.
15 lb. first class from Chicago to Galveston.
20 lb. second class from New York to Denver.
10 lb. first class from New York to New Orleans.

2. Find the total cost of sending the following packages:

15 lb. first class from Chicago to Galveston.
25 lb. second class from Chicago to Portland, Ore.
7 lb. second class from Boston to Chicago.
9 lb. first class from Chicago to New York.
10 lb. first class from Galveston to Chicago.

3. Find the amount of the following freight bill:

Date of W. B., July 15 19 *W. B. No.* 4679 *Albany, N.Y.,* July 20 19

L. N. Palmer & Co.

To THE INTERSTATE TRANSPORTATION COMPANY, *Dr*

For Transportation from Boston *to* Albany

	Weight	Rate	Amount	
200 bbl. Potatoes	32100#			
40 crates Onions	2500#			
	?????#	25¢	??	??
Advance charges			2	15
			??	??

Received payment F. O. Bennett
Freight Agent

No. Car 37145

Bulky goods are generally sent by freight. The articles are divided according to quantity and character, into different classes, and are subject to different rates. All railroads follow some official classification. All official classifications divide freight into six different classes.

Such bulky articles as furniture, uncased, is subject to a classification called *double first-class.* There is so much unoccupied space that the first-class rate is doubled.

Such freight as organs and pianos in cases, furniture, statuary, etc., is generally designated as first-class matter. Baled hay, iron, etc., in car loads, is generally designated as fifth-class matter. Building blocks, brick, etc., in car-load lots, is generally designated as sixth-class matter. First-class rates are the highest and sixth-class rates are the lowest charged.

Between most points, shipments weighing less than 100 lb. are charged as 100 lb., irrespective of weight.

BOSTON & ALBANY RAILROAD

LOCAL FREIGHT TARIFF BETWEEN

BOSTON, MASS.

AND

Miles	Stations	Rate per 100 Lb. Classes						Miles	Stations	Rate per 100 Lb. Classes					
		1	2	3	4	5	6			1	2	3	4	5	6
21	So. Framingham	14¢	12¢	10¢	8¢	6¢	5¢	98	Springfield . .	28¢	24¢	20¢	15¢	11¢	9¢
32	Westboro . .	16¢	14¢	11¢	9¢	6¢	5¢	108	Westfield . .	29¢	25¢	20¢	16¢	12¢	10¢
44	Worcester . .	18¢	15¢	13¢	10¢	7¢	6¢	146	Athol	33¢	28¢	23¢	18¢	13¢	11¢
62	Webster . . .	22¢	19¢	15¢	12¢	9¢	7¢	150	Pittsfield . .	33¢	28¢	23¢	18¢	13¢	11¢
83	Palmer . . .	26¢	22¢	18¢	14¢	10¢	9¢	202	Albany . . .	35¢	30¢	25¢	19¢	14¢	12¢

4. Using the table, find the amount of freight to charge on 27,500 lb. sixth-class matter, from Boston to Pittsfield.

5. Using the above table, find the amount of freight to charge on 27,290 lb. sixth-class matter and 890 lb. first-class matter from Boston to Albany; to Westfield.

6. Using the above table, find the amount of freight to charge on 14,790 lb. fifth-class matter and 2170 lb. second-class matter from Boston to Palmer; to Worcester; to Pittsfield; to Springfield.

7. Using the above table, find the amount of freight to charge on 75 lb. first-class matter, 125 lb. second-class matter, 1250 lb. third-class matter, 7290 lb. fourth-class matter, 21,490 lb. fifth-class matter, and 64,640 lb. sixth-class matter from Boston to South Framingham; to Westboro; to Webster; to Springfield; to Athol; to Albany.

ORAL REVIEW EXERCISE

Each problem on pages 189 and 190 should be completed in approximately four minutes. Without copying, find the cost of:

1.	2.	3.
32 yd. at $6\frac{1}{2}¢$	176 yd. at $12\frac{1}{2}¢$	149 yd. at 25¢
240 yd. at $\$1.12\frac{1}{2}$	177 yd. at 50¢	150 yd. at $\$1.33\frac{1}{3}$
25 yd. at $2.50	210 yd. at $28\frac{4}{7}¢$	120 yd. at $1.25
19 yd. at $4\frac{1}{2}¢$	291 yd. at $33\frac{1}{3}¢$	169 yd. at 11¢
57 yd. at $66\frac{2}{3}¢$	104 yd. at $2\frac{1}{2}¢$	162 yd. at $12\frac{1}{2}¢$
36 yd. at $3\frac{1}{3}¢$	98 yd. at 7¢	48 yd. at $6\frac{1}{4}¢$
$24\frac{1}{2}$ yd. at 8¢	90 yd. at $6\frac{2}{3}¢$	$33\frac{1}{3}$ yd. at 9¢
190 yd. at 25¢	45 yd. at $\$1.33\frac{1}{3}$	174 yd. at 30¢
75 yd. at 28¢	45 yd. at 45¢	$29\frac{1}{2}$ yd. at 3¢
75 yd. at $12\frac{1}{2}¢$	706 yd. at $33\frac{1}{3}¢$	117 yd. at 9¢
42 yd. at $2\frac{1}{2}¢$	$22\frac{1}{2}$ yd. at 8¢	18 yd. at $3\frac{1}{3}¢$
246 yd. at 11¢	146 yd. at 11¢	$182\frac{1}{2}$ yd. at 10¢
48 yd. at $10\frac{1}{2}¢$	74 yd. at $12\frac{1}{2}¢$	179 yd. at 20¢
28 yd. at 6¢	78 yd. at 25¢	64 yd. at $12\frac{1}{2}¢$
144 yd. at $12\frac{1}{2}¢$	44 yd. at $1.25	167 yd. at 50¢

4.	5.	6.
33 yd. at $33\frac{1}{3}¢$	48 yd. at $1.50	688 yd. at $1.10
96 yd. at $37\frac{1}{2}¢$	36 yd. at $1.25	$52\frac{1}{2}$ yd. at 10¢
129 yd. at 11¢	55 yd. at $1\frac{1}{2}¢$	156 yd. at 25¢
75 yd. at $1\frac{1}{3}¢$	143 yd. at 50¢	85 yd. at 85¢
75 yd. at 75¢	36 yd. at 70¢	144 yd. at $16\frac{2}{3}¢$
27 yd. at 80¢	55 yd. at 55¢	34 yd. at 90¢
73 yd. at $1\frac{1}{2}¢$	95 yd. at 30¢	125 yd. at 20¢
65 yd. at 65¢	112 yd. at $14\frac{2}{7}¢$	53 yd. at 25¢
94 yd. at $33\frac{1}{3}¢$	47 yd. at $33\frac{1}{3}¢$	29 yd. at $66\frac{2}{3}¢$
63 yd. at $11\frac{1}{9}¢$	53 yd. at $33\frac{1}{3}¢$	99 yd. at 25¢
139 yd. at 50¢	17 yd. at $66\frac{2}{3}¢$	88 yd. at $37\frac{1}{2}¢$
64 yd. at $62\frac{1}{2}¢$	104 yd. at $12\frac{1}{2}¢$	176 yd. at $12\frac{1}{2}¢$
98 yd. at $16\frac{2}{3}¢$	80 yd. at $6\frac{2}{3}¢$	25 yd. at 25¢
225 yd. at 25¢	77 yd. at 30¢	88 yd. at $9\frac{1}{11}¢$
67 yd. at $33\frac{1}{3}¢$	99 yd. at 40¢	57 yd. at 50¢

7.	8.	9.
128 yd. at 11¢	219 yd. at 11¢	65 yd. at 80¢
73 yd. at 60¢	85 yd. at 30¢	83 yd. at 40¢
76 yd. at 60¢	94 yd. at 25¢	145 yd. at 11¢
177 yd. at 11¢	89 yd. at $1\frac{1}{2}$¢	63 yd. at $1\frac{1}{8}$¢
72 yd. at $1\frac{1}{4}$¢	28 yd. at $2\frac{1}{2}$¢	151 yd. at $33\frac{1}{3}$¢
270 yd. at $11\frac{1}{9}$¢	112 yd. at $6\frac{1}{4}$¢	49 yd. at 75¢
36 yd. at $1\frac{1}{2}$¢	191 yd. at 50¢	124 yd. at $1.25
185 yd. at 25¢	$78\frac{1}{2}$ yd. at 10¢	180 yd. at $12\frac{1}{2}$¢
225 yd. at 20¢	306 yd. at $33\frac{1}{3}$¢	360 yd. at $37\frac{1}{2}$¢
39 yd. at 1.33\frac{1}{3}$	122 yd. at $2\frac{1}{2}$¢	24 yd. at 1.12\frac{1}{2}$
49 yd. at $66\frac{2}{3}$¢	32 yd. at $1.75	165 yd. at $6\frac{2}{3}$¢
42 yd. at $2.50	92 yd. at $12\frac{1}{2}$¢	175 yd. at 8¢
25 yd. at $1.10	224 yd. at 7¢	171 yd. at 11¢
22 yd. at $6\frac{1}{2}$¢	240 yd. at $2\frac{1}{2}$¢	$132\frac{1}{2}$ yd. at 10¢
125 yd. at 36¢	276 yd. at 11¢	110 yd. at 25¢
108 yd. at 9¢	68 yd. at 8¢	51 yd. at $12\frac{1}{2}$¢
216 yd. at $12\frac{1}{2}$¢	125 yd. at 12¢	301 yd. at 25¢

10.	11.	12.
62 yd. at $4\frac{1}{2}$¢	144 yd. at $87\frac{1}{2}$¢	97 yd. at 30¢
189 yd. at $11\frac{1}{9}$¢	25 yd. at $1.62	36 yd. at 1.16\frac{2}{3}$
78 yd. at 40¢	14 yd. at 1.14\frac{2}{7}$	225 yd. at 8¢
334 yd. at 7¢	512 yd. at 6¢	$143\frac{1}{2}$ yd. at 4¢
255 yd. at 5¢	$117\frac{1}{2}$ yd. at 10¢	55 yd. at $10\frac{1}{2}$¢
78 yd. at 11¢	45 yd. at $1\frac{1}{2}$¢	15 yd. at $1\frac{2}{3}$¢
118 yd. at 11¢	155 yd. at 12¢	235 yd. at 20¢
187 yd. at 25¢	247 yd. at 50¢	56 yd. at $33\frac{1}{3}$¢
88 yd. at $33\frac{1}{3}$¢	92 yd. at $12\frac{1}{2}$¢	48 yd. at $12\frac{1}{2}$¢
96 yd. at $37\frac{1}{2}$¢	55 yd. at $16\frac{2}{3}$¢	84 yd. at $12\frac{1}{2}$¢
$405\frac{1}{2}$ yd. at 10¢	37 yd. at $10\frac{1}{2}$¢	165 yd. at 25¢
232 yd. at 11¢	42 yd. at $1.25	192 yd. at $12\frac{1}{2}$¢
36 yd. at 1.33\frac{1}{3}$	24 yd. at 1.66\frac{2}{3}$	145 yd. at 5¢
256 yd. at 6¢	178 yd. at 7¢	231 yd. at 8¢
143 yd. at 9¢	321 yd. at 11¢	148 yd. at $16\frac{2}{3}$¢
64 yd. at $1.25	24 yd. at 1.87\frac{1}{2}$	121 yd. at 25¢
51 yd. at 1.33\frac{1}{3}$	36 yd. at 1.66\frac{2}{3}$	$110\frac{1}{2}$ yd. at 10¢

DENOMINATE NUMBERS

CHAPTER XV

DENOMINATE QUANTITIES

REVIEW OF THE COMMON TABLES[1]

ORAL EXERCISE

1. Which of the following numbers are concrete? which are abstract? which are denominate?

a. 16	*f.* 150	*k.* 36 min.
b. 24 yr.	*g.* 21 yd.	*l.* 5 yd. 2 ft.
c. 64 hr.	*h.* 65 A.	*m.* 3 yr. 4 mo.
d. 12 men	*i.* 17 books	*n.* 10 T. 75 lb.
e. 15 desks	*j.* 34 houses	*o.* 5 A. 61 sq. rd.

2. Define an abstract number; a concrete number; a denominate number; a simple number; a compound number.

3. Which of the numbers in question 1 are simple? which are compound?

ORAL EXERCISE

1. Repeat the table of avoirdupois weight.

2. Repeat the table of long measure; of surveyors' long measure; of square measure; of surveyors' square measure.

3. Repeat the table of cubic measure; of dry measure; of liquid measure; of time; of angular measure; of United States money; of English money.

4. Name a number expressing distance; two numbers expressing area; two expressing value; three expressing capacity.

5. How many statute miles in a degree of the earth's surface at the equator? how many geographical miles? How many feet in a statute mile? how many inches?

[1] Tables of weights and measures may be found in the Appendix B.

REDUCTION

ORAL EXERCISE

1. Change 42 ft. to inches; to yards.
2. Express 15 yd. as feet; as inches.
3. Reduce 80 qt. to gallons; to pints.
4. Change 128 qt. to pecks; to bushels.
5. Express 120 pt. as quarts; as gallons.
6. What part of a yard is 2 ft.? $\frac{1}{2}$ ft.? $\frac{1}{4}$ ft.?
7. Reduce 5 bu. to pecks; to quarts; to pints.

Reduction Descending

206. Example. Reduce 4 T. 75 lb. to ounces.

Solution. Since 1 T. = 2000 lb., 4 T. = 4 times 2000 lb. = 8000 lb.; and with the 75 lb. added this = 8075 lb. Since 1 lb. = 16 oz., 8075 lb. = 8075 times 16 oz. = 129,200 oz., the required result.

8075 times 16 oz. = 16 times 8075 oz.; therefore 8075 times 16 oz. is found as shown in the margin.

$$\begin{array}{r} 2000 \\ 4 \\ \hline 8075 \\ 16 \\ \hline 129200 \end{array}$$ No. of oz

WRITTEN EXERCISE

Reduce:

1. 115′ 6″ to inches.
2. 12 bu. 4 qt. to pecks.
3. £16 15*s.* to shillings.
4. 211 rd. 3 ft. to inches.
5. $3\frac{1}{2}$ rd. to feet.
6. $1\frac{1}{2}$ T. to ounces.
7. 12 A. to square feet.
8. 161 cd. to cubic feet

ORAL EXERCISE

1. How many pecks in $\frac{1}{4}$ bu.? in $\frac{1}{2}$ bu.?
2. Change .25 A. to square rods; .375 A.; 75 A.
3. Reduce $\frac{1}{2}$ gal. to pints. Express $\frac{1}{2}$ rd. as inches; as yards

WRITTEN EXERCISE

Reduce:

1. $\frac{2}{8}$ mi. to feet.
2. .75 cd. to cubic feet.
3. $\frac{119}{160}$ A. to square feet.
4. $\frac{7}{8}$ yd. to inches.
5. .375 mi. to feet.
6. $\frac{1}{12}$ hr. to seconds.

Reduction Ascending

207. Example. Express 176 qt. dry measure in higher denominations.

Solution. Since 8 qt. = 1 pk., divide by 8 and obtain as a result 22 pk. Since 4 pk. = 1 bu., divide by 4 and obtain as a result 5 bu. 2 pk.

$176 \div 8 = 22$, or 22 pk.
$22 \div 4 = 5$ and a remainder of 2, or 5 bu. 2 pk.

WRITTEN EXERCISE

Reduce to higher denominations:

1. 3840 ft.
2. 1054 pt.
3. 14,400 sec.
4. 2000 sq. in.
5. 816 pk.
6. 106,590 ft.
7. 43,560 sq. in.
8. 27,900 lb. avoir.
9. 15,120″
10. 51,200 cu. ft.
11. 145,152 cu. in.
12. 27,900 oz. avoir.

ORAL EXERCISE

1. Reduce $\frac{1}{2}$ ft. to the fraction of a yard.
2. Change .16 cwt. to the decimal of a ton.
3. What part of a yard is 1 in.? 2 in.? $\frac{1}{2}$ in.?
4. What decimal part of an acre is 16 rd.? 40 rd.?
5. What part of 35 bu. is 7 bu.? of $1\frac{1}{2}$ bu. is $\frac{1}{2}$ bu.?

WRITTEN EXERCISE

1. Reduce $1\frac{7}{8}$ in. to the fraction of a foot; of a yard.
2. Reduce 10*s*. 9*d*. to the fraction of a pound sterling.

Solution. There are 12*d*. in a shilling and 20*s*. in a pound sterling, or 240*d*.

To find what fractional part of a pound sterling 10*s*. and 9*d*. are, use the following statement:

10*s*. 9*d*. = 129*d*. £1 = 240*d*. Therefore, 10*s*. 9*d*. = $\frac{129}{240}$ of a £, or £.5375.

10*s*. and 9*d*. = 129*d*.
£1 = 240*d*.
$\frac{129}{240} = .5375$, or £.5375

3. Reduce 4 yd. $1\frac{1}{2}$ ft. to the decimal of a rod.
4. Reduce 10*s*. 6*d*. 2 far. to the decimal of a pound sterling.
5. Reduce 5 T. 721 lb. to tons and decimal of a ton; 6 T. 1750 lb.; 12 T. 290 lb.; 29,240 lb.; 28,390 lb.
6. Find the cost of 1750 lb. of coal at $6.25 per ton; of 2170 lb.; of 690 lb.; of 1360 lb.; of 3240 lb.; of 32590 lb.

ADDITION AND SUBTRACTION

ORAL EXERCISE

State the sum of:

1.	2.	3.	4.
12 ft. 1 in.	5 lb. 8 oz.	15 rd. 5 ft.	10 mi. 8 rd.
6 3	6 3	17 2	8 40

5.	6.	7.	8.
5 rd. 2 ft.	11 ft. 2 in.	5 bu. 1 pk.	5 mi. 20 rd.
8 $2\frac{1}{2}$	8 1	8 0	17 13
7 $2\frac{1}{2}$	3 3	9 1	11 10

State the difference between:

1.	2.	3.	4.
90 mi. 300 rd.	75 rd. $12\frac{1}{2}$ ft.	30 yd. 2 ft.	44 bu. 3 pk.
75 120	26 $4\frac{1}{2}$	17 $1\frac{1}{2}$	29 1

5.	6.	7.	8.
11 mo. 12 da.	12 mo. 31 da.	11 mo. 15 da.	98 gal. 2 qt.
6 6	8 17	2 9	69 1

208. Examples. 1. Three jars of butter weighed 48 lb. 7 oz., 45 lb. 9 oz., and 53 lb. 11 oz. Find the total weight.

SOLUTION. Arrange the numbers as in simple addition, so that units of the same order stand in the same vertical column. Adding the first column at the right, the result is 27 oz. = 1 lb. 11 oz.; write 11 oz. and carry 1 lb. Adding the pounds, the sum is 147.

48 lb.	7 oz.
45	9
53	11
147 lb.	11 oz.

2. From a barrel containing 379 gal. 1 qt. of molasses, 17 gal. 3 qt. were sold. How much remained unsold?

SOLUTION. Arrange the numbers as in simple subtraction, so that units of the same order stand in the same vertical column. 3 qt. cannot be subtracted from 1 qt.; therefore mentally take 1 gal. (4 qt.) from 37 gal. and add it to 1 qt., making 5 qt. 5 qt. − 3 qt. = 2 qt. Inasmuch as 1 gal. was added to 1 qt., there are but 36 gal. remaining in the minuend; 36 gal. − 17 gal. = 19 gal.

37 gal.	1 qt.
17	3
19 gal.	2 qt.

WRITTEN EXERCISE

Find the sum of:

1.	2.	3.	4.
£140 6*s.*	£139 5*s.*	84 T. 75 lb.	279 T. 840 lb.
159 3	214 5	96 14	364 210
162 4	921 3	78 79	872 220
139 2	141 7	37 41	146 140
167 4	10 9	19 63	214 180
129 3	171 8	84 79	926 230
136 4	215 7	97 13	210 420
147 2	321 5	87 125	75 750

Find the difference between:

5.	6.	7.	8.
11 mo. 17 da.	11 mo. 1 da.	8 mo. 14 da.	9 mo. 17 da.
8 31	9 31	2 29	2 31

9. From a pile of wood containing $74\frac{1}{2}$ cd., $28\frac{1}{2}$ cd. and $15\frac{1}{4}$ cd. were sold. How much remained unsold?

10. I owned a farm of 340 A. when I bought an adjoining field of $74\frac{1}{4}$ A. I then sold $140\frac{3}{4}$ A. What is the remainder of the farm worth at \$75 per acre?

11. An English merchant had on hand Jan. 1 goods valued at £5927 10*s.*; during the following six months he bought goods at a cost of £4920 10*s.* and sold goods to the amount of £7926 4*s.* If the value of the goods on hand July 1 of the same year was £4120 10*s.*, what has been the gain or loss in English money? in United States money?

Finding the Difference between Dates

209. In the foregoing problems in addition and subtraction only compound numbers of two denominations were used. These are practically the only compound numbers met with in business, if the case of finding the difference between two dates is excepted.

210. The difference between two dates may be found by compound subtraction, or by counting the actual number of days from the given to the required date.

In business transactions involving long periods of time, the difference is generally found by compound subtraction; but in transactions involving short periods of time, the difference is generally found by counting the exact number of days.

211. Examples. 1. A mortgage dated Oct. 15, 1915, was paid Apr. 6, 1921. How long had it run?

SOLUTION. Write the later date as the minuend and the earlier date as the subtrahend. April being the 4th and October the 10th month, write 4 and 10 respectively instead of the names of the months. Consider 30 da. a month and 12 mo. a year and subtract as usual.

1921 yr.	4 mo.	6 da.	
1915	10	15	
5 yr.	5 mo.	21 da.	

2. Find the difference between Apr. 21 and July 27.

SOLUTION. Write the number of days remaining in April, the number in May and June, and finally the number in July up to and including July 27. The sum of these numbers is the required time expressed with exactness. Observe that *the total time excludes the first and includes the last day of the given dates.*

9 da.	in April
31 da.	in May
30 da.	in June
27 da.	in July
97 da.	from April 21 to July 27

ORAL EXERCISE

State the exact number of days between:

1. Mar. 12 and Apr. 16.
2. Apr. 27 and May 31.
3. May 31 and July 18.
4. June 7 and Aug. 16.
5. July 1 and Oct. 1.
6. June 30 and Sept. 1.
7. July 31 and Nov. 7.
8. Aug. 31 and Dec. 1.

WRITTEN EXERCISE

Find the exact number of days between:

1. Apr. 2 and Nov. 25.
2. Mar. 1 and Sept. 18.
3. Mar. 15 and Nov. 2.
4. Apr. 21 and Dec. 31.
5. Mar. 18 and Nov. 27.
6. Mar. 17 and July 28.
7. June 16 and Sept. 18.
8. June 19 and Nov. 29.

9. Find the difference between Jan. 3, 1915, and each of the following dates: May 15, 1912; Sept. 6, 1913; Apr. 8, 1909; Mar. 12, 1897. Find the difference by compound subtraction.

MULTIPLICATION AND DIVISION

ORAL EXERCISE

Multiply:	*Divide:*
1. 3 ft. by 6.	7. 27 yd. by 9.
2. $1\frac{1}{2}$ mi. by 8.	8. 225 ft. by $7\frac{1}{2}$ ft.
3. 9 lb. 4 oz. by 2.	9. 48 ft. 6 in. by 2.
4. 18 lb. 1 oz. by 9.	10. 540 yd. by 18 yd.
5. 17 yd. 2 in. by 9.	11. 164 lb. 12 oz by 4.
6. 19 gal. 1 qt. by 3.	12. 640 mi. 160 rd. by 20.

212. Examples. 1. How much hay in 8 stacks each containing 5 T. 760 lb.?

SOLUTION. 8 times 760 lb. = 6080 lb. = 3 T. 80 lb.; write 80 in place of pounds and carry 3. 8 times 5 T. = 40 T.; 40 T. + 3 T. carried = 43 T. The required result is therefore 43 T. 80 lb.

$$\begin{array}{r} 5 \text{ T. } 760 \text{ lb.} \\ \times \quad 8 \\ \hline 43 \text{ T. } 80 \text{ lb.} \end{array}$$

2. An importer paid £ 87 10*s.* for 50 pc. of bric-a-brac. What was the cost per piece?

SOLUTION. Since 50 pc. cost £ 87 10*s.*, 1 pc. costs $\frac{1}{50}$ of £ 87 10*s.* $\frac{1}{50}$ of £ 87 = £ 1 with an undivided remainder of £ 37; write £ 1 in the quotient and add £ 37 to the next lower denomination; £ 37 10*s.* = 750*s.* $\frac{1}{50}$ of 750*s.* = 15*s.*

$$50 \overline{)\text{£ } 87 \quad 10s.} \qquad \text{£ } 1 \quad 15s.$$

3. At 10*s.* 6*d.* per yard, how many yards can be bought for £ 15 15*s.*?

SOLUTION. The dividend and divisor are concrete numbers; therefore reduce them to the same denomination before dividing. £ 15 15*s.* = 3780*d.*, 10*s.* 6*d.* = 126*d.* 3780*d.* ÷ 126*d.* = 30; that is 30 yd. can be bought.

£ 15 15*s.* = 3780*d.*
10*s.* 6*d.* = 126*d.*
3780*d.* ÷ 126*d.* = 30, no. of yd.

ORAL EXERCISE

1. At 72 ¢ per gross what will 2 doz. buttons cost? 4 doz.? 7 doz.?

2. How many 3-oz. packages can be put up from 4 lb. of pepper?

3. Find the cost of 3 T. of bran at 30 ¢ per hundredweight; of 5 T. at 50 ¢ per hundredweight.

4. How many 1-lb. packages can be put up from 15 T. of breakfast food?

5. When coal is $6 per ton what will 7000 lb. cost? 6400 lb.? 3600 lb.?

6. Find the cost of 2400 lb. of flour at $2.25 per hundredweight; of 4400 lb.; of 3200 lb.

7. At $12\frac{1}{2}$¢ per quire what will 480 sheets of paper cost? 240 sheets? 2880 sheets? 720 sheets?

8. I buy 3 qt. of milk per day. If I pay 8¢ per quart, what is my bill for July and August?

9. I bought 3 gro. pens at 60¢ a gross and sold them at the rate of 2 for 1¢; what was my gain or loss?

10. I bought $3\frac{1}{2}$ bu. of apples at $1.00 per bu. and sold them at 50¢ a peck. What was my gain?

11. I sold $4\frac{1}{2}$ cd. of wood for $27 and thereby lost $9 on the cost. What was the cost per cord?

WRITTEN EXERCISE

1. Find the cost of 10 pwt. 7 gr. of old gold at $1.25 per pennyweight; of 12 pwt. 4 gr. at $1.10 per pennyweight.

2. I bought $3\frac{1}{8}$ A. of city land at $125 an acre and sold it at 50¢ per square foot. Did I gain or lose and how much?

3. Give the length of a double-track railroad that can be laid with 352,000 rails 30 ft. long.

4. I bought a barrel of cranberries containing $2\frac{1}{2}$ bu. at $4 per bushel and retailed them at 15¢ a quart. Did I gain or lose and how much?

5. From a farm of 375 A. I sold $25\frac{3}{4}$ A. What is the remainder worth at $125 per acre?

6. Find the cost (*a*) in English money and (*b*) in United States money of 360 doz. cotton hose at 5*s*. 2*d*.

SOLUTION. (*a*) 5*s*. 2*d*. = $5\frac{1}{6}$*s*. 360 times $5\frac{1}{6}$*s*. = 1860*s*. = £93, the cost in English money.

(*b*) £1 = $4.8665. 93 times $4.8665 = $452.58, the cost in United States money.

7. Copy and find the amount of the following invoice:

London, England, Apr. 3, 1906

Messrs. W. P. Fisth & Co.
New York City

Terms Net 30 ds. **Bought of E. M. LLOYD & SON**

C	#1	150 doz. Fancy Cotton Hose	5/2	** ***
	#2	250 " Black Cotton Hose	4/3	** **
				*** ***
		2 cases	12/-	* ** *** ***

5/2, 4/3, and 12/- in the price column = 5*s.* 2*d.*, 4*s.* 3*d.*, and 12*s.*, respectively.

8. Reduce $2500 to English money.

SOLUTION. $2500 ÷ $4.8665 = 513.72, or £513.72. .72 × 20*s.* = 14.4*s.* .4 × 12*d.* = 4.8*d.* .8 × 4 far. = 3.2 far. Hence $2500 = £513 14*s.* 4*d.* 3.2 far.

9. Find the value in United States money of a post-office money order for £5 18*s.* 6*d.*; for £3 12*s.*

10. Without copying, find the amount of the following invoice:

Leith, Scotland, May 16, 19

INVOICE OF HOSIERY

Shipped by The J. M. Roberts Co. In the steamship Winifredian
To Edward M. Davidson & Co.
Boston, Mass.

Marks	Number	Quantity		Article and Description	Price	Extension	Amount
D	1611	1	cs	Mdse, as follows:			
	622	51	doz	Black Cotton Hose	6/4		
	670	30	"	" " "	7/2		
	676	6½	"	" " "	6/6		
	650	15	"	Black Lace Hose	12/4		
	680	6	"	" " "	16/6		
	686	3	"	" " "	18/4		

11. A druggist bought by avoirdupois weight 5 lb. of peppermint oil at $2.50 per pound and retailed it at 50¢ an ounce, apothecaries' weight. What was his gain?

213. Farm products which are handled in bulk are frequently bought and sold by the bushel. The statutory weights of the bushel for some of the common commodities are shown in the following table:

STATUTORY WEIGHTS OF THE BUSHEL

COMMODITIES	WEIGHT IN AVOIRDUPOIS POUNDS	EXCEPTIONS
Barley	48	Ala., Ga., Ky., and Penn., 47; Ariz., 45; Cal., 50.
Beans	60	N. H. and Vt., 62.
Clover Seed	60	
Corn, Shelled	56	Ariz., 54; Cal. 52.
Oats	32	Me., N.J., Va., 30; Md., 26.
Potatoes, Irish	60	Md., Penn., and Va., 56.
Rye	56	Cal., 54.
Wheat	60	

214. Example. What will 4260 lb. of wheat cost at 80¢ per bushel?

SOLUTION. In examples of this character the principles of cancellation may be applied to advantage.

$$\frac{\overset{71}{\cancel{4260}} \times 80¢}{\underset{1}{\cancel{60}}} = \$56.80$$

In problems 1–4 in the following exercise the price is per bushel in each case.

WRITTEN EXERCISE

1. Find the total value of:

6640 lb. wheat at 84¢.
4230 lb. wheat at 95¢.
1260 lb. wheat at 90¢.
6120 lb. wheat at $87\frac{1}{2}$¢.

2. Find the total value of:

3264 lb. oats at 25¢.
2400 lb. oats at 48¢.
2560 lb. oats at $37\frac{1}{2}$¢.
6951 lb. oats at 32¢.
1920 lb. oats at $33\frac{1}{3}$¢.
3840 lb. oats at $29\frac{1}{2}$¢.

3. Find the total value of:

3660 lb. clover seed at $4.50.
1200 lb. clover seed at $4.75.
2472 lb. clover seed at $4.20.
5040 lb. shelled corn at $47\frac{1}{2}$¢.
2800 lb. shelled corn at 56¢.
2240 lb. shelled corn at 73¢.

CHAPTER XVI

PRACTICAL MEASUREMENTS

DISTANCES AND SURFACES

Distances

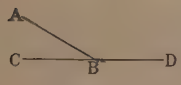
215. An **angle** is the divergence of two lines from a common point.

Thus the divergence of the lines BA and BC from the point B is the angle ABC.

216. A **right angle** is the angle formed when one straight line so meets another as to make the two adjacent angles equal. The lines forming the angles are **perpendicular** to each other.

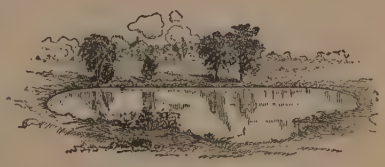
Thus the two angles ABC and ABD are right angles, and the lines AB and CD are perpendicular to each other.

217. An **acute angle** is less than a right angle; an **obtuse angle** is greater than a right angle.



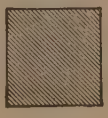
Thus the angle ABC is an acute angle, and the angle ABD is an obtuse angle.

218. A **surface** is that which has *length* and *width*, but not measurable thickness. A level surface, as the surface of still water, is called a **plane surface** or a **plane**.



219. A **rectangle** is a plane figure bounded by four straight lines and having four right angles.

A **square** is a rectangle whose sides are all equal.



220. A **triangle** is a plane figure bounded by three straight lines and having three angles.

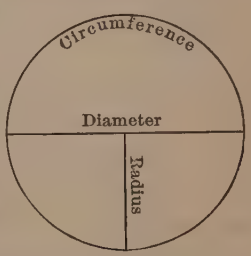
A triangle is called **equilateral** when all its sides are equal; **isosceles** when any two of its sides are equal; **scalene** when no two of its sides are equal.

221. A **right-angled triangle** is a triangle having a right angle.

A triangle containing an acute angle is sometimes called an **acute-angled triangle**; a triangle containing an obtuse angle, an **obtuse-angled triangle.**

222. The **perimeter** of a plane figure is the distance around it.

223. A **circle** is a plane figure bounded by a regularly curved line, every point of which is equally distant from a point within called the *center*. The **circumference** of a circle is the curved line which bounds it; the **diameter** is any straight line passing through the center and terminating in the circumference; the **radius** is one half the diameter. An **arc** is any part of the circumference of a circle.



224. It is proved in geometry that the *circumference of a circle is* 3.1416 *times the diameter.*

225. Therefore, to find the circumference of a circle when the diameter is given, *multiply the diameter by 3.1416.*

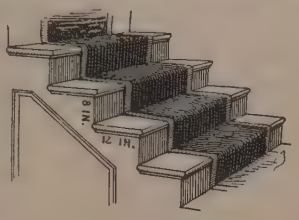
226. And, conversely, to find the diameter of a circle when the circumference is given, *divide the circumference by 3.1416.*

WRITTEN EXERCISE

1. Draw neat figures to represent each of the following: rectangle, triangle, square, circle, right-angled triangle, equilateral triangle, isosceles triangle, scalene triangle, radius of a circle, arc of a circle.

2. A parlor is 18 ft. 6 in. long and 12 ft. 3 in. wide. What will be the cost, at 28¢ per foot, of a molding extending around the room?

3. The circumference of a circle is 113.0976 ft. What is the length of the longest straight line that can be drawn across the circle? Find the circumference of a circle whose radius is 21 ft.



4. What will be the cost, at 75¢ per yard, of carpeting a stairway of 18 steps, the tread of each stair being 12 in. and the riser 8 in.? (Allow for one extra step and one extra riser.)

5. How many telegraph poles 10 rd. apart will be required for 150 mi. of railroad?

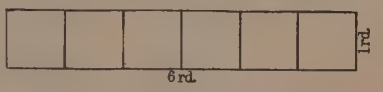
Areas

ORAL EXERCISE

1. What is the area of a square 1 rd. on each side?

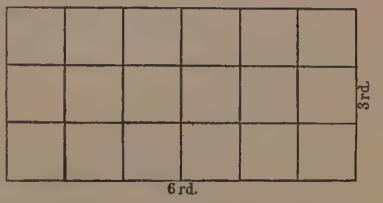
2. How many squares 1 rd. on each side in a rectangle 6 rd. long and 1 rd. wide?

3. How many rectangles, each 6 rd. by 1 rd., in a rectangle 6 rd. by 3 rd.?



4. How many square rods in the area of a rectangle 6 rd. long and 3 rd. wide?

5. How many square rods in the area of a rectangle 16 rd. long and 132 ft. wide?



SOLUTION. 132 ft. = 8 rd. A rectangle 1 rd. on a side contains 1 sq. rd. But the given rectangle is 16 times 1 rd. long and 8 times 1 rd. wide. Therefore the required area is $16 \times 8 \times 1$ sq. rd., or 128 sq. rd.

$$132 \text{ ft.} = 8 \text{ rd.}$$
$$8 \times 16 \text{ sq. rd.} = 128 \text{ sq. rd.}$$

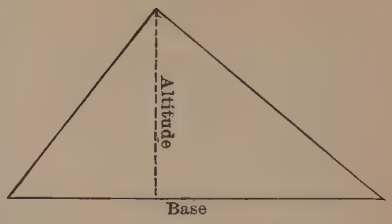
227. *The product of the length and width of a rectangle equals the area.*

ORAL EXERCISE

Find the areas of rectangles having the following dimensions. Make use of the short method explained in §§ 180–182.

1. $6\frac{1}{2}$ ft. by $6\frac{1}{2}$ ft.
2. $7\frac{1}{2}$ rd. by $7\frac{1}{2}$ rd.
3. 9.5 rd. by 9.5 rd.
4. 12.5 ft. by 4.5 ft.

228. The dimensions of a triangle are the **base**, the side on which the triangle appears to stand; the **altitude**, the perpendicular distance from the base to the highest point of the triangle.



ORAL EXERCISE

1. How does the area of the triangle on the right compare with the area of a rectangle 8 ft. by 4 ft.?

2. Compare the area of the triangle on the left with the area of a rectangle 12 rd. by $5\frac{1}{2}$ rd.

3. What is the area of a triangle whose base is 8 ft. and whose altitude is $9\frac{1}{2}$ ft.?

229. In the above exercise it is clear that *one half the product of the base and altitude of a triangle equals the area.*

ORAL EXERCISE

State the areas of triangles whose bases and altitudes, respectively, are as follows:

1. 20 ft., 18 ft.
2. 12 ft., 16 ft.
3. 12 ft., $4\frac{1}{2}$ ft.
4. $19\frac{1}{2}$ ft., 8 ft.

230. If a circle be divided as in the figure on the left and the parts rearranged as in the figure on the right, it will be clear that the area of the circle equals the area of the twelve triangles. The altitude of

each triangle is the radius of the circle, and the sum of the bases the circumference.

231. It is therefore clear that *one half the product of the circumference and radius of a circle equals the area.*

ORAL EXERCISE

1. Find the area of this triangle: base, 8 in.; height, 11 in.

2. A field contains 1280 sq. rd. If the width is 32 rd., what is the length?

3. A man sold a lot 10 rd. long and 8 rd. wide at the rate of $260 per acre. How much did he receive?

WRITTEN EXERCISE

1. Find the area of a circular pavilion with a radius of $56\frac{1}{2}$ ft.

2. A city lot contains $\frac{1}{4}$ A. If it is 200 ft. long, what is its width, and what is its value at 50¢ per square foot?

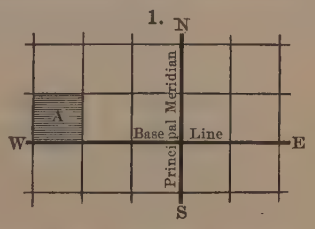
3. The floor of a restaurant 50 ft. long and 40 ft. wide is covered with tiles 8 in. square. How many tiles will be required?

4. A park, 50 rd. long and 40 rd. wide, has a walk 1 yd. wide inclosing it. How many square feet in the walk?

Public Lands

232. In parts of the United States public lands are surveyed by selecting a **principal meridian** which runs north and south, and a **base line** which runs east and west. Other lines divide the land into tracts 6 mi. square called **townships.** Township lines running north and south are called **ranges.**

1.



A in the diagram may be read as Tp. 1 N., R. 3 W.: the first township north of the base line, in the third range west of the principal meridian.

233. Each township is divided into 36 **sections**, each 1 mi. square. The numbering of a section is shown in the diagram at the left.

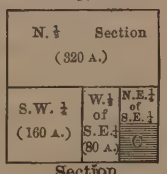
2.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Township

Sections are divided into *halves* and *quarters;* quarter sections are subdivided into *halves* and *quarters.*

3.



Section

If diagram 3 is B of diagram 2, and diagram 2 is A of diagram 1, C of diagram 3 may be described as the S. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$, Sec. 19, Tp. 1 N., R. 3 W.

ORAL EXERCISE

1. How many chains in a mile? how many rods? how many feet? How many rods in a chain? how many feet?

2. How many acres in a field 50 ch. by 40 ch.? in a field 40 ch. square? in a field 80 ch. by 80 ch.?

3. A field has an area of 4 A. If it is 10 ch. long, how wide is it and what will it cost to fence it at 50¢ per rod? at 60¢?

WRITTEN EXERCISE

1. Make a diagram of a township and locate N. $\frac{1}{2}$, Sec. 20.

2. Draw a diagram illustrating principal meridian, base line, range line, and township lines, and mark Tp. 2 S., R. 2 E. and Tp. 1 N., R. 3 W.

3. Find the value, at $12.50 per acre, of Tp. 2 N., R. 3 W.

4. Find the cost at $25 per acre of the E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 20, Tp. 1 N., R. 4 W.

Square Root and Its Applications

ORAL EXERCISE

1. What is meant by *factor?* by *exponent?* by *power of a number?*

2. State the second power of each of the following numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9. How much is 12^2, 13^2, 14^2, 15^2, 16^2?

3. Name *one* of the *two equal* factors of each of the following numbers: 2, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196.

234. The **square** of a number is the product arising from using the number *twice* as a factor. The **square root** of a number is one of the *two equal factors* of the number.

235. The square root of a number may be indicated by writing the number under the *radical sign* $\sqrt{\ }$ or by placing the *fraction* $\frac{1}{2}$ above and to the right of the number.

Thus, $\sqrt{196}$ or $196^{\frac{1}{2}}$ indicates the square root of 196.

236. The square root of a number is readily derived from the process by which the square is formed.

237. Example. What is the square of 42?

SOLUTION. Since $42 = 40 + 2$, the square of 42 may be found as follows:

$$\begin{array}{l} 40 + 2 \\ \underline{40 + 2} \\ (40 \times 2) + 2^2 \\ 40^2 + (40 \times 2) \\ \hline 40^2 + 2(40 \times 2) + 2^2 = \end{array} \qquad \begin{array}{r} 40^2 = 1600 \\ 2(40 \times 2) = \quad 160 \\ 2^2 = \quad 4 \\ \hline 1764 \end{array}$$

238. In the preceding process it is shown that *the square of a number is equal to the square of the tens plus twice the product of the tens by the units, plus the square of the units.*

239. $1^2 = 1$, $10^2 = 100$, $100^2 = 10000$, and so on; $9^2 = 81$, $99^2 = 9801$, $999^2 = 998001$, and so on. It is therefore evident that the square of an integral number contains twice as many figures or one less than twice as many figures as the number. Hence, if an integral number be separated into groups of two figures each, from right to left, there will be as many figures in the square root as there are groups of figures in the number.

240. Examples. 1. What is the square root of 529?

SOLUTION. Beginning at the right, separate the number into periods of two figures each. The greatest square in 5 is 4 and the square root of 4 is 2, the tens' figure of the root. Find the remainder, affix the second period, and the result is 129. This remainder is equal to twice the product of the tens by the units, plus the square of the units (§ 242). Twice 2 tens is 4 tens (40) and 4 tens (40) is contained in 129, 3 times; hence, 3 is the units' figure of the root. Twice the tens multiplied by the units plus the square of the units is the same as twice the tens plus the units multiplied by the units. Therefore, annex 3 units to the 4 tens and multiply by 3; the result is 129. The square root of 529 is thus shown to be 23.

$$\begin{array}{r} 5\ 29(23 \\ 4 \\ \hline 43)1\ 29 \\ 1\ 29 \\ \hline \end{array}$$

2. What is the square root of (*a*) 13.3225; (*b*) of .0961?

$$\begin{array}{r} 13\ .32\ 25(3.65 \\ 9 \\ \hline 6.6)4\ .32 \\ 3\ .96 \\ \hline 7.25)\ .36\ 25 \\ .36\ 25 \\ \hline \end{array} \qquad \begin{array}{r} .09\ 61(.31 \\ .09 \\ \hline .61).00\ 61 \\ .00\ 61 \\ \hline \end{array}$$

241. The process of finding the square root of a number may be summarized as follows:

Beginning at the units, separate the number into groups of two figures each.

Find the greatest square in the left-hand group and write its root for the first figure of the required root.

Subtract the square of the root figure from the left-hand period and annex the second period for a dividend.

Take twice the root figure already found, considered as tens, and divide the dividend by it.

Annex the quotient to both the root and the trial divisor and multiply by the units.

Continue in like manner until all the periods have been used. The result will be the square root.

If a number contains a decimal, begin at the decimal point and indicate groups to the left for the integral part of the root, and to the right for the decimal part of the root. If the last period on the right of the decimal point has but one figure, annex a decimal cipher, as each decimal period *must* contain two figures.

To find the square root of a common fraction, extract the square root of the numerator and denominator separately. If the terms of the fraction are not perfect squares, reduce the fraction to a decimal and then extract the square root.

WRITTEN EXERCISE

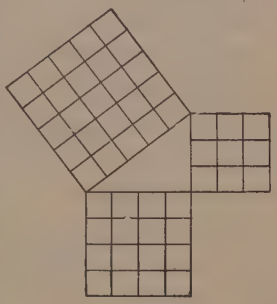
Find the square root of:

1. 324.	**5.** 576.	**9.** 9025.	**13.** $\frac{49}{64}$.
2. 484.	**6.** 1024.	**10.** 3364.	**14.** $\frac{169}{324}$.
3. 676.	**7.** 7225.	**11.** 70.56.	**15.** $\frac{6561}{8281}$.
4. 729.	**8.** 3969.	**12.** 150.0625.	**16.** $\frac{1900969}{7743044}$.

242. It has been seen that the area of a square is the product of its two equal sides. It therefore follows that *the square root of the area of a square equals one of its sides.*

243. The **hypotenuse** is the side opposite the right angle in a right triangle.

244. In the accompanying illustration it will be seen that the square on the hypotenuse is equal to the sum of the squares on the other sides. Hence,



245. To find the hypotenuse *take the square root of the sum of the squares of the base and altitude;* and

246. To find the base or the altitude *take the square root of the difference between the squares of the hypotenuse and the other side.*

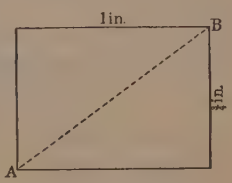
WRITTEN EXERCISE

1. A square field contains 5.625 A. What is the length of one of its sides?

2. Find the side of a square containing the same area as a field 160 rd. long by 90 rd. wide.

3. What is the hypotenuse of a right-angled triangle the base of which is 30 ft. and the altitude 40 ft.?

4. The accompanying diagram represents a piece of land. It is drawn on the scale of $\frac{1}{80}$ in. to the rod. The land is divided into two fields by the line AB. Find the cost, at 50¢ per rod, of fencing the two fields.



5. What will be the cost, at $1.75 per chain, of fencing a square field containing 1.6 A.?

Roofing

247. Roofing is usually measured by the *square* of 100 sq. ft.

248. The **size of slates** used for roofing varies from 6 in. by 12 in. to 16 in. by 24 in.

Contractors and builders generally use prepared tables for estimating the amount of slate to be used. The number of slates per square varies with the size of the slate. Thus, slates 16 in. by 24 in. require 86 per square; slates 6 in. by 12 in. require 533 per square; etc.

249. All shingles average 4 in. in width and are put up in **bundles** of 250. The shingles most commonly used are 16 in. or 18 in. long. 16-inch shingles are generally laid $4\frac{1}{2}$ in. and 18-inch shingles $5\frac{1}{2}$ in. to the weather.

250. A shingle 4 in. wide laid $4\frac{1}{2}$ in. to the weather will cover 18 sq. in. A square contains 14,400 sq. in. 14,400 sq. in. ÷ 18 sq. in. = 800. It is therefore clear that 800 16-*inch shingles will cover a square of roof.*

251. A shingle 4 in. wide laid $5\frac{1}{2}$ in. to the weather will cover 22 sq. in. 14,400 sq. in. ÷ 22 sq. in. = 655. It is therefore clear that 655 18-*inch shingles will cover a square of roof.*

In practice 655 per square is called 700 per square.

ORAL EXERCISE

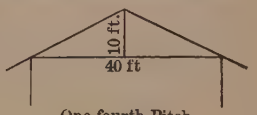
1. How many bundles in 1000 shingles? in 7500 shingles? in 26,000 shingles?

2. What will be the cost, at \$4 per square, of tinning a roof 20 ft. by 15 ft.?

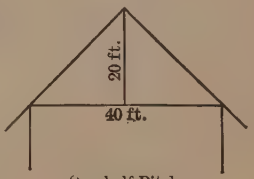
3. A certain roof requires 7610 shingles. How many bundles of shingles must be bought to cover it?

A dealer will not sell a fractional part of a bundle of shingles.

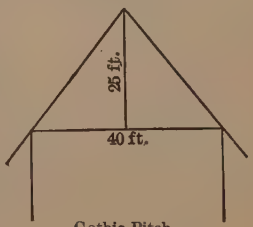
4. How many slates at 300 to the square will be required for a flat roof 30 ft. by 20 ft.?



One fourth Pitch



One half Pitch



Gothic Pitch

252. The rise in the rafters for each foot in the base of the gable is called the **pitch of the roof.**

253. When the rise of the roof is 6 in. per foot, the roof is said to have **one-fourth pitch.**

254. When the rise of the rafters is 12 in. per foot, the roof is said to have **one-half pitch.**

255. When the rise of the rafters is 15 in. per foot, the roof is said to have **five-eighths**, or **Gothic pitch.**

When the rise of the rafters is 6 in. per foot, the perpendicular height of the gable is $\frac{1}{4}$ of the width of the building; when the rise is 12 in. per foot, the height of the gable is $\frac{1}{2}$ the width of the building; when the rise is 15 in. per foot, the height of the gable is $\frac{5}{8}$ of the width, or $1\frac{1}{4}$ times $\frac{1}{2}$ the width of the building. Hence the names one-fourth pitch, one-half pitch, etc.

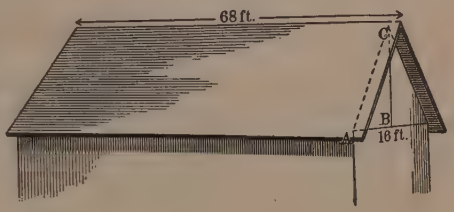
ORAL EXERCISE

Find the height of the gable:

Width of Building	Pitch of Roof	Width of Building	Pitch of Roof
1. 30 ft.	$\frac{1}{2}$	**3.** 24 ft.	Gothic
2. 50 ft.	12 in. per ft.	**4.** 36 ft.	$\frac{1}{4}$

WRITTEN EXERCISE

1. The accompanying diagram represents the roof of a shed 16 ft. wide. If the ridge-pole is 68 ft., the pitch of the roof one half, and the projection of the rafters 18 in., how many shingles 16 in. long, laid $4\frac{1}{2}$ in. to the weather, will be required to cover the roof?



Solution

$\frac{1}{2}$ of 16 ft. = 8 ft. = the base of the triangle ABC.

The pitch of the roof is $\frac{1}{2}$; $\frac{1}{2}$ of 16 ft. = 8 ft. = the altitude of the triangle ABC.

$8^2 + 8^2 = 128$; $128^{\frac{1}{2}} = 11.31$, number of feet in the hypothenuse of ABC.

18 in. = 1.5 ft.; 11.31 ft. + 1.5 ft. = 12.81 ft. = the length of the rafters or the width of each side of the roof.

$2 \times 68 \times 12.81$ ft. = 1742.16 sq. ft. = the entire surface of the roof.

1742.16 sq. ft. = 17.4216 squares; 17.4216×800 shingles = 13937 shingles.

As bundles of shingles are not broken it will be necessary to buy 14000 shingles.

2. A building is 40 ft. wide. If the length of the ridge-pole is 80 ft. and the projection of the rafters 20 in., how many shingles 18 in. long and laid $5\frac{1}{2}$ in. to the weather will be required for the roof, the pitch being $\frac{1}{2}$?

3. A building is 30 ft. wide. If the length of the ridge-pole is 60 ft. and the projection of the rafters 15 in., how many shingles 16 in. long and laid $4\frac{1}{2}$ in. to the weather will be required for the roof, the pitch being $\frac{1}{4}$?

Plastering

256. Plastering is usually measured by the **square yard.**

257. There is no uniform rule with respect to the **allowance** to be made for doors, windows, and other openings.

What allowance, if any, shall be made for openings is usually stated in the contract covering the work. In some sections it is customary to make allowance for one half the area of the openings; in others, for the full area of the openings; in still others, for a stated number of square feet.

In giving the dimensions of a room carpenters, architects, and mechanics write the length first, then the width, and finally the height. They also usually write 5″ for 5 in., 5′ for 5 ft., and 5′ × 5′ for 5 ft. by 5 ft.

ORAL EXERCISE

1. What is the perimeter of a square room 20′ on a side?
2. What is the perimeter of a dining room 18′ × 12′ × 9′?
3. How many square feet in the four walls of the room in problem 2, not allowing for openings? in the ceiling? in the four walls and the ceiling?
4. How many square yards in the four walls of a room 24′ × 16′ × 9′, not allowing for openings?
5. At 25¢ per square yard, what will it cost to plaster 945 sq. ft.? 1080 sq. ft.? 1440 sq. ft.?

WRITTEN EXERCISE

1. What will it cost, at 27 ¢ per square yard, to plaster the walls and ceiling of a hall 60′ × 40′ × 24′, making an allowance of 40 sq. yd. for openings?

2. Find the cost, at 26¢ per square yard, of plastering the walls and ceiling of a room 18′ × 16′ 6″ × 8′ 6″, making full allowance for 2 doors each 7′ 6″ × 4′, 3 windows 6′ × 4′.

3. What will be the cost of plastering, with hard finish, at 34¢ per square yard, the walls of the rooms in the following dwelling?

First Floor. Parlor, 14′ × 12′; sitting room, 12′ × 12′; dining room, 12′ × 10′; kitchen, 12′ × 10′; pantry, 8′ × 6′. All rooms on this floor are uniformly 8′ 6″ high.

Second floor. Front chamber, 14′ × 12′; back chamber, 12′ × 12′; middle chamber, 10′ × 9′; hall, 23′ × 4′. All rooms on this floor are uniformly 8′ high.

Allowance is made for 40 openings of 17 sq. ft. each.

Painting

258. Painting is usually measured by the **square yard.**

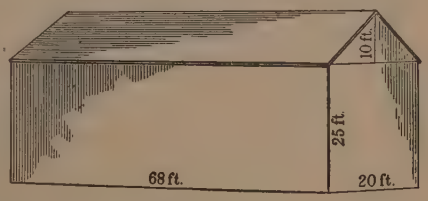
259. It is customary to make no allowance for windows, the painting of window sills and sashes being considered as expensive as the painting of the surface area of the entire window.

WRITTEN EXERCISE

1. What will it cost, at 25¢ per square yard, to paint the walls of a room 20′ × 16′ × 12′, no allowance being made for doors or windows?

2. At $6\frac{1}{4}$¢ per square yard, what will it cost to kalsomine the walls and ceiling of a room 24′ × 18′ × 12′, allowing for a door 9′ × 4′, 2 windows 7′ × 4′, and a wainscot 3′ high around the regular surface of the room?

3. Find cost, at 24¢ per square yard, of two coats of paint on the outside walls of a tobacco barn 68′ × 20′ × 25′ with gables extending 10′ above the ends of the walls.



4. What will be the cost, at 22¢ per square yard, of painting the outside walls of a barn 100′ × 40′ × 20′ with gables extending 10′ above the walls? with gables extending $12\frac{1}{2}$′ above the walls?

Flooring

260. Flooring is measured by the **square** (100 sq. ft.) or by the **thousand square feet.**

Professional floor layers charge by the square, the price being from 75¢ to $1.50 per square. Carpenters usually work by the day in laying floors.

Spruce flooring is 4″ or 5½″ in width; hardwood flooring is 2″ or 2½″ in width. In flooring there is considerable waste in forming the tongue and the groove of the boards. When flooring is 3″ or more in width, it requires about 1¼ sq. ft. of material for every square foot of surface to be covered; when flooring is less than 3″ in width, it requires 1⅓ sq. ft. for every square foot of surface to be covered.

261. Example. How many feet of spruce flooring will be required for a room 32′ × 24′?

Solution. $32 \times 24 = 768$, the number of square feet to be covered.
$1\frac{1}{4} \times 768$ sq. ft. $= 960$ sq. ft., the quantity of flooring required.

WRITTEN EXERCISE

1. Find the cost at $45 per thousand square feet of a hardwood floor for a room 20′ × 16′.

2. A pavilion is 70′ × 50′. If the flooring is of spruce, what will be the cost at $27 per thousand square feet?

3. In a two-story dwelling the floor area measures 35′6″ × 26′. The first floor is to be of hardwood and the second floor of spruce. Find the quantity of flooring needed.

4. What will be the cost of a hardwood floor in a room 30′ × 28′, if the labor and incidentals cost $25.50, the lumber $48 per M.?

5. Find the cost of laying an oak floor 20′ × 15′, reckoning the labor and incidentals at $9.50, the floor boards at $83½ per thousand.

6. The floors in a three-story dwelling are each 55′ 4″ × 33′ 10″. The first floor is to be of hardwood worth $50 per thousand square feet and the other floors of spruce worth $27 per thousand square feet. If it costs $1.10 per square for labor, what will be the total cost of laying the three floors?

Carpeting

262. Carpet is sold by the **yard.** Such floor covering as oilcloth and linoleum are frequently sold by the **square yard.**

263. In determining the number of yards of carpeting required for a room it is necessary to know whether the strips are to run lengthwise or crosswise.

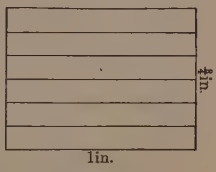
Carpets are generally laid lengthwise of a room; but when the matter of expense is an item, it is sometimes more economical to lay the strips crosswise.

When the length of the strips required is not an even number of yards, there is usually some waste in matching the pattern. Merchants will cut fractional lengths but not fractional widths of carpeting. It is therefore frequently necessary to cut off or turn under a part of a strip.

ORAL EXERCISE

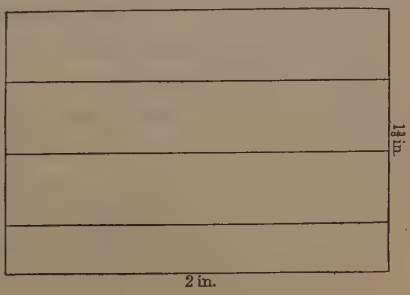
1. How many yards of carpet, 1 yd. wide, must be purchased for a room 5 yd. long by 4 yd. wide?

2. The accompanying diagram represents a room drawn on the scale of $\frac{1}{24}$ of an inch to the foot. Find the dimensions of the room.



3. How many strips of carpet, 1 yd. wide, laid lengthwise of the room, will be required for problem 2? How many feet in each strip? How many yards of carpet will be required for the room?

4. The accompanying diagram represents a room drawn on the scale of $\frac{1}{8}$ in. to the foot. How many strips of carpet, 1 yd. wide, laid lengthwise of the room, will be required to cover it? What part of a strip must be cut off or turned under in this case?



5. How many feet in each strip in problem 4? If there is no waste in matching the pattern, how many feet of carpet will be required? how many yards?

WRITTEN EXERCISE

1. How many yards of carpeting 1 yd. wide will be required to cover the chamber in the accompanying floor plan if the strips are to run lengthwise and there is no waste in matching the pattern?

2. Find the number of yards of carpet required to cover the room in problem 1 if the strips run across the room and there is a waste of 6 in. per strip in matching the pattern.

3. If the chamber is carpeted in the more economical way, what will be the cost at $1.25 per yard?

4. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for the parlor in the foregoing floor plan? The strips are to run lengthwise, and there is no waste in matching the pattern.

The cheaper grades of carpet are usually 1 yd. wide. The expensive grades, such as Brussels, Wilton, etc., are $\frac{3}{4}$ yd. wide.

5. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for the dining room in the foregoing floor plan? The strips are to run lengthwise, and there is a waste of 6 in. per strip in matching the pattern.

PAPERING

264. Wall paper is usually sold in double rolls 18 in. wide and 16 yd. long.

Single rolls 18 in. wide and 8 yd. long are sometimes used, but it is generally found more economical to use double rolls. These dimensions vary more or less.

Allowances for openings, such as doors and windows, are made in different ways by different paper hangers. Some make a uniform allowance for each opening, while others make allowance for the exact measurements of the openings.

Any whole rolls left over after papering may usually be returned to the dealer.

ORAL EXERCISE

1. What will the border for a room 15′ × 18′ cost at $33\frac{1}{3}$¢ per yard?

2. 18 in. = $\frac{3}{2}$ ft. 30 ft ÷ $\frac{3}{2}$ ft. = 30 ft. × $\frac{2}{3}$ ft. = 20. Divide 21 ft. by 18 in.

3. A wall is 15 ft. long and 9 ft. high. If there are no openings, how many strips will be required to cover it? How many full strips can be cut from each double roll of paper? What part of a strip will run to waste? How many rolls will be required for the wall?

4. Suppose that in problem 2 there is a door 3′ × 8′. What is the length of the regular surface of the wall? Fractional strips must be counted as full strips. Why? How many strips of paper will be required to cover the regular surface of the wall? Will dealers sell a fractional part of a roll of paper? How many rolls, then, will be required for the regular surface of the walls?

5. There is a small surface over the door in problem 5 that has not been considered. What may be used to cover this surface?

265. Obviously, to estimate the quantity of paper required for a room:

From the perimeter of the room subtract the width of the openings. Find $\frac{2}{3}$ of this remainder and the result will be the number of strips required. Divide the number of strips required by the number of full strips that can be cut from each roll of paper and the result is the required number of rolls.

By this method the ends of the rolls are supposed to be utilized for the surface above the doors and above and below the windows and other irregular places.

The height of the room, in papering, will be understood to mean the distance from the baseboard to the frieze.

To estimate the paper required for a ceiling, take $\frac{2}{3}$ of the width of the room for the number of strips required. Divide the number of strips required by the number of full strips that can be cut from each roll and the result is the number of rolls of paper required.

266. Example. How many double rolls of paper will be required for the walls and ceiling of a room $21' \times 18' \times 8'$, allowing for 2 doors and 3 windows, each $3\frac{1}{2}$ ft. wide?

SOLUTION

$(21' + 18') \times 2 = 78'$, the perimeter of the room.
$5 \times 3\frac{1}{2}' = 17\frac{1}{2}'$, the total width of the openings.
$78' - 17\frac{1}{2}' = 60\frac{1}{2}'$, the perimeter of the regular surface of the walls.
$\frac{2}{3}$ of $60\frac{1}{2} = 40\frac{1}{3}$, the number of strips of paper necessary for the regular surface.
$48' \div 8' = 6$, the number of strips in each roll.
$40\frac{1}{3}$ strips $\div$ 6 strips $= 6\frac{1}{18}$, or practically 7 rolls of paper required for the walls.
$\frac{2}{3}$ of $18 = 12$, the number of strips required for the ceiling.
$48' \div 21' = 2\frac{2}{7}$, or practically 2, the number of strips in each roll.
12 strips $\div$ 2 strips $= 6$, the number of rolls required for the ceiling.
6 rolls + 7 rolls = 13 rolls required for the walls and ceiling.

WRITTEN EXERCISE

1. The rooms in the floor plan, page 216, are $9'$ high. What will it cost, at 95¢ a roll, to paper the walls and ceiling of the parlor, making allowance for 2 double doors, each $6'$ wide, 1 single door $3\frac{1}{2}'$ wide, and 2 windows, each $3\frac{1}{2}'$ wide?

2. How many rolls of paper will be required for the walls and ceiling of the dining room in the floor plan, page 216, allowing for 1 double door $6'$ wide, 1 single door $3\frac{1}{2}'$ wide, and 2 windows each $3\frac{1}{2}'$ wide?

3. At 43¢ per roll how much will it cost to paper the walls and ceiling of the chamber in the floor plan, page 216, allowing for 2 windows, each $3\frac{1}{2}'$ wide, 1 double door $6'$ wide, and 1 single door $3\frac{1}{2}'$ wide.

SOLIDS

Rectangular Solids

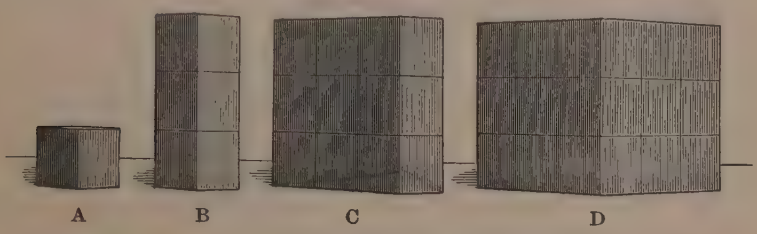
267. A **solid** is that which has *length*, *width*, and *thickness*.

268. A **rectangular solid** is a solid bounded by six rectangular surfaces.

269. A **cube** is a rectangular solid having six square faces.

ORAL EXERCISE

1. If A in the accompanying series of diagrams is 1 cu. ft., how many cubic feet in B? in C? in D?



2. How many cubic feet in a block of granite 6 ft. long, 1 ft. wide, and 1 ft. high? in a block 6 ft. long, 3 ft. wide, and 1 ft. high? in a block 6 ft. long, 3 ft. wide, and 3 ft. high?

3. Find the volume of a rectangular solid 6 ft. by 4 ft. by 2 ft.; a rectangle 10 ft. by 9 ft. by 9 ft.

4. A cellar is 40 ft. square and 6 ft. deep. How many cubic yards of earth were removed in excavating it?

SOLUTION. A cube 1 ft. on the side contains 1 cu. ft. The given cube is 40×1 ft. long, 40×1 ft. wide, and 6×1 ft. high. Therefore, it contains $6 \times 40 \times 40 \times 1$ cu. ft., or 9600 cu. ft.; and 9600 cu. ft. $= 355\frac{5}{9}$ cu. yd., the required result.

$6 \times 40 \times 40 \times 1$ cu. ft. $= 9600$ cu. ft.
9600 cu. ft. $\div 27 = 355\frac{5}{9}$ cu. yd.

270. In the foregoing exercises it is clear that *the product of the three dimensions of a solid equals the volume or solid contents.*

WRITTEN EXERCISE

1. A box car is 50 ft. 6 in. long, 8 ft. 4 in. wide, and 3 yd. high. What is its volume?

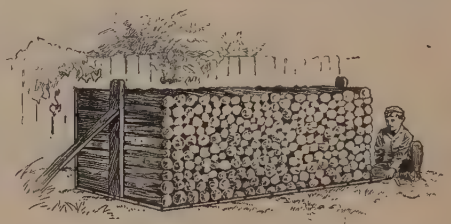
2. A piece of timber is 60 ft. long and 18 in. square. How many cubic feet does it contain?

3. A village constructs a reservoir for a water supply. The length is 100 yd., the width 70 yd., and the depth 15 ft. What will be the cost, at 23¢ per cubic yard, of excavating the reservoir?

Wood

271. Wood is measured by the cord.

272. A **cord** of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It contains 128 cu. ft.



The word "cord," as practically used in wood measure, generally means a pile 8 ft. long and 4 ft. high, the price depending on the length of the stick.

273. Example. How many cords of wood in a pile 32 ft. long, 8 ft. wide, and 4 ft. high?

SOLUTION. $\frac{4 \times 32 \times 8}{128} = 8$; that is, there are 8 cd. in the pile.

WRITTEN EXERCISE

1. How many cords in a pile of wood 60 ft. long, 4 ft. wide, and 6 ft. high?

2. A pile of wood contains 5 cd. If it is 4 ft. wide and 4 ft. high, how long is it?

3. A pile of tan bark contains 150 cd. If it is 4 ft. wide and 8 ft. high, how long is it?

4. A pile of wood contains 8 cd. It is 64 ft. long and as high as it is wide. What is the height of the pile?

Lumber

274. A **foot of lumber**, sometimes called a **board foot**, is a board 1 ft. long, 12 in. wide, and 1 in. thick, or its equivalent. An exception to this is made in the measurement of boards *less than* 1 *in.* in thickness. A square foot of the surface of such boards is regarded as a foot of lumber regardless of the thickness. Boards more than one inch in thickness, planks, joists, beams, scantling, and sawed timber are generally measured by the board foot.

Thus, a board 12 ft. long, 12 in. wide, and 1 in. thick contains 12 *sq. ft. of surface*, or 12 *board feet;* a board 12 ft. long, 12 in. wide, and $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{7}{8}$ in. thick contains 12 *sq. ft. of surface*, or 12 *board feet;* but a board 12 ft. long, 12 in. wide, and $2\frac{1}{2}$ in. thick contains 30 *board feet.*

Scantling is timber $3\frac{7}{8}$ in. wide and from 2 in. to 4 in. thick; *joists* are narrow and deep sticks of lumber; *planks* are thick boards; lumber heavier than joists or scantling is usually called *timber.*

Except when sawed to order and in cherry, black walnut, etc., where the price is 15¢ a board foot and upward, the width of a board is reckoned only the **next smaller half inch.** Thus, a board $10\frac{1}{4}$ in. wide is reckoned as 10 in., and a board $10\frac{5}{8}$ in. wide is reckoned as $10\frac{1}{2}$ in.

The **average width** is used in measuring boards that taper uniformly. Thus, a tapering board 12 ft. long, 8 in. wide, at one end and 6 in. wide at the other and 1 in. thick averages 7 in. wide and contains 7 ft. of lumber.

ORAL EXERCISE

1. How many square feet in the surface of a board 12 ft. long, 8 in. wide, and 1 in. thick? How many board feet?

2. How many board feet in a board 12 ft. long, 4 in. wide, and $\frac{7}{8}$ in. thick?

3. How many feet, board measure, in a board 12 ft. long, 12 in. wide, and 2 in. thick?

4. How many feet of lumber in 65 boards each 12 ft. long, 6 in. wide, and 1 in. thick?

275. In **charging** or **billing lumber** the number of pieces is entered first; then the thickness and width in inches and the length in feet; and finally, the article.

Thus, in billing 12 pc. hemlock, 2 in. thick, 6 in. wide, 12 ft. long, the form would be: 12 pc. $2'' \times 6''$, $12'$, hemlock.

ORAL EXERCISE

1. How many board feet in 6 planks, $1\frac{1}{2}'' \times 12''$, $14'$?

SUGGESTION. By inspection eliminate 12 in the dividend.
Then, $1\frac{1}{2} \times 6 \times 14 = 126$, the required number of board feet.

2. How many feet, board measure, in 6 planks $2'' \times 8''$, $18'$?

SUGGESTION. By inspection cancel a 12 in the dividend (6×2).
Then, $8 \times 18 = 144$, the required number of feet, board measure.

3. How many feet of lumber in 6 pc. of scantling $4'' \times 4''$, $16'$?

SUGGESTION. Mentally picture the problem arranged in form for cancellation $\left(\frac{6 \times 4 \times 4 \times 16}{12}\right)$. Cancel a 12 in the dividend ($\frac{1}{12}$ of $\overline{6 \times 4}$). Then, $2 \times 4 \times 16$, or 128, equals the required number of feet of lumber.

4. How many feet of lumber in 5 sticks, $2'' \times 6''$, $16'$?

SUGGESTION. Mentally picture the problem in form for cancellation $\left(\frac{5 \times 2 \times 6 \times 16}{12}\right)$. Cancel a 12 in the dividend ($\frac{1}{12}$ of $\overline{2 \times 6}$). Then, 5×16, or 80, equals the required number of feet of lumber.

5. How many feet of lumber in a plank $3'' \times 12''$, $16'$? in 6 planks? in 10 planks? How many feet of lumber in a plank $2'' \times 6''$, $12'$? in 5 planks? in 20 planks?

276. Obviously, the number of board feet in lumber 1 in. or less in thickness is $\frac{1}{12}$ *of the product of the length in feet by the width in inches;* and the number of board feet in lumber more than 1 in. in thickness is $\frac{1}{12}$ *of the product of the length in feet by the width and thickness in inches.* But the work may be materially shortened by mentally cancelling 12 from the dividend as illustrated in the foregoing exercise.

ORAL EXERCISE

State the number of feet, board measure, in the following hemlock:

1.	5 pc., $3'' \times 4''$, $14'$.	**13.**	12 pc., $2'' \times 8''$, $18'$.
2.	6 pc., $2'' \times 4''$, $20'$.	**14.**	6 pc., $8'' \times 10''$, $20'$.
3.	6 pc., $2'' \times 6''$, $20'$.	**15.**	30 pc., $2'' \times 6''$, $20'$.
4.	20 pc., $2'' \times 6''$, $14'$.	**16.**	6 pc., $8'' \times 10''$, $21'$.
5.	12 pc., $2'' \times 8''$, $14'$.	**17.**	25 pc., $3'' \times 8''$, $14'$.
6.	25 pc., $3'' \times 4''$, $12'$.	**18.**	10 pc., $2'' \times 6''$, $13'$.
7.	25 pc., $2'' \times 6''$, $20'$.	**19.**	15 pc., $2'' \times 6''$, $18'$.
8.	25 pc., $3'' \times 8''$, $16'$.	**20.**	15 pc., $2'' \times 6''$, $12'$.
9.	10 pc., $3'' \times 4''$, $14'$.	**21.**	16 pc., $2'' \times 6''$, $10'$.
10.	10 pc., $2'' \times 8''$, $18'$.	**22.**	10 pc., $8'' \times 10''$, $15'$.
11.	14 pc., $2'' \times 6''$, $20'$.	**23.**	15 pc., $8'' \times 10''$, $12'$.
12.	10 pc., $3'' \times 6''$, $20'$.	**24.**	200 pc., $2'' \times 6''$, $20'$.

WRITTEN EXERCISE

How many feet, board measure, in each of the following?

1. 100 joists, $4'' \times 4''$, 16′.
2. 65 boards, $\frac{3}{4}'' \times 6''$, 12′.
3. 12 timbers, $8'' \times 8''$, 40′.
4. 70 joists, $2'' \times 10''$, 32′.
5. 8 beams, $10'' \times 10''$, 24′.
6. 10 beams, $12'' \times 12''$, 30′.

7. At \$19 per M, find the total cost of:

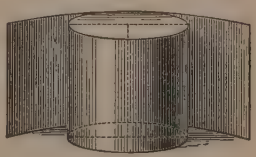
6 joists, $2'' \times 8''$, 12′.
12 joists, $2'' \times 8''$, 13′.
30 joists, $2'' \times 8''$, 15′.
5 joists, $2'' \times 8''$, 18′.
17 joists, $2'' \times 6''$, 16′.
30 joists, $2'' \times 8''$, 16′.

8. At \$26 per M, find the total cost of:

7 beams, $9'' \times 9''$, 20′.
24 joists, $2'' \times 10''$, 18′.
150 boards, $\frac{7}{8}'' \times 5''$, 12′.
16 beams, $9'' \times 9''$, 18′.
75 planks, $2\frac{1}{2}'' \times 8''$, 12′.
576 boards, $1'' \times 9''$, 16′.

CYLINDERS

277. A **cylinder** is a solid bounded by a uniformly curved surface and two equal parallel circles.



Two circles are parallel when all the points of one are equally distant from all the points of the other. The curved surface of a cylinder is called its *lateral surface;* the parallel circles its *bases.*

278. If the lateral surface of a cylinder be exactly covered with paper, it will be found that the paper is in the form of a rectangle whose length and width are equal to the circumference and height, respectively, of the cylinder. Hence,

The product of the circumference and height of a cylinder equals the area of its lateral surface.

ORAL EXERCISE

1. If the accompanying diagram is a solid 4 ft. square and 12 ft. high, what is the area of its six sides?

2. Give a brief rule for finding the entire surface (lateral surface and bases) of a rectangular solid; of a cylinder.

3. How many cubic inches in a block 2 in. square and 1 in. high? in a block 2 in. square and 10 in. high?

279. In the foregoing exercise it is clear that *the area of the base multiplied by the height of the cylinder equals the volume.*

WRITTEN EXERCISE

1. What will be the cost, at 40¢ per cubic yard, of excavating for a cistern 10 ft. in diameter and 23 ft. deep?

2. A man dug a well 6 ft. in diameter and 38 ft. deep. How much should he receive if he was paid $1 for each cubic yard of earth removed?

3. What will be the cost, at $12\frac{1}{2}$¢ per square foot, of a sheet-iron smokestack $2\frac{1}{2}$ ft. in diameter and 30 ft. high?

Cisterns

280. A gallon equals 231 cu. in.

ORAL EXERCISE

1. How many gallons in 462 cu. in.? in 1386 cu. in.?

2. How many gallons of water in a vat 22 in. long, 7 in. high, and 3 in. wide?

3. Give a rule for finding the exact number of gallons in a vessel. How many gallons in a cubic foot?

Solution. 231 cu. in. = 1 gal. 1728 cu. in. = 1 cu. ft. Therefore, 1 cu. ft. $= \frac{1728}{231}$ gal. = 7.48 + gal., or approximately $7\frac{1}{2}$ gal.

4. Find the approximate capacity, in gallons, of a vat 5 ft. square and 4 ft. high.

Solution. 5 ft. × 5 ft. × 4 ft. = 100 cu. ft. 100 times $7\frac{1}{2}$ gal. = 750 gal.

5. State a rule for finding the approximate capacity, in gallons, of a vessel.

WRITTEN EXERCISE

Find the capacity (approximate and exact), in gallons, of:

1. A cistern 6 ft. square and 12 ft. deep.

2. A cistern 6 ft. in diameter and 10 ft. deep.

3. A tank 5 ft. long, 4 ft. wide, and 6 ft. deep.

4. A cistern 15 ft. in diameter and 20 ft. deep.

Stone Work

281. Stone work is usually measured by the **perch**, which is a mass of stone $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, containing $24\frac{3}{4}$ cu. ft.

In some localities the perch contains $16\frac{1}{2}$ cu. ft.

282. Masonry is measured by the *cubic yard* or the *perch*.

In measuring stone work, such as the walls of cellars and buildings, masons take the distance around the outside of the wall (the girt) for the length. In this way the corners are measured twice, but this is considered offset by the extra work required in building the corners.

The work around openings, such as doors and windows, is also more difficult than the straight work and on this account no allowance is usually made for openings, unless they are very large.

WRITTEN EXERCISE

1. How many perches of stone will be required for an 18-in. foundation $72' \times 40' \times 10'$?

2. How many perches of masonry in the 18-in. walls of a cellar $40' \times 30' \times 8'$?

3. How many cubic yards of masonry in the foundation walls of a house $42' \times 32'$ if the walls are $2\frac{1}{2}$ ft. wide and 8 ft. high? (Solve (*a*) by mason's and (*b*) by actual measure.)

Brick Work

283. A common brick is 8 in. long, 4 in. wide, and 2 in. thick.

Bricks vary in size, but the common brick may be taken as a unit for measuring brick work. Contractors and builders do not follow any uniform rule for estimating the number of bricks required for a wall. It is sufficiently accurate, however, to reckon 22 common bricks, laid in mortar, for each cubic foot of wall. **In estimating material** for a brick wall actual measurements are taken and an allowance made for doors and windows and other openings. **In estimating labor** girt measurements are taken and usually a stated allowance made for openings such as doors and windows. The allowance to be made for openings is generally covered by contract. In some localities a uniform number of cubic feet is deducted for each opening; in others one half the volume of all openings is deducted; in still others nothing whatever is deducted.

WRITTEN EXERCISE

1. How many common bricks will be required for a wall 84 ft. long, $16\frac{1}{2}$ ft. high, and $1\frac{1}{2}$ ft. thick?

2. Find the cost of the bricks required to build a wall 300 ft. long, 12 ft. high, and 18 in. thick, at \$6 per thousand.

3. How many bricks will be required for the four walls of a building $80' \times 50' \times 25'$ if the walls are 18 in. thick and 500 cu. ft. is allowed for openings? (Solve (*a*) by mason's measure, making allowance for the openings, and (*b*) by actual measure.)

CAPACITY

Bins

284. The **stricken bushel** is used in measuring grain. The **heaped bushel** is used in measuring such things as large fruits, vegetables, coal, and corn on the cob. A stricken bushel equals 2150.42 cu. in. A heaped bushel equals 2747.71 cu. in.

ORAL EXERCISE

1. How many bushels of wheat in 2,150,420 cu. in.?

2. State a rule for finding the exact number of stricken bushels in a bin. What part of a stricken bushel is 1 cu. ft.?

Solution. 2150.42 cu. in. = 1 bu., stricken measure. 1728 cu. in. = 1 cu. ft. Therefore, 1 cu. ft. = 172800 ÷ 215042, or approximately .8 of a bushel, stricken measure.

$$\begin{array}{r} .8+ \\ 2150.42\overline{)1728.000} \\ 1720\ 336 \\ \hline 7664 \end{array}$$

3. Find the approximate capacity, in stricken bushels, of a cubical bin the inside of which measures 10 ft. on a side; in cubic inches of 800 bu. of wheat.

4. State a brief rule for finding the approximate number of stricken bushels in a bin; the approximate number of cubic feet in any number of stricken bushels.

5. How many bushels of potatoes in a bin containing 2,747,710 cu. in.? State a rule for finding the exact number of heaped bushels in any number of cubic inches. Reduce a cubic foot to a decimal of a heaped bushel.

SOLUTION. 2747.71 cu. in. = 1 bu., heaped measure. Therefore 1 cu. ft. = 172800 ÷ 274771, or approximately .63 of a bushel, heaped measure.

$$
\begin{array}{r}
.63 - \\
2747.71\overline{)1728.0000} \\
\underline{1648\ 626} \\
79\ 3740 \\
\underline{82\ 4313}
\end{array}
$$

6. Find the approximate capacity, in heaped bushels, of 1000 cu. ft.; in cubic feet, of 630 bu.

7. State a short method of reducing cubic feet to heaped bushels; heaped bushels to cubic feet.

8. Find (*a*) the exact capacity and (*b*) the approximate capacity, in stricken bushels, of a bin $10' \times 5' \times 4'$.

SOLUTIONS

(*a*) $10' \times 5' \times 4' = 200$ cu. ft.
200 × 1728 cu. in. = 345,600 cu. in.
345,600 cu. in. ÷ 2150.42 = 165.31 + bu.

(*b*) $10' \times 5' \times 4' = 200$ cu. ft.
.8 of 200 cu. ft. = 160 bu.

ORAL EXERCISE

1. Find the approximate capacity in bushels of a wheat bin 10 ft. long, 8 ft. wide, and 5 ft. high.

2. A square bin 10 ft. high contains, by approximate measurements, 800 bu. What is its width?

3. Approximately, how many bushels of potatoes may be stored in a bin 10 ft. long, 5 ft. wide, and 4 ft. high?

WRITTEN EXERCISE

Find the approximate capacity in stricken bushels of:

1. A bin 12 ft. square and 4 ft. deep.

Inside dimensions are given in all the problems of this and similar exercises.

2. A box 6 ft. long, $2\frac{1}{2}$ ft. wide, and $3\frac{1}{2}$ ft. deep.

3. A farmer wishes to construct a square granary 15 ft. on each side that will hold 800 bu. of grain. How deep must the bin be made? (Approximate rule.)

4-5. Find the exact capacity, in stricken bushels, of problems 1–2.

6-7. Find the approximate capacity, in heaped bushels, of problems 1–2.

CALCULATION TABLES

285. Persons who have a great deal of computing to do frequently use machines (see Appendix A) and calculation tables to aid them in their work. The table on page 229 will give a good idea of the arrangement of calculation tables that are used in making up and proving bills and invoices, computing wages, finding percentages, etc. The following examples will illustrate a few of the many uses of such tables.

286. Examples. **1.** Multiply 58 by 42.

SOLUTION. Under 58 and opposite 42 find 2436.

2. How many square feet in a floor $38' \times 46'$?

SOLUTION. Under 46 and opposite 38 find 1748; that is, 1748 sq. ft.

3. Find the cost of 495 yd. wash silk at 39¢.

SOLUTION. Under 495 and opposite 39 find 19,305; that is, \$193.05.

4. Find the cost of 48,000 bricks at \$4.95 per M.

SOLUTION. Under 495 and opposite 48 find 23,760. Since the zeros in 48,000 have been rejected, there are but two places to point off. Result \$237.60.

5. Find the cost of 46 hr. of labor at $25\frac{3}{4}$¢ per hour.

SOLUTION. Under 46 and opposite 25 find 1150 (\$11.50); under 46 and opposite $\frac{3}{4}$ find 34.50 (35¢). \$11.50 + 35¢ = \$11.85, the required result.

ORAL EXERCISE

By the aid of the table state the product of:

1. 27×26.	**5.** 39×27.	**9.** 87×46¢.	**13.** 35×93¢.
2. 27×58.	**6.** 45×58.	**10.** 93×32¢.	**14.** 93×42¢.
3. 45×46.	**7.** 37×46.	**11** 48×93¢.	**15.** 46×87¢.
4. 47×39.	**8.** 49×58.	**12.** 47×87¢.	**16.** 38×93¢.

17. Find the cost of 49,500 lb. of old rags at $\frac{3}{4}$¢.

18. Find the cost of 93,000 bricks at \$5.25 per M.

19. Find the cost of 37 days' labor at \$1.35 per day; at \$5.25.

20. Find the cost of 109 hours' labor at 27¢; at 39¢; at 46¢.

21. Find the cost of 49,500 lb. freight at 31¢ per hundredweight; of 46,000 lb. at 27¢ per hundredweight.

CALCULATION TABLE

Multiplier	27	39	46	58	Multiplier	87	93	109	128	Multiplier	135	147	495	525	Multiplier
1	27	39	46	58	1	87	93	109	128	1	135	147	495	525	1
2	54	78	92	116	2	174	186	218	256	2	270	294	990	1050	2
3	81	117	138	174	3	261	279	327	384	3	405	441	1485	1575	3
4	108	156	184	232	4	348	372	436	512	4	540	588	1980	2100	4
5	135	195	230	290	5	435	465	545	640	5	675	735	2475	2625	5
6	162	234	276	348	6	522	558	654	768	6	810	882	2970	3150	6
7	189	273	322	406	7	609	651	763	896	7	945	1029	3465	3675	7
8	216	312	368	464	8	696	744	872	1024	8	1080	1176	3960	4200	8
9	243	351	414	522	9	783	837	981	1152	9	1215	1323	4455	4725	9
10	270	390	460	580	10	870	930	1090	1280	10	1350	1470	4950	5250	10
11	297	429	506	638	11	957	1023	1199	1408	11	1485	1617	5445	5775	11
12	324	468	552	696	12	1044	1116	1308	1536	12	1620	1764	5940	6300	12
13	351	507	598	754	13	1131	1209	1417	1664	13	1755	1911	6435	6825	13
14	378	546	644	812	14	1218	1302	1526	1792	14	1890	2058	6930	7350	14
15	405	585	690	870	15	1305	1395	1635	1920	15	2025	2205	7425	7875	15
16	432	624	736	928	16	1392	1488	1744	2048	16	2160	2352	7920	8400	16
17	459	663	782	986	17	1479	1581	1853	2176	17	2295	2499	8415	8925	17
18	486	702	828	1044	18	1566	1674	1962	2304	18	2430	2646	8910	9450	18
19	513	741	874	1102	19	1653	1767	2071	2432	19	2565	2793	9405	9975	19
20	540	780	920	1160	20	1740	1860	2180	2560	20	2700	2940	9900	10500	20
21	567	819	966	1218	21	1827	1953	2289	2688	21	2835	3087	10395	11025	21
22	594	858	1012	1276	22	1914	2046	2398	2816	22	2970	3234	10890	11550	22
23	621	897	1058	1334	23	2001	2139	2507	2944	23	3105	3381	11385	12075	23
24	648	936	1104	1392	24	2088	2232	2616	3072	24	3240	3528	11880	12600	24
25	675	975	1150	1450	25	2175	2325	2725	3200	25	3375	3675	12375	13125	25
26	702	1014	1196	1508	26	2262	2418	2834	3328	26	3510	3822	12870	13650	26
27	729	1053	1242	1566	27	2349	2511	2943	3456	27	3645	3969	13365	14175	27
28	756	1092	1288	1624	28	2436	2604	3052	3584	28	3780	4116	13860	14700	28
29	783	1131	1334	1682	29	2523	2697	3161	3712	29	3915	4263	14355	15225	29
30	810	1170	1380	1740	30	2610	2790	3270	3840	30	4050	4410	14850	15750	30
31	837	1209	1426	1798	31	2697	2883	3379	3968	31	4185	4557	15345	16275	31
32	864	1248	1472	1856	32	2784	2976	3488	4096	32	4320	4704	15840	16800	32
33	891	1287	1518	1914	33	2871	3069	3597	4224	33	4455	4851	16335	17325	33
34	918	1326	1564	1972	34	2958	3162	3706	4352	34	4590	4998	16830	17850	34
35	945	1365	1610	2030	35	3045	3255	3815	4480	35	4725	5145	17325	18375	35
36	972	1404	1656	2088	36	3132	3348	3924	4608	36	4860	5292	17820	18900	36
37	999	1443	1702	2146	37	3219	3441	4033	4736	37	4995	5439	18315	19425	37
38	1026	1482	1748	2204	38	3306	3534	4142	4864	38	5130	5586	18810	19950	38
39	1053	1521	1794	2262	39	3393	3627	4251	4992	39	5265	5733	19305	20475	39
40	1080	1560	1840	2320	40	3480	3720	4360	5120	40	5400	5880	19800	21000	40
41	1107	1599	1886	2378	41	3567	3813	4469	5248	41	5535	6027	20295	21525	41
42	1134	1638	1932	2436	42	3654	3906	4578	5376	42	5670	6174	20790	22050	42
43	1161	1677	1978	2494	43	3741	3999	4687	5504	43	5805	6321	21285	22575	43
44	1188	1716	2024	2552	44	3828	4092	4796	5632	44	5940	6468	21780	23100	44
45	1215	1755	2070	2610	45	3915	4185	4905	5760	45	6075	6615	22275	23625	45
46	1242	1794	2116	2668	46	4002	4278	5014	5888	46	6210	6762	22770	24150	46
47	1269	1833	2162	2726	47	4089	4371	5123	6016	47	6345	6909	23265	24675	47
48	1296	1872	2208	2784	48	4176	4464	5232	6144	48	6480	7056	23760	25200	48
49	1323	1911	2254	2842	49	4263	4557	5341	6272	49	6615	7203	24255	25725	49
50	1350	1950	2300	2900	50	4350	4650	5450	6400	50	6750	7350	24750	26250	50

Multiplier	27	39	46	58	Multiplier	87	93	109	128	Multiplier	135	147	495	525	Multiplier
1/8	3 38	4 88	5 75	7 25	1/8	10 88	11 63	13 63	16 00	1/8	16 88	18 38	61 88	65 63	1/8
1/4	6 75	9 75	11 50	14 50	1/4	21 75	23 25	27 25	32 00	1/4	33 75	36 75	123 75	131 25	1/4
3/8	10 13	14 63	17 25	21 75	3/8	32 63	34 88	40 88	48 00	3/8	50 63	55 13	185 63	196 88	3/8
1/2	13 50	19 50	23 00	29 00	1/2	43 50	46 50	54 50	64 00	1/2	67 50	73 50	247 50	262 50	1/2
5/8	16 88	24 38	28 75	36 25	5/8	54 38	58 13	68 13	80 00	5/8	84 38	91 88	309 38	328 13	5/8
3/4	20 25	29 25	34 50	43 50	3/4	65 25	69 75	81 75	96 00	3/4	101 25	110 25	371 25	393 75	3/4
7/8	23 63	34 13	40 25	50 75	7/8	76 13	81 38	95 38	112 00	7/8	118 13	128 63	433 13	459 38	7/8

22. Find the cost of 48,000 ft. of lumber at \$16 per M; of 93,000 ft.; of 52,500 ft.; of 49,500 ft.; of 58,000 ft.

23. An agent sold 240 (10×24) excursion tickets at \$4.95. How much did he receive? $360 \times \$5.25 = ?$ $310 \times \$1.47 = ?$

24. Find the cost of 45 rm. of paper at \$1.35; at \$1.28; at \$1.09; at 93¢; at \$4.95. Also find the cost of 38 rm. at each of the above prices; of 29 rm.; of 37 rm.; of 46 rm.

25. Find the cost of 4600 lb. of coal at \$6.40 per ton (\$3.20 per thousand pounds); at \$8.40; at \$4.60; at \$6.80; at \$7.20; at \$7.40; at \$9.20; at \$5.60. Also find the cost of 2700 lb. at each of the above prices; of 3900 lb.; of 8700 lb.; of 9300 lb.; of 10,900 lb.; of 12,800 lb.; of 13,500 lb.; of 14,700 lb.; of 49,500 lb.; of 52,500 lb.

26. By the aid of the table find the total cost of:

525 bolts at \$1.70 per C. 128 bolts at \$1.90 per C.
495 bolts at \$2.40 per C. 525 bolts at \$2.70 per C.
135 bolts at \$1.60 per C. 495 bolts at \$3.50 per C.

27. By the aid of the table find the total cost of:

1280 ft. lumber at \$28 per M. 5250 ft. lumber at \$27 per M.
1350 ft. lumber at \$29 per M. 3800 ft. lumber at \$27 per M.
4950 ft. lumber at \$19 per M. 4600 ft. lumber at \$18 per M.

A REVIEW EXERCISE

1. Without copying, extend and find the total of each invoice on pages 189 and 190. Time for each invoice, approximately. 3 min.

2. How many acres in each of the following fields:

a. A field 60 rd. long and 40 rd. wide.

Suggestion. As the field is 40 rd. wide, each 4 rd. of length makes an acre; hence, there are as many acres as 4 is contained times in 60. Ans. 15 A.

b. A field 50 rd. long and 32 rd. wide.
c. A field 80 rd. long and 16 rd. wide.
d. A field 96 rd. long and 20 rd. wide.
e. A field 75 rd. long and $53\frac{1}{3}$ rd. wide.
f. A field 80 rd. long and 80 rd. wide.
g. A field 120 rd. long and 40 rd. wide.

PERCENTAGE AND ITS APPLICATIONS

CHAPTER XVII

PERCENTAGE

ORAL EXERCISE

1. .50 may be read *fifty hundredths*, *one half*, or *fifty per cent.* Read each of the following in three ways: .25, .30, $12\frac{1}{2}\%$.

2. Read each of the following in three ways: $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{25}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{4}$, 2 %, $2\frac{1}{2}\%$, 125 %, $6\frac{1}{4}\%$, $8\frac{1}{3}\%$, $66\frac{2}{3}\%$, 250 %, 375 %.

3. 50 % of a number is .50 or $\frac{1}{2}$ of the number. What is 50 % of \$600? 25 %? $12\frac{1}{2}\%$? 10 %? 40 %? 20 %? 75 %?

287. **Per cent** is a common name for hundredths.

288. The **symbol** % may be read *hundredths* or *per cent.*

289. **Percentage** is the process of computing by *hundredths* or *per cents.*

ORAL EXERCISE

Express as per cents:

1. .28.	3. $.00\frac{1}{2}$.	5. $.33\frac{1}{3}$.	7. $.62\frac{1}{2}$.	9. .5.
2. .37.	4. $.14\frac{2}{7}$.	6. $.28\frac{4}{7}$.	8. .0075.	10. .2.

Express as decimal fractions:

11. 20 %.	13. 72 %.	15. $\frac{1}{2}\%$.	17. 125 %.	19. $\frac{1}{50}\%$.
12. 45 %.	14. 18 %.	16. $\frac{1}{4}\%$.	18. 250 %.	20. 375 %.

Express as common fractions:

21. 1 %.	23. $2\frac{1}{2}\%$.	25. $133\frac{1}{3}\%$.	27. $87\frac{1}{2}\%$.	29. $\frac{1}{2}\%$.
22. 2 %.	24. $3\frac{1}{2}\%$.	26. $266\frac{2}{3}\%$.	28. $112\frac{1}{2}\%$.	30. 175 %.

Express as per cents:

31. $\frac{1}{5}$.	33. $\frac{1}{11}$.	35. $1\frac{1}{4}$.	37. $\frac{6}{5}$.	39. $\frac{5}{6}$.
32. $\frac{1}{9}$.	34. $\frac{9}{10}$.	36. $2\frac{3}{4}$.	38. $1\frac{7}{8}$.	40. $\frac{24}{6}$.

IMPORTANT PER CENTS AND THEIR FRACTIONAL EQUIVALENTS

PER CENT	FRACTIONAL VALUE	PER CENT	FRACTIONAL VALUE	PER CENT	FRACTIONAL VALUE	PER CENT	FRACTIONAL VALUE
$12\frac{1}{2}\%$	$\frac{1}{8}$	75 %	$\frac{3}{4}$	$83\frac{1}{3}\%$	$\frac{5}{6}$	$6\frac{1}{4}\%$	$\frac{1}{16}$
25 %	$\frac{1}{4}$	100 %	1	20 %	$\frac{1}{5}$	$6\frac{2}{3}\%$	$\frac{1}{15}$
$37\frac{1}{2}\%$	$\frac{3}{8}$	$16\frac{2}{3}\%$	$\frac{1}{6}$	40 %	$\frac{2}{5}$	$8\frac{1}{3}\%$	$\frac{1}{12}$
50 %	$\frac{1}{2}$	$33\frac{1}{3}\%$	$\frac{1}{3}$	60 %	$\frac{3}{5}$	$11\frac{1}{9}\%$	$\frac{1}{9}$
$62\frac{1}{2}\%$	$\frac{5}{8}$	$66\frac{2}{3}\%$	$\frac{2}{3}$	80 %	$\frac{4}{5}$	$14\frac{2}{7}\%$	$\frac{1}{7}$

290. The **terms** used in percentage are the base, the rate, and the percentage. The **base** is the number of which a per cent is taken; the **rate**, the number of hundredths of the base to be taken; the **percentage**, the result obtained by taking a certain per cent of the base.

In the expression "12 % of \$50 is \$6," \$50 is the base, 12 %, the rate, and \$6, the percentage.

291. The base plus the percentage is sometimes called the **amount**; the base minus the percentage, the **difference**.

FINDING THE PERCENTAGE

292. Example. What is 15 % of \$660?

SOLUTION. 15 % of a number equals .15 of it. .15 of \$660 = \$99, the required result.

\$660
.15
\$99.00

293. Obviously, *the product of the base and rate equals the percentage.*

The *base* may be either *concrete* or *abstract*. The *rate* is always *abstract*. The *percentage* is always of the *same name as the base.*

ORAL EXERCISE

1. What aliquot part of 1 is $.12\frac{1}{2}$? .25? .50? $.16\frac{2}{3}$? $.33\frac{1}{3}$? .20? $.06\frac{1}{4}$? $.06\frac{2}{3}$? $.08\frac{1}{3}$? $.11\frac{1}{9}$? $.14\frac{2}{7}$? $37\frac{1}{2}\%$? $62\frac{1}{2}\%$? $66\frac{2}{3}\%$?

2. Formulate a short method for finding $12\frac{1}{2}\%$ of a number.

SOLUTION. $12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$; hence, to find $12\frac{1}{2}\%$ of a number, *divide by 8.*

3. State a short method for finding 25 % of a number; 50 %; $16\frac{2}{3}\%$; $33\frac{1}{3}\%$; 20 %; $6\frac{1}{4}\%$; $6\frac{2}{3}\%$; $8\frac{1}{3}\%$; $11\frac{1}{9}\%$.

To guard against absurd answers in exercises of this character estimate the results in advance as explained on pages 58 and 146.

4. Find 50 % of 960. Also 25 %; $37\frac{1}{2}$ %; $12\frac{1}{2}$ %; $62\frac{1}{2}$ %; 75 %; $16\frac{2}{3}$ %; $33\frac{1}{3}$ %; $66\frac{2}{3}$ %; $83\frac{1}{3}$ %; 20 %; 40 %; 60 %; $6\frac{1}{4}$ %.

5. By inspection find:

a. 50 % of \$792.
b. $37\frac{1}{2}$ % of \$320.
c. $12\frac{1}{2}$ % of \$880.
d. $16\frac{2}{3}$ % of \$669.
e. 25 % of \$1729.
f. $6\frac{2}{3}$ % of \$6600.
g. $6\frac{1}{4}$ % of 3296.
h. $83\frac{1}{3}$ % of 4560.
i. $66\frac{2}{3}$ % of 2460.
j. $33\frac{1}{3}$ % of 2793.
k. $8\frac{1}{3}$ % of 24,960.
l. 20 % of 12,535.

ORAL EXERCISE

1. Find 10 % of 720; of \$15.50; of 120 men; of \$127.50.
2. What aliquot part of 10 % is 5 %? $2\frac{1}{2}$ %? $1\frac{1}{4}$ %? $3\frac{1}{3}$ %? $1\frac{2}{3}$ %?
3. Formulate a short method for finding $1\frac{1}{4}$ % of a number.

SOLUTION. $1\frac{1}{4}$ % of a number is $\frac{1}{8}$ of 10 % of the number; hence, to find $1\frac{1}{4}$ % of a number, *point off one place to the left and divide by 8.*

4. State a short method for finding 5 % of a number; $2\frac{1}{2}$ %; $3\frac{1}{3}$ %; $1\frac{2}{3}$ %.

5. By inspection find:

a. 5 % of 720.
b. $2\frac{1}{2}$ % of 840.
c. $3\frac{1}{3}$ % of 1560.
d. $1\frac{1}{4}$ % of 1840.
e. $1\frac{2}{3}$ % of \$366.
f. $2\frac{1}{2}$ % of \$720.
g. $3\frac{1}{3}$ % of \$3900.
h. $1\frac{2}{3}$ % of 120 mi.
i. $1\frac{1}{4}$ % of 1632 A.

ORAL EXERCISE

1. Compare 24 % of \$25 with 25 % of \$24; 24 % of \$2500 with 25 % of \$2400. What is 32 % of \$25?

SOLUTION. 32 % of \$25 = 25 % of \$32 = $\frac{1}{4}$ of \$32 = \$8, the required result.

2. What is 125 % of \$880?

SOLUTION. 125 % = 1.25 = $\frac{1}{8}$ of 10; $\frac{1}{8}$ of \$8800 (10 times \$880) = \$1100.

3. Find 125 % of 400; of 640; of 3200; of 160; of 1280.
4. Formulate a short method for finding $166\frac{2}{3}$ % of a number; $333\frac{1}{3}$ % of a number; 250 % of a number.
5. Compare 88 % of 12,500 bu. with 125 % of 8800 bu.
6. Find 32 % of \$125; of \$1250; of \$12,500; of \$125,000.
7. Find 250 % of \$720; of \$3200; of \$28,800; of \$64,800.

ORAL EXERCISE

By inspection find:

1. 48 % of 250.
2. 32 % of 125.
3. 128 % of 250.
4. 16 % of 2500.
5. 180 % of 625.
6. 160 % of 875.
7. 240 % of 7500.
8. 125 % of $240.40.

WRITTEN EXERCISE

1. A farmer sold 640 bu. wheat, receiving $1.05 per bushel for 87½ % of it and 85¢ per bushel for the remainder. What was the total amount received?

2. A grocer compromised with his creditors, paying 60 % of the amount of his debts. If he owed A $756, B $1250, and C $3750, how much did each receive?

3. A merchant sold 360 bbl. apples for $1200. If he received $3.50 per barrel for 66⅔ % of the apples, what was the price received per barrel for the remainder?

4. A man bought a house for $12,864.75; he expended for improvements 33⅓ % of the first cost of the property, and then sold it for $20,000. Did he gain or lose, and how much?

5. A commission merchant bought 1200 bbl. apples and after holding them for 3 mo. found that his loss from decay was 10 %. If he sold the remainder at $3.75 per barrel, how much did he receive?

6. A merchant prepaid the following bills and received the per cents of discount named: 4 % on bill of $875.50; 6 % on bill of $378.45; 2 % on bill of $940.50; 3½ % on bill of $400. What was the net amount paid?

FINDING THE RATE

ORAL EXERCISE

1. 8 is what part of 40? what per cent of 40?
2. 90 is what per cent of 270? of 360? of 450?
3. 70 is what per cent of 560? of 630? of 700?
4. The base is 900 and the percentage 450; what is the rate?

294. Example. $35.50 is what per cent of $284?

SOLUTIONS. *a.* $35.50 is $\frac{3550}{28400}$ or $\frac{1}{8}$ of $284. $284 is 100% of itself; hence, $35.50, which is $\frac{1}{8}$ of $284, must be $\frac{1}{8}$ of 100%, or $12\frac{1}{2}$%. Or,

(*a*)

$$\frac{3550}{28400} = \frac{1}{8} = 12\frac{1}{2}\,\%$$

b. Since the product of the base and the rate is the percentage, the quotient obtained by dividing the percentage by the base is the rate.

(*b*)

$$284\overline{)35.50}\quad .125 = 12\tfrac{1}{2}\,\%$$

295. Obviously, *the percentage divided by the base equals the rate.*

ORAL EXERCISE

What per cent of:

1. 95 is 19?
2. 4.8 is 1.2?
3. $35 is $17\frac{1}{2}$?
4. 225 A. are 75 A.?
5. 34 bu. are 34 bu.?
6. 34 bu. are 68 bu.?
7. 1.6 is .008?
8. 1 yd. is 1 ft.?
9. 2 da. are 8 hr.?
10. 4 T. are 3000 lb.?
11. 1 yr. are 4 mo.?
12. 2 mi. are 80 rd.?

WRITTEN EXERCISE

1. A man bought a house for $7500 and sold it for $8700. What per cent did he gain?

2. In a certain city, school was in session 190 da. A lost 38 da. What per cent of the school year did he attend?

3. An agent sold a piece of property for $8462.50 and received $338.50 for his services. What per cent did he receive?

4. A commission agent sold 28,600 bu. of grain at 50¢ per bushel and received for his services $357.50. What per cent did he receive on the sales made?

5. Smith and Brown engaged in business, investing $18,000. Smith invested $10,440, and Brown the remainder. What per cent of the total capital did each invest?

6. An agent for a wholesale house earned $165.55 during the month of May. If the goods sold amounted to $1505, what per cent did he receive on the sales made?

FINDING THE BASE

ORAL EXERCISE

1. What is 5% of 240 bu.?

2. 12 bu. is 5% of how many bushels?

3. 160 is 8% of what number? 4%? 2%? 1%? $\frac{1}{2}$%? $\frac{1}{4}$%?

4. The multiplicand is 400 and the multiplier 10; what is the product? The product is 2000 and the multiplicand 100; what is the multiplier? The product is 4000 and the multiplier 20; what is the multiplicand?

5. In percentage what name is given to the product? to the multiplicand? to the multiplier? When the base and rate are given, how is the percentage found? When the percentage and base are given, how is the rate found? When the percentage and rate are given, how is the base found?

296. Example. 37.5 is 25% of what number?

SOLUTION. 25% or $\frac{1}{4}$ of the number = 37.5
∴ the number = 37.5 ÷ $\frac{1}{4}$ = 150.

297. Obviously, *the quotient of the percentage divided by the rate equals the base.*

WRITTEN EXERCISE

1. N invested 30% of the capital of a firm, H 35%, and W the remainder, $1400. What was the capital of the firm?

2. During the month of May the sales of a clothing merchant amounted to $4864.24, which was 8% of the total sales for the year. What were the total sales for the year?

3. B sold his city property and took a mortgage for $4375, which was 17$\frac{1}{2}$% of the value of the property. If the balance was paid in cash, what was the amount of cash received?

4. In compromising with his creditors, a man finds that his assets are $270,900, and that this sum is 43% of his entire indebtedness. What will be the aggregate loss to his creditors?

5. The aggregate attendance in the schools of a certain city for 1 da. was 43,225 students. If this number was 95% of the number of students belonging, how many students were absent?

PER CENTS OF INCREASE

ORAL EXERCISE

1. If $2\frac{1}{2}$ times a number is 50, what is the number?
2. If 2.5 times a number is 75, what is the number?
3. If 250% of a number is \$1250, what is the number?
4. If 250% of a number is 150, what is the number? If 250% is 125, what is the number?
5. If 300% of a number is \$5400, what is the number?

298. Examples. **1.** A man sold a farm for \$3900 and thereby gained 30%. How much did the farm cost?

SOLUTION. 1.30 of the cost = \$3900.
∴ the cost = \$3900 ÷ 1.30 = \$3000.

2. What number increased by $33\frac{1}{3}$% of itself equals 180?

SOLUTION. $\frac{4}{3}$ of the number = 180.
∴ the number = 180 ÷ $\frac{4}{3}$ = 135.

ORAL EXERCISE

What number increased by:

1. 10% of itself is 220?
2. 25% of itself is 125?
3. 50% of itself is 300?
4. 75% of itself is 700?
5. $6\frac{1}{4}$% of itself is 170?
6. $12\frac{1}{2}$% of itself is 180?
7. $66\frac{2}{3}$% of itself is 135?
8. $6\frac{2}{3}$% of itself is 480?
9. 125% of itself is 900?
10. $37\frac{1}{2}$% of itself is 440?
11. $11\frac{1}{9}$% of itself is 300?
12. $14\frac{2}{7}$% of itself is 328?
13. 200% of itself is 2700?
14. 300% of itself is 2800?

WRITTEN EXERCISE

1. I sold 375 bu. of wheat for \$427.50, thereby gaining 20%. How much did the wheat cost me per bushel?
2. A fruit dealer sold a quantity of oranges for \$6.75. If his gain was $12\frac{1}{2}$%, what did the oranges cost him?
3. My savings for March increased $33\frac{1}{3}$% over February. If my savings for March were \$84.36, what were my savings for February and March?

PER CENTS OF DECREASE

ORAL EXERCISE

1. What per cent of a number is left after taking away $33\frac{1}{3}\%$ of it? What fractional part?

2. If $\frac{2}{3}$ of a number is 600, what is the number? If $66\frac{2}{3}\%$ of a number is 75, what is the number?

3. A man spent 40 % of his money and had \$60 remaining. How much had he at first? How much did he spend?

299. Examples. 1. A man sold a horse for \$332, thereby losing 17 %. What was the cost?

SOLUTION. 0.83 of the cost = \$332.
∴ the cost = \$332 ÷ 0.83 = \$400.

2. What number decreased by 25 % of itself equals \$375?

SOLUTION. $\frac{3}{4}$ of the number = \$375.
∴ the number = \$375 ÷ $\frac{3}{4}$ = \$500.

ORAL EXERCISE

What number diminished by:

1. $6\frac{1}{4}\%$ of itself equals 75?
2. $8\frac{1}{3}\%$ of itself equals 440?
3. $6\frac{2}{3}\%$ of itself equals 280?
4. $\frac{1}{4}$ of itself equals 750?
5. $\frac{1}{2}\%$ of itself equals 99.5?
6. 1 % of itself equals 49.5?

WRITTEN EXERCISE

1. Of what number is 9581.88 77 %?

2. A merchant sold 1200 bu. of potatoes for \$640, which was $16\frac{2}{3}\%$ less than he paid for them. What was the cost per bushel?

3. In selling a carriage for \$75 a merchant lost 25 % on the cost. What was the asking price if the carriage was marked to gain 25 %?

4. A newsboy sold 92 papers on Tuesday. If this number was $23\frac{1}{3}\%$ less than the number sold on Monday, how many papers were sold on the two days?

5. A dealer sold a quantity of apples at \$6 per barrel, and by so doing lost $16\frac{2}{3}\%$. If he paid \$309.60 for the apples, how many barrels did he buy?

ORAL REVIEW EXERCISE

1. By inspection find $12\frac{1}{2}\%$ of the following numbers:

a. \$872.	*e.* \$2464.	*i.* \$1688.	*m.* \$24.72.
b. 648 bu.	*f.* 2696 A.	*j.* 2072 A.	*n.* \$168.48.
c. 1264 A.	*g.* 1624 ft.	*k.* 11,464 mi.	*o.* \$176.24.
d. 960 mi.	*h.* 1832 mi.	*l.* 37,128 mi.	*p.* \$2184.32.

2. By inspection find 10% of each of the above numbers; 25%; 125%; 20%.

3. State the missing term in each of the following:

No.	Base	Rate	Percentage	No.	Base	Rate	Percentage
a.	\$600	$7\frac{1}{2}\%$	?	*f.*	966	$16\frac{2}{3}\%$	?
b.	\$650	?	\$39	*g.*	?	$8\frac{1}{3}\%$	15 bu.
c.	?	4%	\$18	*h.*	1275	$6\frac{2}{3}\%$	?
d.	900	?	720	*i.*	?	$6\frac{1}{4}\%$	21 mi.
e.	?	4%	20	*j.*	400	?	600

4. By inspection find 10% of each of the following:

a. \$264.	*d.* \$840.	*g.* \$232.	*j.* \$2448.
b. \$920.	*e.* \$750.	*h.* \$144.	*k.* \$1432.
c. \$720.	*f.* \$364.	*i.* \$288.	*l.* \$3624.

5. By inspection find $1\frac{1}{4}\%$ of each of the above numbers; $1\frac{2}{3}\%$; 1000%; 125%; $166\frac{2}{3}\%$.

WRITTEN REVIEW EXERCISE

1. A collector charged 4% on all amounts collected. If he remitted to his customers in one month \$3720.48, how much did he receive for his services?

2. A father left to his son 60% of his estate and to his daughter the remainder, \$9390.88. What was the value of the estate and how much did the son receive?

3. A farmer planted 1 bu. 3 pk. of oats on an acre of ground and harvested 56 bu. What per cent of the yield was the planting? What per cent of the planting was the yield?

4. A merchant paid the following charges on a bill of goods: cartage \$12.45, freight \$65.32, insurance \$41. If the charges represent 5% of the face of the bill, what was the gross cost of the goods?

5. A man had 6 A. of land; to one party he sold a piece 25 rd. by 20 rd., and to another party 140 sq. rd. What per cent of the field remained unsold?

6. In a recent year 191,571,750 lb. of fish were landed in Boston and Gloucester, and of this quantity 103,460,410 lb. were landed in Gloucester and 88,111,340 in Boston. What per cent of the total was furnished by each city? (Correct to the nearest .01.)

7. A owned property valued at $12,000 from which he received a yearly rental of $960. If he paid taxes amounting to $160, insurance $75.50, and made repairs amounting to $184.50, what per cent net income did he receive?

8. B owns a field 80 rd. square. During a certain year this field yielded on an average 25 bu. of wheat to an acre. The wheat when sold at $1 a bushel produced an amount equal to 25% of the value of the field. What was the value of the field?

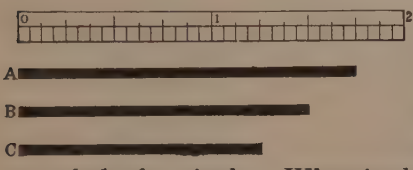
9. A landowner rented a field to a tenant and was to receive as rent $16\frac{2}{3}\%$ of the grain raised. The owner of the field sold his share of the grain for 84¢ per bushel, receiving $298.20. If the tenant sold his share of the grain for the same price per bushel, how much did he receive?

10. In a single year the cost of the cotton yarn used in the manufacture of hosiery and knit goods in the state of New York, in round numbers, was $13,825,000; in the state of Illinois, $1,550,000. The cost of the cotton yarn used in Illinois was what per cent less than the cost of the cotton yarn used in New York, in a year? (Correct to the nearest .01.)

11. By a recent census report it was shown that the value of all personal property in the state of New York was approximately $500,000,000 and the value of all the real estate approximately $10,000,000,000. Draw parallel lines making a comparison of the personal property and the real estate. The real estate is what per cent greater than the personal property? The personal property is what per cent less than the real estate?

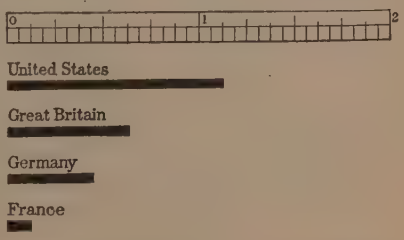
12. A young man entered a bank as cashier and at the end of the first year his salary was increased 25%; at the end of the second year he was given an increase of 20%; and at the end of the third year he was given an increase of 25%, which made his salary $4500. What salary did he receive at first?

13. A government statistician collected facts regarding wages and income from nearly two thousand private manufacturing concerns, and reported the following: the average wages of all employees, men, women, and children, per year was $263.06, and the average net profit for each employer was $2273. What per cent greater was the income of each employer than of each employee? (Correct to the nearest .01.)



14. The population of three cities during a certain year is illustrated by the accompanying lines, which are drawn on a scale of 12,500 inhabitants to each $\frac{1}{8}$ of an inch. What is the population of A, B, and C, respectively? The population of each city is what per cent of the population of the three cities?

15. The annual coal production in the United States, Great Britain, Germany, and France for a certain year is illustrated in the accompanying rectangles, drawn on the scale of 50,000,000 short tons to each $\frac{1}{8}$ of an inch. During that year, how many tons did the United States, Great Britain, Germany, and France, respectively, produce? The production of each country is what per cent of the production of the four countries? In the same year the rest of the world produced approximately 200,000,000 short tons. Illustrate graphically the world's coal production for that year. What was the world's approximate production this year?



16. The total value of the cotton crop to farmers in a recent year was $920,630,000 and the value of the cotton exported to Great Britain in the same year was $231,817,000. What per cent was exported to Great Britain? (Correct to the nearest .01.)

17. A saleswoman in a city store receives $9 per week. She pays $3.50 per week for board and room, 10¢ per day for car fare 6 da. of the week, 20¢ per day for 6 da. of each week for luncheon, and has incidental expenses amounting to $1.70. If she saves the remainder, what per cent of her weekly wages does she save? What per cent does she spend?

18. The production, in bushels, of grain in the United States in two recent years was approximately as follows:

Cereals	1912	1913
Corn	3,124,000,000	2,370,000,000
Wheat	730,000,000	753,000,000
Oats	1,418,000,000	1,122,000,000
Barley	223,000,000	175,000,000
Rye	35,000,000	34,000,000
Buckwheat	19,000,000	14,000,000

Find the per cent of increase or decrease of each cereal for 1913 as compared with the previous year. Then draw a series of parallel rectangles to compare the production of 1913 with the production of 1912. Also draw a series of rectangles to compare the production of 1913 with the production of a later year.

Suggestion. This may be represented by one series of rectangles. Each rectangle may be divided into two parts — one shaded and the other unshaded. The shaded part may be made to represent the yield for 1913 and the unshaded part the yield for 1912.

19. The silver produced by the leading sources in a recent year was approximately as follows:

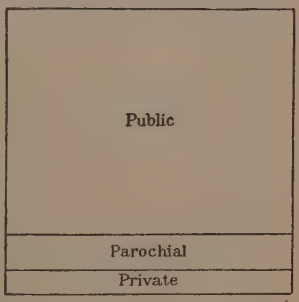
Mexico	80,000,000 oz.	Canada	32,000,000 oz.
United States	60,000,000 oz.	Peru	6,000,000 oz.
Turkey	1,500,000 oz.	Spain	4,000,000 oz.
Australia	16,000,000 oz.	Japan	4,500,000 oz.
Germany	6,000,000 oz.	Austria-Hungary	1,500,000 oz.

Draw a set of parallel rectangles to represent graphically the above numbers.

20. In the following table is shown the population in the United States in a certain year, men and women, engaged in gainful occupations, classified according to geographic divisions. Supply the missing terms. Check the work.

Geographic Divisions	Engaged in Gainful Occupations					
	Number			Per Cent of Total		
	Total	Men	Women	Total	Men	Women
North Atlantic . . .	8,274,869	6,539,941	1,734,928	30.3	29.1	35.9
South Atlantic . . .	3,553,985	2,781,825	772,160	?	?	?
North Central . . .	9,211,119	7,895,395	1,315,724	?	?	?
South Central . . .	4,610,924	3,792,422	818,502	?	?	?
Western	1,672,158	1,479,842	192,316	?	?	?
Totals	?	?	?	100.0	100.0	100.0

21. Suppose the accompanying diagram illustrates the distribution of school enrollment in the public, private, and parochial schools of the United States during a certain year. The private and parochial schools are what per cent of the public schools? of the entire school enrollment? The public schools are what per cent of the total enrollment? of the private and parochial schools combined?



22. The gold production in the eight principal gold-producing states in the United States in a recent year was approximately as follows: Colorado, 900,000 oz.; California, 960,000 oz.; Arizona, 180,000 oz.; Montana, 170,000 oz.; Nevada, 800,000 oz.; South Dakota, 350,000 oz.; Utah, 200,000 oz.; Idaho, 60,000 oz. Compare these values by drawing a series of parallel rectangles.

A REVIEW EXERCISE

Illustrate the following problems by the use of graphs. Graph forms are given on pages 144, 153, 243. Use the form suggested by the instructor.

1. Illustrate graphically problem 3, page 85. Use the even number of thousands for each month.

2. In a recent year the railway mileage, single-track, of the world was as follows: America, 325,000 mi.; Europe, 200,000 mi.; Asia, 63,000 mi.; Africa, 23,000 mi.; Australia, 19,000 mi. Illustrate graphically, showing the total mileage, and the relation that each country bears to the total.

3. In a recent year there were enrolled in the schools and colleges of the United States 20,000,000 students, grouped according to ages as follows: 5 yr., 400,000; 6 to 9 yr., 6,200,000; 10 to 14 yr., 9,000,000; 15 to 17 yr., 3,000,000; 18 to 20 yr., 1,000,000; 21 to 24 yr., 400,000. Illustrate graphically. Each group is what per cent of the total?

4. The number of cattle, other than milch cows, on farms and ranches in the United States, as reported by the decennial censuses, for the years named were as follows: 1870, 13,500,000; 1880, 22,500,000; 1890, 33,500,000; 1900, 50,000,000; 1910, 41,000,000. Illustrate graphically. What do these figures suggest regarding the cost of living as applied to beef?

5. The following figures represent the latest estimates of the wealth of the nations named. The figures given represent billions of dollars: United States, 130; Great Britain and Ireland, 80; France, 65; Germany, 60; Russia, 40; Austria-Hungary, 25; Italy, 20; Belgium, 9; Spain, $5\frac{1}{2}$; Netherlands, 5; Portugal, $2\frac{1}{2}$. Switzerland, $2\frac{1}{2}$. Illustrate graphically.

6. In a recent year the cities of the United States which had a population of over 100,000 expended $100,000,000 in various school expenses, according to the following geographical divisions: North Atlantic Division, $54,000,000; North Central Division, $30,000,000; South Atlantic Division, $2,700,000; South Central Division, $3,000,000; Western Division, $10,300,000. Illustrate graphically.

A WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

1. A gardener planted 1 qt. of corn and harvested 5 bu. What per cent of the planting was the harvest?

2. A bookkeeper made an investment on which he lost 15%. If the sum returned to him was $1912.50, what was the investment?

3. A piece of cloth, unfinished, cost 6¢ per yard. It costs .75¢ per yard to bleach it, and then it sells for $7\frac{3}{4}$¢ per yard. The selling price is what per cent advance over the total cost?

4. A merchant paid the following bills less the discounts named: $85.50 less 2%; $141.50 less 3%; $117.95 less 1%; $225.40 less $1\frac{1}{2}$%; $47.50 less $2\frac{1}{2}$%. What was the total sum paid? What was the total discount allowed?

5. On Monday a man deposited in the bank $184.96. On Wednesday he deposited a sum $12\frac{1}{2}$% greater than the deposit of Monday; he then drew a check for 50% of his total deposit. What was the amount of the check?

6. A merchant's sales increased the second month of his business 25% over the first month; the third month they increased 20% over the second month; the fourth month they decreased 10% from the sales of the third month. What were the sales for each month if they were $3240 for the fourth month?

7. A farmer used 1200 lb. of potato fertilizer per acre, on a 16-acre field of potatoes. The fertilizer cost $24.125 per ton, less 5% for cash payment. If the unfertilized land produced 60 bu. of potatoes per acre, and the fertilized land produced 150 bu. per acre, what per cent of increase was realized by using the fertilizer if the potatoes sold for 80¢ per bushel?

8. A man bought a piece of land, and at the end of the first year it had increased in value 25%; at the end of the second year it had increased an additional 8% in value; at the end of the third year it had increased an additional 5% in value. What did he pay for the property if at the end of the third year it was worth $2551.50?

CHAPTER XVIII

COMMERCIAL DISCOUNTS

ORAL EXERCISE

1. A set of Scott's works is marked \$12. If I buy it at this price, less $16\frac{2}{3}\%$, what does it cost me?

2. I buy \$90 worth of goods on 30 da. time, or 5% off for cash. What cash payment will settle the bill?

3. I owe B \$600, due in 30 da. He offers to allow me 5% discount if I pay cash to-day. I accept his offer and give him a check for the amount. What was the amount of the check?

300. A reduction from the catalogue (list) price of an article, from the amount of a bill of merchandise, or from the amount of a debt, is called a **commercial** or **trade discount.**

Business houses usually announce their terms upon their bill heads. The space allowed for recording the terms is usually limited, and bookkeepers find it necessary to use symbols and abbreviations to indicate them. Thus, if a bill is due in 30 da. without discount, the terms may be written $^{N}/_{30}$, or Net 30 da.; if the bill is due in 30 da. without discount, but an allowance of 2% is made for payment within 10 da., the terms may be written $^{2}/_{10}$, $^{N}/_{30}$, or 2% 10 da., net 30 da.

301. Manufacturers, jobbers, and wholesale dealers usually have printed price lists for their goods. To obviate the necessity of issuing a new catalogue every time the market changes, these lists are frequently printed higher than the actual selling price of the goods, and made subject to a trade discount.

302. The fluctuations of the market and the differences in the quantities purchased by different customers frequently give rise to two or more discounts called a **discount series.**

Large purchasers sometimes get better prices and terms than small purchasers. Thus, the average customer might be quoted the regular prices less a trade discount of 25%, while an especially large buyer might be quoted the regular prices less trade discounts of 25% and 10%.

303. When two or more discounts are quoted, one denotes a discount off the list price, another, a discount off the remainder, and so on.

The order in which the discounts of any series is considered is not material. Thus, a series of 25%, 20%, and 10% is the same as one of 20%, 10%, and 25%, or one of 10%, 25%, and 20%.

304. Catalogue prices are generally estimated on the basis of credit sales, and a cash purchaser is given the usual trade discount and a special discount for early payment. This latter discount has the effect of encouraging prompt payments.

The list price is sometimes called the **gross price** and the price after the discount has been deducted the **net price.**

FINDING THE NET PRICE

305. Example. The list price of a dozen pairs of shoes is \$45. If this price is subject to a discount series of 20% and 10%, what is the net selling price?

SOLUTION. 20% or $\frac{1}{5}$ of \$45 = \$9, the first discount.
\$45 − \$9 = \$36, the price after the first discount.
10% or $\frac{1}{10}$ of \$36 = \$3.60, the second discount.
\$36 − \$3.60 = \$32.40, the net selling price.

ORAL EXERCISE

Find the net price:

	List Price	Trade Discount		List Price	Trade Discount		List Price	Trade Discounts
1.	\$4	25%	8.	\$6	40%	15.	\$4	25% and $33\frac{1}{3}$%
2.	\$15	20%	9.	\$4	$12\frac{1}{2}$%	16.	\$30	$33\frac{1}{3}$% and 25%
3.	\$90	$33\frac{1}{3}$%	10.	\$24	$8\frac{1}{3}$%	17.	\$35	20% and 25%
4.	\$20	10%	11.	\$42	$16\frac{2}{3}$%	18.	\$45	20% and $16\frac{2}{3}$%
5.	\$50	50%	12.	\$35	20%	19.	\$50	20% and 25%
6.	\$2.50	20%	13.	\$100	25%	20.	\$100	20% and 10%
7.	\$4.50	$16\frac{2}{3}$%	14.	\$720	$33\frac{1}{3}$%	21.	\$600	$16\frac{2}{3}$% and 20%

22. A piano listed at \$750 is sold less $33\frac{1}{3}$%, 20%, and 10%. What is the net cost to the purchaser?

23. A dealer offers cloth at \$3.50 per yard subject to a discount of 20%. How many yards can be bought for \$56?

WRITTEN EXERCISE

Find the net price:

	Gross Selling Price	Trade Discounts		Gross Selling Price	Trade Discounts
1.	$3360	25% and 10%	4.	$2500	20%, 10%, and 5%
2.	$3510	$33\frac{1}{3}$% and 20%	5.	$5400	25%, 20%, and 10%
3.	$4500	20% and $16\frac{2}{3}$%	6.	$3960	$33\frac{1}{3}$%, 20%, and $16\frac{2}{3}$%

7. The list price of cloth is $4.80 per yard, but this price is subject to discounts of 25% and 20%. How many yards can be bought for $288?

8. A hardware dealer sold 25 doz. 5-in. files at $2.50 and 25 doz. 12-in. files at $7.50, less 50% and 10%; 150 machine bolts at $21.50 per C, less 20% and 10%. What was the net amount of the bill?

9. Study the following model. Copy and find the net amount of the bill, using the discounts named in the bill, and the following prices: 5-in. pipe, $1.45; 1-in. pipe, 17¢; valves, $2.67.

Chicago, Ill., July 16, 19

Mrs. E. M. Farnsworth
Toledo, O.

Bought of GEORGE W. MUNSON & CO.

Terms 60 da. net

300	ft. 5 in. Iron Pipe	1.45	420					
200	" 1 in. " "	16¢	32					
			452					
	Less 50%		226					
			226					
	Less 10%		22 60			203	40	
20	1 in. Valves	2.67	48					
	Less 25%		12					
			36					
	Less 16⅔%		6			30		233 40

10. One firm offers a piano for $400, subject to discounts of 20% and 20%; another offers the same piano for $400 less discounts of 25% and 15%. Which is the better offer? How much better?

11. A jobber bought a quantity of goods listed at $3600, subject to discounts of 25% and 20%. He sold the goods at the same list price, subject to discounts of 20% and 10%. Did he gain or lose, and how much?

12. Make out bills for the following, using the current date and the name and address of some dealer whom you know. Terms in each case, 60 da. net.

a. You bought 12 doz. hand saws, #27, at $18.50; $7\frac{1}{2}$ doz. mortise locks, #271, at $4.25; 25 doz. pocket knives, #27, at $7.50; and $1\frac{3}{4}$ doz. cheese knives at $8.25. Discount: 25%, 10%.

b. You bought $41\frac{1}{2}'$ of 2″ extra strong iron pipe at 70¢; $94\frac{1}{4}'$ of $1\frac{1}{4}''$ extra strong iron pipe at $31\frac{1}{2}$¢; $153\frac{1}{4}'$ of $\frac{1}{4}''$ iron pipe at $6\frac{1}{2}$¢; $88\frac{1}{2}'$ of $\frac{3}{8}''$ iron pipe at $7\frac{3}{4}$¢. Discount: 25%, 10%.

c. You bought 25 kitchen tables at $3.25; 25 dining-room tables at $8.75; 15 doz. dining-room chairs at $12.50; 12 antique rockers at $12.25; and 15 oak bedroom sets at $32.50. Discount: $16\frac{2}{3}$%, 5%.

FINDING A SINGLE RATE OF DISCOUNT EQUIVALENT TO A DISCOUNT SERIES

306. Example. What single rate of discount is equivalent to a discount series of 25%, $33\frac{1}{3}$%, and 10%?

SOLUTION. Represent the list price by 100%. Then, 75% equals the price after the first discount, 50% the price after the second discount, and 45% equals the net selling price. 100%, the list price, minus 45%, the net selling price, equals 55%, the single rate of discount equivalent to the given discount series.

A single discount equivalent to a discount series may often be determined mentally (see §§ 307–308).

$$\begin{array}{l} 1.00 \\ \underline{.25} \ (25\% \text{ of } 100\%) \\ .75 \\ \underline{.25} \ (33\frac{1}{3}\% \text{ of } 75\%) \\ .50 \\ \underline{.05} \ (10\% \text{ of } 50\%) \\ .45 \end{array}$$

$$100\% - 45\% = 55\%$$

WRITTEN EXERCISE

1. Find a single rate of discount equivalent to a discount series of 50 %, 25 %, 20 %, and 10 %.

2. Which is the better for the buyer and how much, a single discount of 65 % or a discount series of 25 %, 20 %, and 20 %?

3. The net amount of a bill of goods was $450 and the discounts allowed were 25%, 33⅓%, and 10 %. Find the total discount allowed.

4. I allowed a customer discounts of 50%, 10 %, and 10 % from a list price. What per cent better would a single discount of 65 % have been?

5. Goods were sold subject to trade discounts of 25 %, 20 %, and 10 %. If the total discount allowed was $460, what was the net selling price of the goods?

6. A quantity of goods was sold subject to trade discounts of 20 % and 20 %. The terms were 60 da. net or 5 % off for payment within 10 da. If a cash payment of $1026 was required 3 da. after the date of the bill, what was the list price of the goods sold?

307. Since the first of a series of discounts is computed on 100 % of the list price, and the second on 100 % minus the first discount, it follows that *the sum of any two separate discounts exceeds the equivalent single discount by the product of the two rates per cent.*

Thus, in a discount series of 20 % and 20 % the apparent single discount is the sum of the two separate discounts or 40 %; but since the second discount is not computed on 100 %, but on 80%, 40% exceeds the true single discount by 20 % of 20 %, or 4 %; and the equivalent single discount is 40 % minus 4 %, or 36 %. Hence,

308. To find the single discount equivalent to a series of two discounts:

From the sum of the separate discounts subtract their product, and the remainder will be the equivalent single discount.

When two separate discounts cannot be reduced to a single discount *mentally*, proceed as in § 306; when they can, proceed as in § 308.

ORAL EXERCISE

State a single rate of discount equivalent to a discount series of:

1. 10% and 10%.
2. 20% and 20%.
3. 30% and 30%.
4. 40% and 40%.
5. 50% and 50%.
6. 20% and 10%.
7. 30% and 10%.
8. 40% and 10%.
9. 50% and 10%.
10. 60% and 10%.
11. 30% and 20%.
12. 40% and 20%.
13. 50% and 20%.
14. 60% and 20%.
15. 25% and 10%.
16. 35% and 40%.
17. 50% and 5%.
18. 10% and 5%.
19. 20% and 5%.
20. 40% and 5%.
21. 25% and 30%.
22. 25% and 40%.
23. 20% and 15%.
24. 40% and 15%.
25. 35% and 20%.
26. 45% and 20%.
27. 55% and 20%.
28. 60% and 25%.
29. 40% and 25%.
30. 60% and 15%.
31. 25% and $33\frac{1}{3}$%.
32. 45% and $33\frac{1}{3}$%.
33. 25% and 8%.
34. $8\frac{1}{3}$% and 24%.
35. $8\frac{1}{3}$% and 36%.
36. 35% and 10%.
37. 20% and $12\frac{1}{2}$%.
38. 40% and $12\frac{1}{2}$%.
39. 60% and $12\frac{1}{2}$%.
40. 12% and $12\frac{1}{2}$%.
41. 24% and $16\frac{2}{3}$%.
42. $16\frac{2}{3}$% and 20%.
43. $14\frac{2}{7}$% and 35%.
44. $16\frac{2}{3}$% and 25%.
45. $33\frac{1}{3}$% and 15%.
46. $66\frac{2}{3}$% and 15%.
47. $11\frac{1}{9}$% and 18%.
48. 36% and $11\frac{1}{9}$%.

309. When a discount series consists of three separate rates, the first two may be combined as in § 308 and then the result and the third may be combined in the same manner.

310. Example. Find a single rate of discount equivalent to a discount series of 25%, 20%, and 20%.

SOLUTION. — Combine the first two by thinking $25\% + 20\% - 5\% = 40\%$, the single discount equivalent to the series 25% and 20%. $20\% + 40\% - 8\% = 52\%$, or the single rate equivalent to the discount series 25%, 20%, and 20%.

ORAL EXERCISE

State a single rate of discount equivalent to a discount series of:

1. 20%, 25%, and 20%.
2. 20%, 15%, and 10%.
3. 20%, 20%, and 20%.
4. 10%, 10%, and 10%.
5. 20%, 20%, and 10%.
6. 25%, $33\frac{1}{3}$%, and 10%.
7. 20%, 10%, and 10%.
8. 40%, 10%, and 10%.
9. 50%, 10%, and 10%.
10. 30%, 10%, and 10%.
11. 20%, 25%, and 10%.
12. 20%, 20%, and 25%.

311. When it is not desirable to show the separate discounts, the net selling price may be found as shown in the following example.

312. Example. A mahogany sideboard listed at \$175 is sold subject to trade discounts of 20% and 25%. Find the net cost to the purchaser.

SOLUTION. By inspection determine that a discount of 40% is equivalent to a series of 25% and 20%. Represent the gross cost by 100%.

$100\% - 40\% = 60\%$
60% of \$175 = \$105

Then $100\% - 40\% = 60\%$, the net cost to the purchaser; that is, the net cost of the sideboard is 60% of the list price. 60% of \$175 = \$105, the net cost to the purchaser.

ORAL EXERCISE

By inspection find the net cost of articles listed at:

1. \$400, less 20% and 25%.
2. \$300, less 20% and 20%.
3. \$600, less 10% and 10%.
4. \$200, less 30% and 30%.
5. \$1000, less 50% and 50%.
6. \$1000, less 30% and 10%.
7. \$200, less 60% and 25%.
8. \$400, less 20% and 40%.

WRITTEN EXERCISE

1. Find the net selling price of a piano listed at \$450, less 20% and 20%.

2. Find the net selling price of an oak sideboard listed at \$125, less 25%, $33\frac{1}{3}\%$, and 10%.

3. I bought 125 cultivators listed at \$8.50, each subject to trade discounts of 20% and 25%. If I paid freight \$30.50 and drayage \$7.90, how much did the cultivators cost me?

4. The net cost of an article was increased \$30 by freight, making the actual cost of it \$630. What was the list price of the article, the rates of discount being 25% and $33\frac{1}{3}\%$?

5. You desire to buy 24,000 ft. choice cypress: one firm quotes you \$60 per thousand feet, less trade discounts of 20% and 5%; another firm offers you the same lumber at \$75 per thousand feet, less $33\frac{1}{3}\%$ and 8%. The terms offered by both firms are $^1/_{10}$, $^N/_{30}$. You accept the better offer and pay cash. How much does the lumber cost you?

WRITTEN REVIEW EXERCISE

1. Find the cost of 125 $1\frac{1}{2}''$ brass ells at \$1.25 each, less 25%, 20%, and 10%.

2. An agent bought 10 pianos listed at \$450 each, less $33\frac{1}{3}$% and 10%, and sold them for \$400 each, less 10% and 5%. Did he gain or lose and how much?

3. Apr. 15, E. L. Gano bought of W. L. Cunningham & Co. 5 phaetons listed at \$450 each, less 25% and 20%. Terms: $^{2}/_{30}$, $^{N}/_{60}$. How much ready money would settle the bill?

4. Study the following bill. Copy and find the net amount of it, using the discounts indicated in the bill, and the following prices: windmills, \$675; pumps, \$610; 1-in. iron pipe, $17\frac{1}{2}¢$; 4-in. iron pipe, 73¢; hose, 97¢; elbows, $21\frac{1}{2}¢$; valves, \$1.49.

Boston, Mass., Oct. 24, 19—

Messrs. Cooper, Benedict & Co.
Springfield, Mass.

Bought of E. M. McGREGOR & CO.

Terms 3/10, 1/30, N/60

	3	Excelsior Windmills	650	1950						
		Less 25% and 20%		780		1170				
	5	Eureka Elevator Pumps	600	3000						
		Less 33⅓% and 25%		1500		1500				
	1800	ft. Iron Pipe, 1 in.	15¢	270						
	200	" " " 4 in.	80¢	160						
				430						
		Less 20% and 25%		172		258				
	300	ft. Engine Hose, 4 ply	1.00	300						
		Less 50% and 10%		165		135				
	100	Elbows	20¢	20						
	50	Eureka Valves	1.50	75						
				95						
		Less 40% and 10%		43	70	51	30	3114	30	

5. How much cash would settle the model bill (page 253) Oct. 30? Nov. 8? How much cash would settle the bill called for in problem 4, if it is paid for on the day it is written? If it is paid Nov. 15? Copy the model bill in the form that it would be written if cash accompanied the order; that is, copy it deducting the 3 % allowed for immediate payment.

6. Copy and find the net amount of the following bill:

Leith, Scotland, May 10, 19

Invoice of Wire Cloth

Shipped by the J. M. ROBERTS COMPANY

In the Steamship Winifredian To Edward M. Davidson & Co.
Philadelphia, Pa.

D	6 pc.,	each	34' x 5' 6"	1122 sq. ft.	1/3	70	2	6	
	6 "	"	40' x 6' 6"	****	1/4	***	*	*	
	6 "	"	42' x 7' 4"	****	1/5	***	**	*	
	3 "	"	48' x 7' 2"	****	1/5	**	**	*	
						***	**	*	
			Less 10%			**	**	*	*** ** *

7. E. M. French & Co., Albany, N.Y., bought of Austin Bailey & Co., Boston, Mass., Apr. 12, 3 doz. pr. hinges, 8 in., at $4.20, and 3 doz. pr. hinges, 4 in., at $2.10, less 60 %, 10 %, and 10 %; 50 lb. brads, $\frac{5}{8}$ in., at 90¢, and 50 lb. brads, $\frac{3}{4}$ in., at 80¢, less 50 %, 10 %, and 5 %. Terms: $^{2}/_{10}$, $^{N}/_{30}$. Find the net amount of the bill Apr. 15.

8. D. M. DeLong, Portland, Me., sold S. H. Shapleigh & Co., Concord, N.H., on account 30 da., 2 % 10 da.: 35 cultivators listed at $7.50 each, less 20 % and 10 %; 15 doz. table knives listed at $9.75, less 10 %; 15 doz. hair curlers at 90¢, less 5 %; 15 doz. locks, No. 534, at $3.75, less 10 % and 5 %; $\frac{3}{4}$ doz. steel squares, No. 8, at $36, less 25 % and 10 %; $\frac{1}{3}$ gro. knives and forks, No. 760, at $12, less 20 % and 10 %; $\frac{3}{4}$ doz. cheese knives at $9.75, less $16\frac{2}{3}$ %. Find the net amount of the bill 5 da. after date.

WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

1. If goods are bought 25% below the list price and sold at the list price, what is the advance per cent over the cost?

2. If goods are bought 20% below the list price and sold at the list price, what is the advance per cent over the cost?

3. If goods are bought 10% below the list price and sold at 10% above the list price, what is the advance per cent over the cost?

4. If goods are bought at 20% and $12\frac{1}{2}\%$ below the list price, and sold at 10% below the list price, what is the advance per cent over the cost?

5. A hardware dealer bought a machine listed at \$24, less $16\frac{2}{3}\%$ and 10%, and sold it at the list price. At what per cent above cost did he mark the selling price?

6. A jobber wished to buy at such a discount from the manufacturer's list price that he could make an advance of 25% over cost, and still sell at the manufacturer's list price. What would the jobber pay for \$1000 worth of goods?

7. A gentleman wished to buy a carriage. One dealer offered him a discount of $33\frac{1}{3}\%$ and 10% from the list price, and another dealer offered him 20%, 10%, and 10% from the list price. If the list price is \$450, what will be the cost of the carriage if it is bought at the better discount?

8. Aug. 5, you buy of Gray, Salisbury & Son, New York City, 4000 lb. raisins at 16¢, less trade discounts of 25%, 20%, and 10%. Terms: $^{2}/_{10}$, $^{N}/_{30}$. You pay cash for freight \$3.20. If you pay the bill Aug. 7, what will the raisins cost you?

9. You desire to buy 200 lb. nutmeg. You find that S. S. Pierce Co., of your city, offer this article at 75¢ per lb., less a discount of 25%, and that Smith, Perkins & Co., New York City, offer it at 70¢ per lb., less discounts of 15% and 10%. The freight from New York to your city on a package of this kind is \$1.50. The terms offered by both firms are: $^{1}/_{10}$, $^{N}/_{30}$. You accept the better offer and pay cash. How much does the nutmeg cost you?

CHAPTER XIX

GAIN AND LOSS

ORAL EXERCISE

1. What is $33\frac{1}{3}\%$ of \$660? How much is gained on goods bought for \$900 and sold at a profit of $33\frac{1}{3}\%$?

2. What per cent greater is \$75 than \$60? what per cent less is \$60 than \$75? Goods bought for \$100 are sold for \$150. What is the gain per cent?

3. What per cent less is \$80 than \$100? what per cent more is \$100 than \$80? Goods bought for \$100 are sold for \$90. What is the loss per cent?

4. If \$800 is increased by 25% of itself, what is the result? Goods bought for \$1400 are sold at a profit of 25%. What is the selling price?

5. If \$1500 is decreased by $33\frac{1}{3}\%$ of itself, what is the result? Goods bought for \$2700 are sold at a loss of $33\frac{1}{3}\%$. What is the selling price?

6. State a brief method for finding a gain of $6\frac{1}{4}\%$; a gain of $6\frac{2}{3}\%$; a gain of $8\frac{1}{3}\%$; a gain of 10%; a gain of $1\frac{1}{4}\%$; a gain of $1\frac{2}{3}\%$; a gain of $2\frac{1}{2}\%$; a gain of $3\frac{1}{3}\%$.

7. State a brief method for finding a loss of $11\frac{1}{9}\%$; a loss of $12\frac{1}{2}\%$; a loss of $14\frac{2}{7}\%$; a loss of $16\frac{2}{3}\%$; a loss of 20%; a loss of 25%; a loss of $9\frac{1}{11}\%$; a loss of $37\frac{1}{2}\%$.

8. State a brief method for finding a gain of $33\frac{1}{3}\%$; a gain of $22\frac{2}{9}\%$; a gain of 50%; a gain of $66\frac{2}{3}\%$; a gain of 75%.

313. The **gains** and **losses** resulting from business transactions are frequently estimated at some *rate per cent* of the cost, or of the money or capital invested.

Since no new principles are involved in this subject, illustrative examples are unnecessary.

FINDING THE GAIN OR LOSS

ORAL EXERCISE

By inspection find the gain or loss:

	Cost	Per Cent of Gain		Cost	Per Cent of Loss		Cost	Per Cent of Gain
1.	\$2900	50%	9.	\$1500	10%	17.	\$7500	20%
2.	\$1600	75%	10.	\$1600	$1\frac{1}{4}\%$	18.	\$1400	25%
3.	\$5600	$28\frac{4}{7}\%$	11.	\$3000	$1\frac{2}{3}\%$	19.	\$2200	$9\frac{1}{11}\%$
4.	\$2700	$33\frac{1}{3}\%$	12.	\$4800	$2\frac{1}{2}\%$	20.	\$8100	$11\frac{1}{9}\%$
5.	\$2400	$37\frac{1}{2}\%$	13.	\$3600	$3\frac{1}{3}\%$	21.	\$6400	$12\frac{1}{2}\%$
6.	\$1400	$42\frac{6}{7}\%$	14.	\$3200	$6\frac{1}{4}\%$	22.	\$2800	$14\frac{2}{7}\%$
7.	\$3200	$62\frac{1}{2}\%$	15.	\$4500	$6\frac{2}{3}\%$	23.	\$9600	$16\frac{2}{3}\%$
8.	\$2100	$66\frac{2}{3}\%$	16.	\$8400	$8\frac{1}{3}\%$	24.	\$3600	$22\frac{2}{9}\%$

25–48. Find the selling price in each of the above problems.

WRITTEN EXERCISE

1. An importation of silks invoiced at £40 10*s.* was sold at a profit of 25%. Find the amount (in United States money) of the gain.

2. An importation of German toys invoiced at 43,750 marks was sold at a gain of $33\frac{1}{3}\%$. Find the amount (in United States money) of the gain.

3. An article that cost \$1 was marked 10% above cost. In order to effect a sale, it was afterward sold for 10% below the marked price. Find the gain or loss on 250 of the articles.

4. A man bought a city lot for \$1150 and built a house on it costing \$2650. He then sold the house and lot at a gain of 5%. How much did he gain and what was his selling price?

5. A man bought a quantity of silk for \$450, a quantity of fancy plaids for \$120, and a quantity of velvet for \$90. He sold the silk at a gain of 25%, the plaids at a loss of 5%, and the velvet at a gain of $33\frac{1}{3}\%$. What was his gain, and how much did he realize from the sale of the three kinds of material?

FINDING THE PER CENT OF GAIN OR LOSS

ORAL EXERCISE

By inspection find the per cent of gain or loss:

	Cost	Gain		Cost	Loss		Cost	Selling Price		Selling Price	Gain
1.	$100	$10	7.	$60	$15	13.	$80	$90	19.	$300	$60
2.	$150	$50	8.	$40	$10	14.	$90	$80	20.	$115	$23
3.	$140	$70	9.	$90	$45	15.	$60	$75	21.	$102	$17
4.	$140	$140	10.	$70	$14	16.	$75	$60	22.	$420	$60
5.	$200	$400	11.	$80	$16	17.	$10	$50	23.	$300	$200
6.	$300	$750	12.	$15	$10	18.	$50	$10	24.	$700	$100

WRITTEN EXERCISE

1. A milliner bought hats at $15 a dozen and retailed them at $3 each. What per cent was gained?

2. A stationer bought paper at $2 a ream and retailed the same at a cent a sheet. What was his per cent of gain?

3. A dry-goods merchant bought gloves at $7.50 a dozen pair and retailed them at $1.25 a pair. What was his per cent of gain?

4. A merchant imported 50 gro. of table knives at a cost of $1125. Two months later he found that the sales of table knives aggregated $920 and that the value of the stock unsold was $435. Did he gain or lose, and what per cent?

5. An importer bought a quantity of silk goods for £400 5*s*. After disposing of a part of the goods for $1200 he took an account of the stock remaining unsold and found that at cost prices it was worth $1047.82. Did he gain or lose, and what per cent?

6. Jan. 1, F. E. Smith & Co. had merchandise on hand valued at $2500. During the month they purchased goods costing $6000 and sold goods amounting to $7500. If the stock on hand at cost prices Feb. 5 was worth $2500, what was the per cent of gain on the sales?

FINDING THE COST

ORAL EXERCISE

By inspection find the cost:

	Loss	Rate of Loss		Gain	Rate of Gain
1.	\$150	10 %	7.	\$35	20 %
2.	\$100	$1\frac{1}{4}$ %	8.	\$79	25 %
3.	\$200	$1\frac{2}{3}$ %	9.	\$12	$11\frac{1}{9}$ %
4.	\$450	$2\frac{1}{2}$ %	10.	\$19	$16\frac{2}{3}$ %
5.	\$220	$6\frac{1}{4}$ %	11.	\$44	$22\frac{2}{9}$ %
6.	\$115	$8\frac{1}{3}$ %	12.	\$15	$33\frac{1}{3}$ %

	Selling Price	Rate of Gain		Selling Price	Rate of Loss
13.	\$1050	5 %	19.	\$950	5 %
14.	\$2040	2 %	20.	\$900	50 %
15.	\$3600	20 %	21.	\$150	$6\frac{1}{4}$ %
16.	\$1400	$16\frac{2}{3}$ %	22.	\$550	$16\frac{2}{3}$ %
17.	\$1800	$12\frac{1}{2}$ %	23.	\$240	$33\frac{1}{3}$ %
18.	\$2400	$33\frac{1}{3}$ %	24.	\$490	$22\frac{2}{9}$ %

25. A man bought a machine for \$240.48. For how much must he sell it to gain $12\frac{1}{2}$ % ?

26. B sold a farm for \$2400, thereby losing 25 %. For how much should he have sold it to have gained 10 % ?

27. By selling a piano at \$400 a dealer realizes a gain of $33\frac{1}{3}$ %. What would be the selling price of the piano if sold at a gain of 25 % ?

WRITTEN EXERCISE

1. A sleigh was sold for \$64.80, which was 10 % below cost. What was the cost ?

2. An office safe was sold at \$102, which was 20 % above cost. What was the cost ?

3. A merchant marks goods $16\frac{2}{3}$ % above cost. What is the cost of an article that he has marked \$21.70 ?

4. An owner of real estate sold 2 city lots for $12,000 each. On one he gained 25% and on the other he lost 25%. What was his net gain or loss from the two transactions?

5. A merchant sold a quantity of goods to a customer at a gain of 25%, but owing to the failure of the customer he received in settlement but 88¢ on the dollar. If the merchant gained $645.15, what did the goods cost him?

6. A manufacturer sold an article to a jobber at a gain of 25%, the jobber sold it to a wholesaler at a gain of 20%, and the wholesaler sold it to a retailer at a gain of $33\frac{1}{3}$%. If the retailer paid $28 for the article, what was the cost to manufacture it?

7. A manufacturer sold an article to a wholesaler at a gain of 20%, the wholesaler sold the same article to a retailer at a gain of $33\frac{1}{3}$%, and the retailer to the consumer at a gain of 25%. If the average gain was $40, what was the cost to manufacture the article?

WRITTEN REVIEW EXERCISE

1. A merchant bought goods at 40% off from the list price and sold the same at 20% and 10% off the list price. What was his gain per cent?

2. I bought goods at 50% off from the list price and sold them at 25% and 25% off from the list price. Did I gain or lose, and what per cent?

3. Apr. 15 you bought of Baker, Taylor & Co., Rochester, N.Y., 4000 bbl. Roller Process flour listed at $4.50 a barrel, and 2000 bbl. of Searchlight pastry flour listed at $4.75 a barrel. Each list price was subject to trade discounts of 20% and 10%. You paid cash $16,000 and gave your note at 30 da. for the balance. What was the amount of the note?

4. May 18 you sold to F. H. Clark & Co., New York City, 2000 bbl. of the Roller Process flour, bought in problem 3, at $33\frac{1}{3}$% above cost. Terms: $^{2}/_{10}$, $^{N}/_{30}$. F. H. Clark & Co. paid cash. Find the cash payment.

5. May 30 you sold Smith, Perkins & Co., Albany, N.Y., the balance of the flour bought in problem 3, at an advance of $33\frac{1}{3}\%$ on the cost. Terms: $^{2}/_{10}$, $^{N}/_{30}$. The flour was paid for June 8. Find the cash payment.

6. What is the net gain on the transactions in problems 3, 4, and 5? the net gain per cent?

7. Dec. 15 you bought of E. B. Johnson & Co. 400 bbl. of apples at \$2.50 per barrel. Terms: $^{1}/_{10}$, $^{N}/_{30}$. You paid cash. Find the amount of your payment.

8. May 15 you sold F. E. Redmond the apples bought in problem 7, at \$4 a barrel. Terms: $^{1}/_{10}$, $^{N}/_{30}$. At the maturity of the bill Redmond refused payment and you placed the account in the hands of a lawyer who succeeded in collecting 75% of the amount due. If the lawyer's fee for collecting was 4%, what was your net gain or loss?

9. A tailor made 25 doz. overcoats with cloth worth \$2 a yard. 4 yd. were required for each coat and the cost of making was \$48 per dozen. He sold the overcoats so as to gain $33\frac{1}{3}\%$. How much did he receive for each?

10. Apr. 12 J. D. Farley & Son, Trenton, N.J., bought of Cobb, Bates & Co., Boston, Mass., a quantity of green Java coffee sufficient to yield 2400 lb. when roasted. If the loss of weight in roasting averages 4%, what will the green coffee cost at 30¢ a pound, less a trade discount of 10%? Arrange the problem in bill form.

11. If the coffee in problem 10 is retailed $33\frac{1}{3}\%$ above cost, and there is a loss of 1% from bad debts, what is the gain on the transactions in coffee? the gain per cent?

12. The Metropolitan Coal Co., of Boston, Mass., decides to bid on a contract for supplying 2240 T. of coal for the public schools of the city. It can buy the coal at \$4.50 per long ton delivered on board track, Boston. It costs on an average 75¢ per short ton to deliver the coal, and there is a waste of $\frac{1}{4}\%$ from handling. Name a bid covering a profit of 20%. Terms: cash.

FINDING THE PER CENT OF GAIN OR LOSS ON THE SELLING PRICE

ORAL EXERCISE

1. An article cost \$80 and it is sold for \$100. What is the sum gained? The gain is what per cent of the cost? of the selling price?

2. An article costs \$60 and it is sold for \$75. What is the sum gained? The gain is what per cent of the cost? of the selling price?

3. An article is sold for \$90. If the gain on the selling price is $33\frac{1}{3}\%$, what was the cost, and what is the gain per cent on the cost price?

314. Find by inspection the gain per cent on the selling price:

	Cost	Selling Price		Cost	Selling Price
a.	\$20	\$30	*f.*	\$120	\$150
b.	\$30	\$40	*g.*	\$125	\$150
c.	\$45	\$60	*h.*	\$140	\$160
d.	\$60	\$75	*i.*	\$150	\$175
e.	\$50	\$60	*j.*	\$160	\$180

This principle may be applied effectively when goods have been marked by a merchant at a certain per cent on the advance of the cost, and then marked down to sell at cost.

315. If an article that costs \$1 is marked to sell at \$1.10, what per cent of reduction will restore the original cost price?

Suggestion. It is evident that a reduction of 10% on the selling price will not restore the original marking of \$1.

316. Find by inspection the per cent of reduction that must be made to reduce the marked price to the cost price.

	Cost	Marked Price		Cost	Marked Price
a.	\$1.00	\$1.25	*d.*	\$1.50	\$1.80
b.	\$1.25	\$1.50	*e.*	\$2.00	\$2.50
c.	\$1.60	\$2.00	*f.*	\$3.00	\$4.00

317. Business men are continually dealing with the problem of *overhead charges;* that is, the *cost of doing business.* Overhead

charges include such expenses as employees' salaries, rent, insurance, taxes, light and heat, postage, advertising, depreciation, telephone, and many others. To the invoice charges there must be added a certain per cent to cover the cost of doing business.

318. The following principle applies to subsequent problems:

Divide the invoice cost plus the freight by *100 % minus the overhead charges plus the per cent of profit* (100 % − charges + profits); the result will be the selling price. (This statement is based on reckoning the overhead expenses and the gain as a per cent of the selling price.)

WRITTEN EXERCISE

1. An article was invoiced at $33.50; freight charges, $1.50. If the overhead charges amounted to 15 % and the gain to 10 %, what was the selling price?

SOLUTION. 15% + 10% = 25%.
100% − 25% = 75%.
$33.50 + $1.50 = $35, the cost.
$35 ÷ .75 = $46.67, the selling price.

PROOF. 25% of $46.67 = $11.67, overhead charges and gain.
$46.67 − $11.67 = $35, the cost.

2. A merchant sold goods amounting to $22,500. If the overhead charges amounted to 18 % and the profits to 8 %, what was the invoice price of the goods if the freight amounted to $350?

3. A merchant marked a lot of goods $33\frac{1}{3}$ % above cost, but as he was unable to sell them at the marked price, he decided to reduce the marking to cost. What per cent reduction must be made?

4. A machine was invoiced at $53.50; freight charges, $3.50. If the overhead charges of the business amounted to 20 %, and the gain to 10 %, what must be the selling price of the goods?

5. An invoice of merchandise amounted to $1204.50; freight charges, $10.50. If the overhead charges amounted to $17\frac{1}{2}$ % and the gain to $7\frac{1}{2}$ %, what must be the selling price?

6. A merchant marked a lot of goods at 25 % above cost, but as the goods did not sell at the marked price, he reduced it 25 %, and announced that he was selling at cost. What per cent represents the amount of his error? If the goods thus marked cost $1760.48, what did the merchant lose by his blunder?

CHAPTER XX

MARKING GOODS

319. Merchants frequently use some private mark to denote the cost and the selling price of goods. The word, phrase, or series of arbitrary characters employed for private marks is called a **key**.

Many houses use two different keys in marking goods, one to represent the cost and the other the selling price. In this way the cost of an article may not be known to the salesmen, and the selling price may not be known to any except those in some way connected with the business. In large houses, when but one key is used, only the selling price is indicated on the article, it being deemed best to keep the actual cost of the article a secret with the buyers. In small houses, when but one key is used, both the cost and the selling price are frequently written on the article.

320. If letters are used to mark goods, any word or phrase containing *ten different letters* may be selected for a key. If arbitrary characters are used, any *ten different characters* may be selected for a key.

Some methods of marking are so complicated that it is necessary to always have a key of the system at hand for reference. Goods are so marked in order that important facts, such as the cost of goods, may be kept strictly private.

321. When a figure is repeated one or more times, one or two extra letters called **repeaters** are used to make the key word more secure as a private mark.

322. The following illustrates the method of marking goods by letters:

Cost Key	Selling-price Key
REPUBLICAN	PERTHAMBOY
1 2 3 4 5 6 7 8 9 0	1 2 3 4 5 6 7 8 9 0
Repeaters: S and Z	Repeaters: W and D

The cost is generally written above and the selling price below a horizontal line on a tag, or on a paster or box. Gloves No. 271, costing \$5 a dozen and selling for \$6.25 a dozen, might be marked as shown in the margin. Fractions may be designated by additional letters or characters. Thus, W may be made to represent $\frac{1}{2}$, K $\frac{1}{4}$, etc. in the above key. In marking goods for the retail trade, *all fractions* of a cent are called another whole cent.

WRITTEN EXERCISE

323. *Using the keys given in § 322, write the cost and the selling price in each of the following problems:*

	First Cost of Article	Freight	Gain	Loss		First Cost of Article	Freight	Gain	Loss
1.	\$2.50	10%	20%		**5.**	\$16.00	$2\frac{1}{2}$%	$37\frac{1}{2}$%	
2.	\$1.00	10%	20%		**6.**	\$40.00	5%	$16\frac{2}{3}$%	
3.	.50		$33\frac{1}{3}$%		**7.**	\$ 3.60	$2\frac{1}{2}$%		
4.	\$4.80	20%		25%	**8.**	\$24.00			10%

324. *Using the following key, write the cost and the selling price in each of the following problems:*

Cost Key

1	2	3	4	5	6	7	8	9	0
┌	└	┐	┘	├	┤	⊏	⊐	⌠	+

Repeaters: □ ◺

Selling-price Key

1	2	3	4	5	6	7	8	9	0
T	⊥	⊔	⊓	E	∃	⊓	Ш	I	#

Repeaters: X —

	First Cost of Article	Charges	Gain	Loss		First Cost of Article	Charges	Gain	Loss
9.	\$10.00	5%	20%		**12.**	\$15.00	$6\frac{2}{3}$%	25%	
10.	\$20.00	10%	50%		**13.**	\$18.00	10%	25%	
11.	\$30.00	$6\frac{2}{3}$%		25%	**14.**	\$12.00	5%	$33\frac{1}{3}$%	

325. Wholesalers and jobbers buy and sell a great many articles by the dozen. Retailers buy a great many articles by the dozen, but generally sell them by the piece. In marking goods, therefore, it is highly important that the student be able to divide by 12 with great rapidity.

To divide by 12 with rapidity, the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, should be *memorized*.

TABLE OF TWELFTHS

TWELFTHS	SIMPLEST FORM	DECIMAL VALUE	TWELFTHS	SIMPLEST FORM	DECIMAL VALUE
$\frac{1}{12}$		$.08\frac{1}{3}$	$\frac{7}{12}$		$.58\frac{1}{3}$
$\frac{2}{12}$	$\frac{1}{6}$	$.16\frac{2}{3}$	$\frac{8}{12}$	$\frac{2}{3}$	$.66\frac{2}{3}$
$\frac{3}{12}$	$\frac{1}{4}$	.25	$\frac{9}{12}$	$\frac{3}{4}$	.75
$\frac{4}{12}$	$\frac{1}{3}$	$.33\frac{1}{3}$	$\frac{10}{12}$	$\frac{5}{6}$	$.83\frac{1}{3}$
$\frac{5}{12}$		$.41\frac{2}{3}$	$\frac{11}{12}$		$.91\frac{2}{3}$
$\frac{6}{12}$	$\frac{1}{2}$	.50	$\frac{12}{12}$	1	1.00

326. Example. What is the cost of one shirt when a dozen shirts cost $19?

SOLUTION. Divide by 12 the same as by any number of one digit and mentally reduce the twelfths in the remainder to their decimal equivalent. Thus, say or think $1\frac{7}{12}$, $1.58\frac{1}{3}$, practically $1.58.

ORAL EXERCISE

State the cost per article when the cost per dozen articles is:

1. $25.00.	7. $7.00.	13. $23.20.	19. $9.00.
2. $37.00.	8. $3.60.	14. $19.20.	20. $7.00.
3. $42.00.	9. $2.40.	15. $66.60.	21. $5.00.
4. $64.00.	10. $5.60.	16. $38.00.	22. $7.50.
5. $80.00.	11. $3.40.	17. $17.00.	23. $8.40.
6. $13.00.	12. $13.20.	18. $11.00.	24. $17.50.

ORAL EXERCISE

1. Hats costing $48 a dozen must be sold for what price each to gain 25%?

2. Rulers bought at $2 a dozen must be retailed at how much each to gain 50%?

3. Note books costing $1.60 per dozen must be retailed at what price each to gain $12\frac{1}{2}$%?

4. Erasers bought at $3.24 per gross must be retailed at how much each to gain $111\frac{1}{9}$%?

5. Matches costing $3.60 per gross boxes must be retailed at what price per box to gain 100%?

6. Envelopes bought at $2 per M must be sold at what price per package of 25 to gain 100%?

7. Pickles bought at $1.80 per dozen bottles must be sold at what price per bottle to gain $33\frac{1}{3}$%?

8. Mustard costing $14.40 per gross packages must be retailed at what price per package to gain 20%? to gain 50%?

LISTING GOODS FOR CATALOGUES

327. In listing goods for catalogues dealers generally mark them so that they may allow a discount on the goods and still realize a profit.

328. Example. What should be the catalogue price of an article costing $24 in order to insure a gain of 25% and allow the purchaser a discount of 20%?

SOLUTION. $\frac{1}{4}$ of $24 = $6, the gain.
$30 = the selling price, which is 20% below the catalogue price.
.80 of the catalogue price = $30.
∴ the catalogue price = $30 ÷ .80 = $37.50.

WRITTEN EXERCISE

1. At what price must you mark an article costing $.400 to gain 25% and provide for a 20% loss through bad debts?

2. What should be the catalogue price of a library table costing $25 in order to insure a gain of 20% and allow the purchaser a discount of 25%?

3. You list tea costing 30¢ a pound in such a way that you gain $33\frac{1}{3}$% after allowing the purchaser a trade discount of 20%. What is your list price?

4. You buy broadcloth at $3.80 per yard. At what price must you mark it in order that you may allow your cash customers 5% discount and still realize a gain of 20%?

5. Having bought a quantity of oranges for $3.00 per C you mark them so as to gain $33\frac{1}{3}$% and allow for a 20% loss through bad debts. What will be your asking price per dozen?

6. At what price must the articles in the following invoice be listed to gain 20 % and allow discounts of 25 % and 20 % ?

Boston, Mass., Nov. 24, *19*

Mr. Edgar C. Townsend
Rochester, N.Y.

Bought of *WELLS, FOWLER & CO.*

Terms Net 30 da.

#721	50	Oak Bookcases	$8.00	400			
#924	25	Gentlemen's Chiffoniers	12.00	300			
				700			
		Less 10%		70		630	

WRITTEN REVIEW EXERCISE

1. Using the word *importance*, with repeaters *s* and *w*, for the buying key, and the words *buy for cash*, with repeaters *t* and *m*, for the selling key, write the cost and selling price of the articles in the following bill. It is desired to gain 25 % on the pens and pencils, 20 % on the cards, and to provide for a loss of 12½ % through bad debts.

Boston, Mass., Oct. 18, **19**

Messrs. WHITE & WYCKOFF
Holyoke, Mass.

Bought of C. E. Stevens & Co.

Terms Net 30 da.

100	gro.	Pens	$0.80	80			
25	"	Lead Pencils	3.20	80			
50	pkg.	Record Cards	.40	20			
				180			
		Less 12 1/2%		22	50	157	50

2. At what price must I mark the following shoes to gain 20%?

Detroit, Mich., Sept. 7, 19

Mrs. M. I. Fisher
Buffalo, N. Y.

Bought of ATWOOD & RANDALL

Terms 5/10, 2/60

300	prs. Children's Lace Shoes 2.00	600		
	Less 5%	30	570	
	Paid Sept. 7, 19 Atwood & Randall, per A.			
	Freight and other charges		17	50
			587	50

3. You list tea bought for 30¢ at an advance of $33\frac{1}{3}$% on the cost. Finding small sale for the article you determine to sell so as to gain but $16\frac{2}{3}$%. What trade discount should you allow?

4. What price per pound must be obtained for the following invoice of coffee to gain 25% and allow 10% for loss in roasting and $16\frac{2}{3}$% for loss through bad debts?

Boston, Mass., Nov. 25, *19*

Messrs. Merchant & Co.
120 Main St., City

Bought of Cobb, Bates & Co.

Terms 30 da.

2000 lb. Green Java Coffee	24¢	480	00	
Cartage		2	50	482 50

CHAPTER XXI

COMMISSION AND BROKERAGE

ORAL EXERCISE

1. A collected a bill of $350 and received 6 % for his services. How much did he make?

2. B bought $80 worth of eggs for a dealer who paid him $7\frac{1}{2}$ % for his services. How much did B make?

3. C receives $12 a week, and 5 % of his weekly sales. If he sold $350 worth of goods in a week, what was his income for the week?

329. An **agent** is a person who undertakes to transact business for another called the **principal.**

330. A great deal of the produce of the country and a large variety of manufactured articles are bought and sold through agents called commission merchants and brokers.

331. A **commission merchant** (sometimes called a factor) is an agent who has actual possession and control of the goods of his principal; a **broker** is an agent who arranges for purchases or sales of goods without having actual possession of them.

332. The sum charged by an agent for transacting business for his principal is called **commission** or **brokerage.**

Commission and brokerage are frequently computed at a certain per cent of the amount of property bought or sold, or of the amount of business transacted. Brokerage is also often a fixed rate per bushel, barrel, tierce, or other standard measure.

333. Agents frequently charge an additional commission, called **guaranty,** for assuming any risk or guaranteeing the quality of goods bought or sold.

The person who ships goods is sometimes called the **consignor**; the person to whom the goods are shipped, the **consignee.**

A quantity of goods sent away to be sold on commission is called a **shipment**; a quantity of goods received to be sold on commission, a **consignment**.

334. An **account sales** is an itemized statement rendered by a commission merchant to his principal. It shows in detail the sales of the goods, the charges thereon, and the net proceeds remitted or credited.

Buffalo, N.Y., June 18, *19*

Sale of Merchandise for Account of

E. H. Barker & Co., Poughkeepsie, N.Y.

By Hogg, Taylor & Hogg

June	5	200 bbl. Roller Process Flour	$6.00			1200	00
	12	300 " Old Grist Mill Flour	6.10			1830	00
		Charges					
June	2	Freight and Drayage		40	75		
	12	Commission 5%		151	50		
	18	Net proceeds remitted		2837	75		
				3030	00	3030	00

335. An **account purchase** is a detailed statement rendered by a purchasing agent to his principal. It shows in detail the quantity, grade, and price of goods purchased, the expenses incurred, and the gross (total) cost of the transaction.

CHICAGO, ILL., Mar. 15, 19—

Purchase of Merchandise for Account of F. N. Carpenter
Cincinnati, O.

By GRAY, DUNKLE & CO.

200	bbl. Greening Apples	3.00	600	—	
200	" Russet Apples	3.00	600	—	
200	" Baldwin Apples	3.25	650	—	1850 —
	Charges				
	Drayage		10	50	
	Commission, 2%		37	—	47 50
	Amount charged to your acct.				1897 50

ORAL EXERCISE

1. I sold 100 A. of land at $50 per acre on a commission of 3%. What was my commission?

2. A lawyer collected an account of $1000 and received for his services $40. What was his rate of commission?

3. A book agent received 25% on all books sold. In one week, after paying his expenses, $25, he netted $75. What was the gross amount of the week's sales?

4. I bought through a broker 1000 bu. of wheat quoted at $89\frac{7}{8}$¢ per bushel. If the broker charged $\frac{1}{8}$¢ per bushel for buying the wheat, what was his brokerage? How much did the wheat cost me?

SELLING ON COMMISSION

WRITTEN EXERCISE

1. Copy and complete the following letter:

JOHNSON & CO.
Produce Merchants

Boston, Mass., May 10, 19—

(STUDENT'S NAME)

(STUDENT'S ADDRESS)

Dear ——:

We have today shipped to your address, to be sold on our account and risk the following:

200 crates Eggs
100 boxes Cheese
200 tubs Creamery Butter

Please dispose of these goods to the best advantage.

Yours truly
Johnson & Co.

2. May 15 you sell F. E. Spencer, Brattleboro, from Johnson & Co.'s consignment: 200 tubs, 10,000 lb., creamery butter at 23¢, and 100 crates, 3000 doz., eggs at 20¢, f.o.b. cars Brattleboro. You pay freight $16 and drayage $2.50. The terms are $^2/_{10}$, $^N/_{30}$. F. E. Spencer pays cash. Make a receipted bill for the transaction.

3. May 23 you sell Comstock & Co., Montpelier, from Johnson & Co.'s consignment: 100 crates, 3000 doz., eggs at 20¢, and 100 boxes, 6000 lb., cheese at 12¢, f.o.b. Montpelier. You pay freight $25 and drayage $4.50. Terms: $^2/_{10}$, $^N/_{30}$. Comstock & Co. pay cash. Make a receipted bill for the transaction.

4. Render Johnson & Co. an account sales under date of May 24 for the goods shipped May 10. The net proceeds are remitted by New York draft. Commission, 5%.

5. Find for Johnson & Co., the net gain on the shipment in problem 1. The eggs were bought at 12¢, the creamery butter at 15¢, and the cheese at 8¢. Johnson & Co. prepaid freight on shipment to you, $38.50.

6. Pay freight $20.50 on the merchandise enumerated in the following shipping invoice. This sum is 5% of the cost of the goods. Find the gross cost of the goods.

New York, Dec. 8, 19—

Invoice of Merchandise shipped to _____
(STUDENT'S NAME)

_____
(STUDENT'S ADDRESS)

To be sold for account of C. L. BROWN & CO.

	60	bx. Lemons				
	50	" Oranges				

7. Dec. 15 you sell Morgan & Co., Albany, N.Y., 60 bx. lemons at $4. Terms: $^2/_{10}$, $^N/_{30}$. Morgan & Co. pay cash. What is the amount of the cash payment?

8. Dec. 18 you sell Meachum & Co., Troy, N.Y., 50 bx. oranges at $4.50. Terms: $^{2}/_{10}$, $^{n}/_{30}$. Meachum & Co. pay for the goods Jan. 12. What is the amount of their payment?

9. Render C. L. Brown & Co. an account sales for the goods received Dec. 8, commission, 5%. Assume that on Dec. 5 you advanced them $50 on the consignment. Find C. L. Brown & Co's net gain or loss on the shipment in problem 6.

10. Prepare an account sales, under the current date, for the following, sold by you, for the account of Lewis, Grayson & Co., Rochester, N.Y.: 60 bbl. Pillsbury's flour at $6.25; 75 bbl. XXXX flour at $5.75; 45 bbl. star brand flour at $5; 100 bbl. XXX flour at $4.90; 50 bbl. peerless flour at $5.15. Charges: freight, $38.95; cartage, $12.60; cooperage, $6.25; commission, $3\frac{1}{2}$%; guaranty, $\frac{1}{2}$%.

BUYING ON COMMISSION

WRITTEN EXERCISE

1. B, a broker, bought for C, a speculator, 3000 bu. wheat at $90\frac{7}{8}¢$, on a commission of $\frac{1}{8}¢$ per bushel. What was the broker's commission, and what did the wheat cost C?

2. I bought through a broker 5000 bags coffee, each containing 130 lb., at $12\frac{1}{2}¢$. If the broker charged $10 for each 250 bags, how much did he earn on the transaction, and what did the coffee cost me?

3. I bought through a broker 20,000 bu. of wheat at $87\frac{7}{16}¢$, and three weeks later sold it through the same broker at $92\frac{7}{8}¢$. If the broker charged me $\frac{1}{8}¢$ per bu. for buying and the same for selling, what was my gain?

4. A firm of produce dealers bought through a broker 1500 bbl. pork at $12.50, and immediately sold it through another broker at $12.72\frac{1}{2}$. If each broker charged a commission of $2\frac{1}{2}¢$ per barrel, what was gained by the produce dealers?

5. You buy for your principal 1500 bbl. flour at $4.50, on a commission of 3%, and pay drayage $18.50. What is the cost of the purchase to your principal?

6. By your principal's instructions you put the flour (problem 5) in storage and later sold it at $5.25 a barrel, on a commission of 3%. The storage charges were 5¢ per barrel. What amount should you remit to your principal?

7. A broker bought cotton for a manufacturer as follows: 750 bales, 375,000 lb. at $10\frac{1}{2}$¢; 1500 bales, 750,000 lb. at $10\frac{3}{4}$¢; and 1000 bales, 500,000 lb. at $10\frac{3}{5}$¢. The broker's charges were $7.50 for each 100 bales. How much did he earn on the transaction, and what did the cotton cost the manufacturers?

8. Find the amount to be charged to Roe & Co.:

NEW YORK, N.Y., Mar. 15, 19

Purchased by ARAULT & Co.

For the account and risk of ROE & Co.

TELEPHONE, 690 MAIN — Poughkeepsie, N.Y.

20	hf. ch. Japan Tea	1200#	.30¢			
20	hf. ch. Oolong Tea	1000#	45¢			
	Charges					
	Drayage			4	50	
	Commission, 2%, $; guaranty, ½%, $					
	Amount charged to your account					

9. Find the rate of commission and the amount due Brown Bros. Co. in the following account purchase.

ROCHESTER, N.Y., Apr. 20, 19

Purchased by BROWN BROS. Co.

For the account and risk of W. D. SNOW

Telephone, 1291 Main — Springfield, Mass.

	600 bbl. Pillsbury's Best Flour	6.00				
	100 bbl. XXXX Flour	5.50				
	200 bbl. Peerless Flour	5.25				
	Charges					
	Cartage			15	00	
	Commission ? %			104	00	
	Amount due us					

A WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

Copy problems 1–12 and complete the work in each one:

	Face of the Debt	Rate of Commission	Amount received by the Agent	Amount received by the Principal
1.	$457.75	2%	?	?
2.	?	1%	$2.59	?
3.	?	?	$5.27	$170.23
4.	$325.45	?	?	$318.94
5.	$182.40	5%	?	?
6.	$255.50	?	$10.22	?
7.	$112.75	?	?	$108.24
8.	$282.00	?	$4.23	?
9.	?	$2\frac{1}{2}$%	?	$251.55
10.	?	?	$14.84	$409.16
11.	$455.95	2%	?	?
12.	?	?	$6.60	$125.40

13. A commission merchant sold 5000 bu. grain and charged $1\frac{1}{2}$¢ per bushel for selling. If the grain was sold at 49¢ per bushel, what sum did he remit to his principal?

14. The net proceeds of a consignment were $593.75. The following were the different charges: commission, $26; freight, $8.55; drayage, $3.40; storage, $9.20; advertising, $3; insurance, $6.10. What was the rate of commission?

15. A firm of contractors employed an agent to collect their overdue accounts. As a special inducement for closing the accounts, they were to give him 6% on all collections made the first month, and $3\frac{1}{2}$% on all collections made the second month. The first month he returned to the firm $4013.80; the second month he returned $2798.50. The returns were made after taking out his commission. What was the agent's commission?

CHAPTER XXII

PROPERTY INSURANCE

FIRE INSURANCE

ORAL EXERCISE

1. One hundred persons have property valued at $500,000. They pay into a common fund 60¢ per $100 of this sum. What is the amount of the fund?

2. These one hundred persons live in widely separated parts of the country. Is it likely that many of them will suffer losses by fire in the same year?

3. Suppose the losses to this property by fire for a year amount to $2500. What portion of the common fund will remain on hand as a surplus? (No interest.)

4. If this surplus is divided among the hundred persons at the end of the year, how much should A, who paid in $30, receive?

5. What are the companies organized to receive and distribute the fund in problem 1 called?

336. Insurance is a contract whereby for a stipulated consideration one party agrees to indemnify another for the loss or damage on a specified subject by specified perils, according to certain prescribed terms and conditions.

The best-known forms of property insurance are *fire* insurance and *marine* insurance.

There are also property-insurance companies which insure against loss due to steam-boiler explosions, failure of crops, death of live stock, burglary, injury to business by strikes among employees, and numerous other hazards.

337. Fire insurance is insurance against loss of property or damage to it by fire.

A contract of fire insurance frequently covers loss by lightning or tornado. It also covers damage resulting from or consequent on a fire, such as the loss

resulting from water applied for the purpose of extinguishing flames, also, for the loss when such destruction has been ordered by the proper authorities.

338. The **insurer,** also called the **underwriter,** is the one who agrees to indemnify. The **insured** is the one to whom the promise of indemnity is made. The **premium** is the consideration agreed upon to be paid by the insured. The **policy** is the written contract between the insurer and the insured.

339. Fire insurance is usually conducted under the joint stock or the mutual plan.

In a *joint stock company* capital is subscribed, paid for, and owned by persons called stockholders, who share in the gains and are liable, to the extent of their subscriptions, for all the losses.

A *mutual insurance company* is one in which all the policy holders share the gains and bear the losses in proportion to the amount of the premiums they pay to that particular company, and their fire funds consist of the reserve earnings and the results of investments.

340. Policies of insurance are of various kinds. The *ordinary* policy is a contract of indemnity, that is, a contract in which the amount paid in case of loss does not exceed a certain specified sum; this sum is determined by evidence after the loss occurs. A *valued* policy is one that states in advance the amount to be paid in case of loss.

Further subdivisions of policies are as follows: *specific*, one that covers a particular kind of property, as a single building; *blanket*, one that covers several items of property, as a group of buildings and the contents; *fixed*, one that covers property at some particular defined location; *floating*, one that covers specified property while in transit or in various defined localities; *open*, one which, while it affixes the extreme limit of the amount and duration of the risk, is yet open to secure endorsements granting insurance in various amounts and places at any time and for any period that may be agreed upon at the time of the endorsement; this policy is used largely to protect such stocks as grain in elevators or as the contents of warehouses, and the records are usually kept in a book known as an *open policy book*.

341. The **standard forms of contract** used in fire insurance policies are prescribed by the state.

These forms not only define the maximum amount and the term for which the company is liable but also the consideration paid by the insured,

the conditions under which the contract will become void, the methods to be followed in the settlement of a loss, and the procedure to effect the cancellation of the contract.

If a loss either total or partial occurs under such a policy, the company is bound to pay only so much of the sum stated in the policy as will indemnify the insured; *e.g.* if a building insured for $3000 is damaged by fire $400, only the actual loss, $400, can be recovered; but if the same building were damaged by fire $3500, the company could not be held for more than $3000, the sum stated in the policy.

342. Average and co-insurance clauses. Where a number of detached properties are insured under one policy, it is customary to attach what is known as an **average clause** which specifies that the amount of insurance covering any one particular piece of property shall bear such proportion to the total amount of insurance on the whole as the value of that special piece of property bears to the value of all of the properties so covered.

343. Many fire-insurance policies contain what is known as a **co-insurance, or a reduced-rate, clause.** Under this clause the insured party agrees to keep his property insured for a certain percentage of its value; failing to do this, the company or companies insuring him are liable only for that proportion of a loss which the amount they insure bears to the specified percentage of the sound value of the property covered.

Thus, the value of a piece of property is $10,000, and the insured agrees to keep it insured for 80% of its value, or $8000, but fails to do so and carries only $6000 insurance. Should a loss occur, the company will pay only three fourths ($\frac{6000}{8000}$) of the amount of such loss.

344. The **rate** in fire insurance is the amount to be paid to secure $100 of indemnity for one year.

The rate is based on the character of the risk; the greater the likelihood of fire the higher the rate.

When policies are written for a period of more than one year, a reduction is usually made in figuring the premium. Illustrations: on city dwellings the premium for five years is charged for four times the annual rate; if written for three years, for two and one-half times the annual rate.

Rates are expressed by the number of cents charged for $100 of insurance. When over $1 per hundred, the rate is often stated in dollars and cents.

Short rates are those used for a term of less than one year; they are proportionately higher than the annual rates.

ORAL EXERCISE

1. What is the cost of $6500 insurance at 80¢ per $100?
2. What is the premium on a $4000 policy at $1.50 per $100?
3. What is the cost of $6000 insurance at 75¢ per $100?
4. B insures a $6000 barn for $\frac{2}{3}$ value at 50¢ per $100. What quarterly premium should he pay?
5. A insures a $6000 house for $\frac{4}{5}$ value, at 50¢ per $100. What is the semiannual premium?
6. Goods worth $3000 are insured for $\frac{2}{3}$ value. If the annual premium is $30, what is the rate?
7. I insure $2400 worth of merchandise for $\frac{5}{6}$ of its value at 60¢ per $100. What premium must I pay?
8. I insure a stock of goods worth $8000 for $6000 at 2%. The goods become damaged by fire to the extent of $3000. Under an ordinary policy how much can I recover? What will be my net loss, premium included?
9. A brick schoolhouse is insured at 50¢ per $100, the annual premium is $50, and the face of the policy $\frac{5}{8}$ of the value of the building. What is the value of the building?

ORAL EXERCISE

State the premium in each of the following problems:

	Face of Policy	Rate		Face of Policy	Rate
1.	$1600	$1\frac{1}{2}$%	3.	$3500	$1.10 per $100
2.	$1000	$1\frac{1}{4}$%	4.	$5000	$1.20 per $100

State the face of the policy in each of the following problems:

	Premium	Rate		Premium	Rate
5.	$9	2%	7.	$13.50	$1.35 per $100
6.	$15	$1\frac{1}{2}$%	8.	$24.00	$1.60 per $100

State the rate of insurance in each of the following problems:

	Face of Policy	Premium		Face of Policy	Premium
9.	$1700	$25.50	11.	$3200	$130.00
10.	$1850	$37.00	12.	$6500	$40.00

345. The following is an extract from a tariff, or rate, book for the properties shown on the map which follows this schedule.

MAIN STREET, SOUTH SIDE

No.		Flat Rate	80% Rate
189	John Smith & Co.		
	Frame carriage factory	$1.75 *c*	$1.23 *c*
	Contents	1.75 *c*	1.23 *c*
193	John Smith		
	Frame dwelling	$0.25 *a*	——
197	Frame stable (private)	1.00 *a*	——
199	William Brown		
	Frame store and dwelling	$0.40 *c*	$0.28 *c*
	Contents of grocery store	0.40 *c*	0.28 *c*
	Contents of dwelling	0.40 *a*	0.28 *c*
203–205	James Robinson		
	Brick mercantile building	$0.70 *a*	$0.50 *a*
	Robinson & Co.		
	Department store	$0.70 *c*	$0.50 *c*
	Offices second and third floors	0.70 *c*	0.50 *c*

STATE STREET, NORTH SIDE

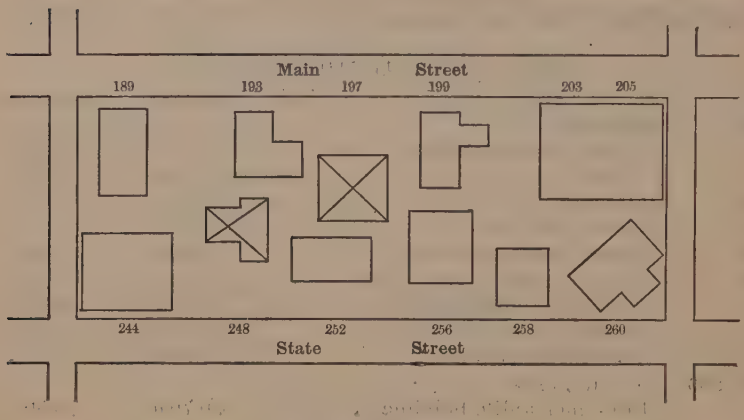
244	James Green		
	Brick store and dwelling	$0.25 *a*	$0.17½ *a*
	National Butter Co.	0.25 *c*	0.17½ *c*
	Dwelling	0.25 *c*	——
248	Thomas White		
	Frame stable	$1.00 *c*	$0.70 *c*
	White's Livery	1.00 *c*	0.70 *c*
252	Thomas White		
	Frame dwelling and contents	$0.25 *a*	$0.17½ *a*
256	Town of Jonesville		
	Brick high school	$0.50 *a*	$0.35 *a*
	Contents	0.50 *a*	0.35 *a*
258	Samuel Parker		
	Brick dwelling	$0.17 *a*	$0.12 *a*
260	State Street Baptist Society		
	Brick church building	$0.50 *a*	$0.35 *a*
	Organ and other contents	0.50 *a*	0.35 *a*

The letter *a* after the rate indicates that the insurance on this property can be written for more than one year; that is, at two and one-half times the rate, for a three-year policy, and at four times the rate, for a five-year policy.

The letter *c* after the rate indicates that the insurance on this property can be written for one year, or for a number of years, at yearly rates.

The city block, page 282, contains properties insured under the above schedule of rates.

DIAGRAM OF A CITY BLOCK



WRITTEN EXERCISE

These problems apply to the properties shown on the above diagram; also to the tariff of rates in the preceding schedule.

The flat rate is used unless the co-insurance clause is mentioned.

1. The frame carriage factory at 189 Main Street is worth $7000. The contents are worth $8000; both are insured at $\frac{4}{5}$ of their value. What is the amount of the annual premium?

2. The frame dwelling at 193 Main Street is worth $3400, and the contents, $1200. The frame stable owned by the same party at 197 Main Street is worth $1500, and the contents, $1100. All of this property is insured for 1 yr. at a $\frac{5}{6}$ valuation. What is the annual premium? What will it cost to insure it for 3 yr.?

3. The store and dwelling at 199 Main Street are worth $4800. The contents of the store are worth $2400, and of the dwelling, $800. What will it cost to insure the property for 1 yr.?

4. The brick mercantile building at 203–205 Main Street is worth $20,000. The contents of the first floor are worth $4500, and of the second and third floors, $7500. All are insured at a 75% valuation for 1 yr. What is the amount of the premium? A fire occurs, and the building and the contents are damaged to the extent of $4500. If the policies contained an 80% co-insurance clause, how much will the insuring company have to pay?

5. Suppose that the building described in problem 4 was insured in Company A for \$18,000 at the tariff rate, and the contents in Company B for \$10,000 at a rate of 75¢; that each company had an 80% co-insurance clause attached to its policy; that the building was damaged to the extent of \$3000, and the contents, \$2500. How much would each company have to pay? What would be the net loss to the owner of the building? to the owner of the contents? (Premium included in each case, but no interest.)

6. The brick church at 260 State Street is worth \$10,000, and the contents, \$3500. The property is insured for 1 yr. for \$8100. If the policy contains an 80% co-insurance clause, what is the net loss to the insurance company (premium included) if the property is wholly destroyed by fire?

7. If the brick school building at 256 State Street is worth \$15,000 and the contents are worth \$7500, what will it cost under the term rule to insure it for 5 yr. for 80% of its value?

8. For insuring the frame buildings at 252 and 248 State Street, and the contents of each for $\frac{3}{4}$ of their value, the owner pays an annual premium of \$22.50. If the frame stable and the contents are worth $\frac{2}{7}$ of the frame dwelling and the contents, what is the value of each building, including the contents?

9. The brick store and the dwelling at 244 State Street are worth \$15,000; the property is insured in three companies for $\frac{4}{5}$ of its value. Company A carries $\frac{1}{3}$ of the line at the tariff rate; Company B, $\frac{2}{5}$ of the line at a 50¢ rate; Company C, the remainder of the line at a $66\frac{2}{3}$¢ rate. What is the total premium paid? The building is damaged by fire to the amount of \$6000. What amount will each company pay?

10. I insured a block of buildings in the Ætna Insurance Company for \$75,000 at an annual rate of 75¢. The Ætna afterwards reinsured \$15,000 of its liability under my policy in the Continental Insurance Company at 75¢, and \$20,000 in the German American Insurance Company at the same rate. The building was damaged by fire \$20,000. What was the net loss of each of the three companies?

11. All of the Main Street buildings shown on the preceding diagram were purchased by one party for the following sums:

Frame carriage factory No. 189	$7,000
Frame dwelling at No. 193	3,400
Frame stable at No. 197	1,500
Frame store and dwelling at No. 199	4,500
Brick mercantile building at 203–205	20,000
	$36,400

It is proposed to insure them under a blanket form of policy at a 60¢ rate for $\frac{7}{8}$ of the cost. The policies have an average clause attached: Formerly these properties were insured separately at the tariff rates for $\frac{3}{4}$ of the above values. Will the proposed plan of insurance cost more or less, and how much for 1 yr.?

Information regarding the different kinds of policies is given on page 278; the student is referred to this page for a suggestion regarding the *blanket* form of policy, the form used in problem 11.

12. A certain man owns four grain warehouses, and carries an insurance of $20,000 on the contents of them all, with an average clause attached. At the time of a fire which damaged the contents of warehouse B $1500, and the contents of warehouse D $7000, it was found that the grain in each warehouse was of the following values: warehouse A, $3000; warehouse B, $6000; warehouse C, $8000; warehouse D, $10,000. What must the insuring company pay on the damaged stock in warehouse B? in warehouse D?

MARINE INSURANCE

346. Marine insurance is insurance against loss to ships and cargoes by perils of navigation.

347. In marine insurance, the policies usually contain a clause to the effect that if a vessel or cargo, or both, are valued at more than the amount insured, the insurers will pay only such part of the loss, either partial or total, as the amount insured bears to the full valuation. This clause is called an **average clause.**

Thus, should a vessel valued at $20,000, and insured for $15,000, become damaged by fire to the extent of $8000, under an average clause policy the company will pay three fourths ($\frac{15000}{20000}$) of the loss, or $6000. Should the same vessel and cargo be wholly destroyed, the company will pay the full $15,000, which is three fourths of the entire valuation. In order to be fully protected in a marine risk, the insured must insure his property for full value. Some fire insurance policies contain a clause similar to the average clause of marine insurance policies.

WRITTEN EXERCISE

1. A vessel valued at $50,000 is insured (average clause policy) for $18,000 in Company A, and for $17,000 in Company B. A fire occurs by which the vessel is damaged $15,000. What is the amount to be paid by each company?

2. I paid $25.40 for insuring a shipment of goods by steamer from Boston to Manila. If the rate was $1\frac{1}{4}\%$, less 20%, what was the face of the policy? If the face of the policy was equal to the value of the goods, what would it cost to make the shipment by sailing vessel at $1\frac{7}{8}\%$, less 20%?

3. You take out a $7500 average clause policy on your stock of merchandise worth $9000. The premium is 75¢ per $100, which you pay in advance. A fire occurs by which the stock is damaged $3000. Estimate your total loss and the net loss to the company. (Premium included in each case.)

4. A of Boston instructed B of Sidney, Australia, to purchase $25,000 worth of hides. B made the investment as instructed and charged $1\frac{7}{8}\%$ commission. The hides were then shipped by steamer and insured at $1\frac{1}{2}\%$ for enough to cover the value of the hides and all charges. What was the amount of the policy and what was the premium?

5. A of New York ordered B of Duluth to buy on commission 6000 bu. of wheat and 6000 bu. of corn. B bought the wheat at 92¢ and the corn at 57¢ per bushel, and charged $1\frac{1}{2}$¢ per bushel commission. Before shipping the grain to A by boat, B took out a policy of insurance at $1\frac{1}{2}\%$ to cover the cost of the goods and all charges. What was the agent's commission? the insurance premium? What did the grain cost A?

CHAPTER XXIII

STATE AND LOCAL TAXES

ORAL EXERCISE

1. How are the expenses of towns, cities, counties, and states met?

2. A has property worth $5000 and B property worth $10,000. How should the taxes of these two men compare?

3. Mention several purposes for which taxes are raised in your city or town.

348. A **tax** is a sum levied for the support of government, or for other public purposes. Taxes are of two kinds: **direct taxes**, which are taxes levied on a person, his property, or his business; **indirect taxes**, which are taxes levied on imported goods, and on tobacco, liquors, etc., produced and consumed in the United States.

The expenses of town, county, city, and state governments are met by *capitation* or *poll taxes*, *property taxes*, and *license fees*. The expenses of the National Government are met chiefly by *import duties*, or *customs*, and *excise duties*.

349. A **capitation**, or **poll tax**, is a tax sometimes levied on each male inhabitant who has attained his majority. A **property tax** is a tax levied on real estate or on personal property. A **license fee** is a tax paid for permission to engage in certain kinds of business.

Real estate and personal property belonging to religious or charitable organizations are frequently exempt from taxation.

350. Property taxes are imposed in nearly all the states by practically the same method, namely:

1. Officers called assessors are elected in every city and town, whose business it is to set a valuation upon all property subject to taxation.

2. In most of the states a County Board of Equalization reviews the original assessments, and the judgment of this body is subsequently passed upon by the State Board of Equalization.

3. All the taxes for state purposes are then equitably apportioned among the different counties, cities, and towns. Each county, city, town, and school district also levies taxes for its own local expenses.

351. The **tax rate** is expressed as so many mills on the dollar or so many dollars on a hundred or a thousand dollars.

The **Federal Income Tax** law was approved Oct. 13, 1913. It is a new provision for raising revenue to support the National Government. Only a few leading provisions are noted here.

Every citizen of the United States, whether residing at home or abroad, and every person residing in the United States, though not a citizen thereof, who has an annual net income from all sources in excess of $3000 will be required to pay a normal tax of 1% on such entire net income in excess of $3000 or in excess of $4000 if a person is married and living with a wife or a husband. The normal tax of 1% is also imposed on corporations, joint-stock companies or associations, and insurance companies.

For special information address any collector of internal revenue.

ORAL EXERCISE

1. If the rate of taxation is 12 mills on a dollar, how much tax must I pay on property assessed at $5000?

2. The tax rate is 13 mills on a dollar. B has property valued at $8000 and assessed at $\frac{3}{4}$ value. What is his tax?

3. C pays $1\frac{1}{4}\%$ tax on a city lot 100 ft. by 150 ft., valued at $1 per square foot, and assessed at $\frac{2}{3}$ value. What is the amount of his tax?

4. What tax must I pay on $80,000, at 5 mills on $1, the collector's commission being 1%?

SOLUTION. .005 of $80,000 = $400, the property tax.
1% of the tax = 4, the collector's commission.
$404, my total tax.

5. What tax must I pay on $10,000 at $4\frac{1}{2}$ mills on $1, the collector's commission being 1%?

6. An unmarried man has a net annual income of $4568. If exemptions are allowed amounting to $215, what income tax will he have to pay?

7. A married man living with his wife has a net annual income of $5432.50. If exemptions are allowed amounting to $384.25, what income tax will he pay?

8. A collector turns over to the county treasurer $8000. If his commission was $1\frac{1}{2}\%$ what amount did he collect? If the property taxed was worth $800,000, what was the rate of taxation? Express this rate in three ways.

9. The assessed valuation of real and personal property in a certain city is $400,000,000. The city has a bonded indebtedness of $2,000,000, on which it pays 4% interest. Find the tax rate necessary to pay the interest.

WRITTEN EXERCISE

Find the total tax:

1. Valuation, $3600; rate, $0.016; 3 polls at $2.

2. Valuation, $4550; rate, $9\frac{1}{2}$ mills; 1 poll at $1.50.

3. Valuation, $2875; rate, $0.0175; 1 poll at $1.75.

4. Valuation, $5600; rate, $\$1.12\frac{1}{2}$ per $100; 1 poll at $2.

5. Valuation, $6000; rate, $13.40 per $1000; 2 polls at $1.00.

Find the valuation:

6. Total tax, $3800; rate, $0.015; 100 polls at $2.00.

7. Total tax, $11,295; rate $1.40 per $100; 250 polls at $1.50.

8. Total tax, $8850; rate, $15.00 per $1000; 225 polls at $1.00.

9. In a town 1040 persons were subject to a poll tax; the assessed valuation of real estate was $3,209,400, and of personal property $265,100. The polls were taxed $1.25 each. The tax levy was $42,994. What was the tax rate? What was the total tax of Charles B. Lester, who owned real estate valued at $6450, and personal property valued at $1250, and who paid for 2 polls?

10. In a town taxes were levied as follows: state tax, $4287; county tax, $9312.50; town tax, $93,156.20. There were 1850 polls assessed at $2 each. If the total property valuation was $6,245,800, what was the tax rate per thousand?

11. A town made provision by taxation for the following expenses: public schools $18,180; interest on borrowed money $2106; public highways $4720; officials' salaries $4620; general expenses $11,746; sinking fund $8000. The value of real and personal property was $2,450,600, and 2120 polls were assessed $1.50 each; $4531.80 was collected from license fees. What was the tax rate?

12. A died leaving property valued at $47,950 to B, his son, and property valued at $17,500 to C, a friend. The statutes of the state in which these three live provide that B, a lineal heir, and C, a collateral heir, shall pay to the state an **inheritance tax.** The rate for lineal heirs is 1%, and for collateral heirs 5%. What inheritance tax must B and C, respectively, pay when they come into possession of the property?

13. A city made the following appropriation for its public schools: teaching and supervision, $36,000; care and cleaning, $3360; fuel, $3000; repairs, $2000; text-books, $1700; supplies, $1700; printing, $300; contingent fund, $775; truant officer, $500; evening schools, $1305; transportation of pupils, $600; kindergarten, $1100; manual training, $700. The assessed value of real estate was $6,709,998 and of personal property $2,130,002. What was the tax rate for school purposes?

14. An agent made the following report of his income as a basis for computing his income tax:

Salary per year	$3500	Interest on money loaned . .	$415
Commissions	985	Dividends on bank stock . .	50
Dividends on preferred stock in a corporation			$250

If the dividends received were exempt, the tax having been paid by the corporations, what would be his income tax if he were a married man living with his wife? What would be the income tax of an unmarried man having the same income?

352. In order to facilitate clerical work a table may be used for computing taxes. The following table was made from the published tax lists of a city in Massachusetts:

TAX TABLE. RATE $18.60 PER $1000

	0	1	2	3	4	5	6	7	8	9
0	.0000	.0186	.0372	.0558	.0744	.0930	.1116	.1302	.1488	.1674
1	.1860	.2046	.2232	.2418	.2604	.2790	.2976	.3162	.3348	.3534
2	.3720	.3906	.4092	.4278	.4464	.4650	.4836	.5022	.5208	.5394
3	.5580	.5766	.5952	.6138	.6324	.6510	.6696	.6882	.7068	.7254
4	.7440	.7626	.7812	.7998	.8184	.8370	.8556	.8742	.8928	.9114
5	.9300	.9486	.9672	.9858	1.0044	1.0230	1.0416	1.0602	1.0788	1.0974
6	1.1160	1.1346	1.1532	1.1718	1.1904	1.2090	1.2276	1.2462	1.2648	1.2834
7	1.3020	1.3206	1.3392	1.3578	1.3764	1.3950	1.4136	1.4322	1.4508	1.4694
8	1.4880	1.5066	1.5252	1.5438	1.5624	1.5810	1.5996	1.6182	1.6368	1.6554
9	1.6740	1.6926	1.7112	1.7298	1.7484	1.7670	1.7856	1.8042	1.8228	1.8414

In the table the rate on each $1000 was made up as follows: state tax $.0807; county tax, $.5643; state highways, $.003; city tax, $17.952. The first figure of the number of dollars assessed is given at the left, and the second one at the top.

353. Example. What is the tax on a valuation of $16,400?

SOLUTION. Tax on $16,000 = $297.60 (1000 times .2976)
Tax on 400 = 7.44 (100 times .0744)
Tax on $16,400 = $305.04

WRITTEN EXERCISE

Using the table, find the tax on the following valuations:

1. $2485.	**5.** $8,478.	**9.** $34,500.	**13.** $20,000.
2. $1200.	**6.** $13,200.	**10.** $82,500.	**14.** $27,800.
3. $1050.	**7.** $14,700.	**11.** $98,250.	**15.** $71,690.
4. $4630.	**8.** $18,400.	**12.** $21,850.	**16.** $89,800.

Find the tax on the following valuations when the collector's commission is 1 %:

17. $5500.	**21.** $9500.	**25.** $19,000.	**29.** $21,000.
18. $7500.	**22.** $8700.	**26.** $26,000.	**30.** $89,000.
19. $2900.	**23.** $6500.	**27.** $85,000.	**31.** $10,000.
20. $4700.	**24.** $7250.	**28.** $78,000.	**32.** $21,000.

CHAPTER XXIV

CUSTOMS DUTIES

ORAL EXERCISE

1. The expenses of the National Government average about $2,250,000 per day. What is this per year?

SUGGESTION. To multiply by 15, multiply by 10 and add $\frac{1}{2}$ of the result.

2. Name five sources of income to the National Government.

3. Name ten expense items of the National Government.

354. Duties, or **customs,** are taxes levied by the National Government on imported goods. They are imposed in two forms: ad valorem and specific. An **ad valorem duty** is a certain *per cent* levied on the net cost of the importation. A **specific duty** is a *fixed sum* levied on each article, or on each pound, ton, yard, or other standard measure, without regard to the cost.

Ad valorem duties are not computed on fractions of a dollar. If the cents of the net cost are less than fifty, they are rejected; if fifty or more than fifty, one dollar is added before computing the duty.

Some articles are subjected to both ad valorem and specific duties. Before specific duties are estimated allowance is usually made for tare and breakage. Specific duties are not computed on fractions of a unit. Fractions less than $\frac{1}{2}$ of a unit are rejected; fractions $\frac{1}{2}$ or more are counted a whole unit. The **long ton** of 2240 lb. is used in computing specific duties.

355. A **tariff** is a schedule exhibiting the different rates of duties imposed by Congress on imported articles. A **free list** is a schedule of imported articles exempt from duty.

356. A **customhouse** is an office established by the National Government for the collection of duties and the entry and clearance of vessels. A port at which a customhouse is established is called a **port of entry**; ports of entry and other ports are called **ports of delivery.**

The United States is divided into *customs districts*, each with a headquarters port. Goods arriving must be entered at the original port of entry; if consigned to an interior port, this entry is a transportation entry, but at the other port it may be entered either for consumption or the warehouse.

357. In the most important ports of the United States the customhouse business is distributed among three departments:

1. The **collector's office**, which takes charge of the entries and papers, issues the permits, and collects the duties.

2. The **surveyor's office**, which takes charge of the vessel and cargo, receives the permits, ascertains the quantities, and delivers the merchandise to the importer.

3. The **appraiser's office**, which examines imported merchandise and determines the dutiable value and the rate of duty on same.

One package of every invoice and one package, at least, out of every ten similar packages is sent to the appraiser's store for examination. Merchandise in bulk and all heavy and bulky packages uniform in size and quantity of contents are generally examined on the wharf.

358. A **manifest** is a memorandum, signed by the master of the vessel, showing the name of the vessel, its cargo, and the names and addresses of the consignors and consignees. An **invoice** is a detailed statement showing the particulars of the goods imported.

All invoices should be made out in the weights and measures of the country in which the goods are purchased; and if the goods are subject to an ad valorem duty, they must be invoiced in the currency of the country into which they are imported. Invoices over $100 must be certified before a United States consul, who causes three copies of the invoice to be made. One is sent to the collector of the port at which the goods are to be entered, one is kept on file in the consul's office, and one is sent to the importer.

When the merchandise is loaded on board the vessel the shippers are given a **bill of lading** which acknowledges the receipt of the several packages and agrees to deliver the same at destination. The vessel's commander keeps a copy of the bill of lading and from the several that have been issued makes out his manifest of cargo. The shippers mail the invoice and bill of lading to the purchaser, who fills out an entry therefrom and presents it and the invoice at the customhouse where the duties imposed by law on the several classes of merchandise are collected and a permit issued for the landing and delivery of the merchandise, subject to examination.

359. The **values of foreign coins** are periodically proclaimed by the Secretary of the Treasury, and these values must be taken in estimating duties unless a depreciation of the value of the foreign currency expressed in an invoice shall be shown by the consular certificate thereto attached. The following estimate of the values of foreign coins was recently proclaimed.

VALUES OF FOREIGN COINS

COUNTRY	STANDARD	MONETARY UNIT	VALUE IN U. S. GOLD
Brazil	Gold	Milreis	$ .546
Denmark, Norway, Sweden	Gold	Crown	.268
France, Belgium, Switzerland	Gold	Franc	.193
German Empire	Gold	Mark	.238
Great Britain	Gold	Pound sterling	4.866½
Japan	Gold	Yen	.498
Mexico	Gold	Peso	.498
Netherlands	Gold	Florin	.402
Philippine Islands	Gold	Peso	.500
Russia	Gold	Ruble	.515

The *lira* of Italy, and the *peseta* of Spain, are of the same value as the franc. The *dollar*, of the same value as our own, is the standard of the British possessions of North America, except Newfoundland.

360. Depositing goods in a government or bonded warehouse is called **warehousing.**

Many importers buy foreign goods in large quantities, withdraw a part of them, and store the remainder in the government warehouse. The goods so deposited may be taken out at any time in quantities not less than an entire package, or in bulk, if not less than one ton, by the payment of duties, storage, and labor charges. Foreign goods are sometimes bought three or four months earlier than they can be placed on the market and are stored in the government warehouse until they are seasonable. In this way importers are able to make better selections and they also get better terms and prices.

361. A **bonded warehouse** is a building provided for the storage of goods on which duties have not been paid.

The importer must give bond for the payment of duties on all goods stored in a bonded warehouse. Goods left in the government warehouse

beyond 3 yr. unclaimed are forfeited to the government and sold under the direction of the Secretary of the Treasury. Goods may be withdrawn from a bonded warehouse for export, or for transfer to a warehouse in another district, without the payment of duty.

362. The two common forms of entry under which duties are collected are known as **consumption entry** and **warehouse entry**. The former is used for merchandise entered for consumption; the latter for merchandise that is placed in a bonded warehouse under charge of the government storekeeper.

363. Excise duties are taxes levied on certain goods produced and consumed in the United States. If goods, on which either excise or import duties have been paid, are exported, the amount so paid is refunded. The amount refunded is called a **drawback.**

TABLE OF DUTIES ON CERTAIN IMPORTS

ARTICLE AND DESCRIPTION	DUTY	
	Specific	Ad Valorem
Axminster rugs		50%
Barley, 48 lb. to the bushel	15¢ per bu.	
Barley malt, 34 lb. to the bushel	25¢ per bu.	
Beans, 60 lb. to the bushel	25¢ per bu.	
Brussels carpets		25%
Books		15%
Butter	$2\frac{1}{2}$¢ per lb.	
Castile soap		10%
Cheese		20%
China and porcelain, undecorated		50%
Clocks		60%
Cotton tablecloths		35%
Hay	$2 per T.	
Ingrain carpets		20%
Knit woolens		35%
Manufactures of leather		30%
Manufactures of marble		45%
Plate glass, 16″ × 24″	6¢ per sq. ft.	
Pocket knives, value not over $1 per doz.		35%
Potatoes, 60 lb. to the bushel		10%
Saccharin	65¢ per lb.	
Silk dress goods		50%
Toilet soap, unperfumed		10%
Wheat	10¢ per bu.	
Window glass	$\frac{7}{8}$¢ per lb.	

FINDING A SPECIFIC DUTY

ORAL EXERCISE

Using the table on page 294, find the duty on:

1. 67,200 lb. of hay.
2. 48,000 lb. of barley.
3. 100 pc. plate glass 16″ x 24″.
4. 2400 lb. of window glass 10″ x 15″.
5. A quantity of butter weighing 1000 lb.
6. A shipment of wheat weighing 240,000 lb.
7. A quantity of saccharin weighing 2100 lb.; tare 100 lb.

WRITTEN EXERCISE

1. *Using the table on page 294, find the total duty on:*

2500 bu. potatoes; value, $1200.
1275 lb. toilet soap; value, $425.
30,000 bu. potatoes; value, $15,000.
96,000 lb. barley.
24,000 lb. beans.
136,000 lb. barley malt.

2. What is the duty on 175 bx. castile soap, each weighing 110 lb., if 5% is allowed for tare? Invoiced at 20¢ per pound.

3. Calculate the duty on 10 hogsheads of saccharin weighing 1060–105, 1040–105, 1160–112, 1240–120, 1180–116, 1100–102, 1090–101, 1100–100, 1005–100, 1210–118 lb., respectively.

4. Richard Roe & Co. imported from Canada 3750 bu. of potatoes invoiced at 20¢ per bushel. If the transportation and other charges amounted to $187.50, how much must be received per bushel for the potatoes in order to gain 25%?

FINDING AN AD VALOREM DUTY

ORAL EXERCISE

Find the total duty:

1. On 40 clocks invoiced at $4.50 each.
2. On 12 books invoiced at $1.50 each.
3. On 25 doz. pocket knives invoiced at 50¢ per doz.
4. On 100 sq. yd. ingrain carpet invoiced at $1 per yard.

WRITTEN EXERCISE

Find the duty on:

1. An Axminster rug, $12' \times 18'$, invoiced at £10.

For the values of foreign coins, see page 293.

2. A 200 lb. box of knit woolen goods invoiced at £100.

3. An importation of cotton table cloths invoiced at £100.

4. An importation of cotton table cloths invoiced at £ 255.

5. 300 bx. plate glass, each containing 25 plates $16'' \times 24''$.

6. 20 Axminster rugs, each $12' \times 18'$, invoiced at £8 6*s.* per rug.

7. An importation of Dresden china, undecorated, invoiced at 100 fr.

8. An invoice of knit woolens weighing 600 lb. and valued at £315 12*s.*

9. 200 blocks of marble, each $10' \times 4' \times 2'$, invoiced at 328,000 lira.

10. An importation of leather from Sweden invoiced at 6750 crowns.

11. 400 yd. of Brussels carpeting, $\frac{3}{4}$ yd. wide, invoiced at $2 per yard.

12. 4000 meters of Brussels carpeting, $\frac{3}{4}$ yd. wide, invoiced at 5 francs per meter.

A meter equals approximately 1.1 yd.

13. 4800 meters of silk dress goods, $\frac{3}{4}$ yd. wide, invoiced at 3.75 marks per meter.

14. A case of silk dress goods containing 200 yd., 1 yd. wide, invoiced at 1000 marks.

15. An invoice of leather goods from the Netherlands invoiced at 12,520 florins.

16. 5 cs. of silk dress goods, each containing 200 yd., $\frac{3}{4}$ yd. wide, invoiced at 20 marks per yard.

17. I bought an invoice of Swiss clocks, paying 10,750 fr. for them in Geneva. What was the total cost of the clocks, including the duty?

INVOICES AND ENTRIES

WRITTEN EXERCISE

1. At what price per pair must the lace curtains in the following invoice be sold in order to realize a gain of $33\frac{1}{3}\%$?

No. 427 *Manchester, England,* Dec. 15, *19*

Invoice of Lace

Shipped by WILLIAM P. FIRTH & CO.

In the Steamer Catalonia *To* R. H. White Company
Boston, Mass.

Marks	No.	Quantity	Articles and Description	Price	Extension	Amount
W	317	50 doz. pr.	Lace Curtains	3/2/6	***-**-**	
			Less 2%		*-**-**	
					***-**-**	
			Insurance and Freight		4-10-6	
			Packing and Carting		16-6	
					***-**-**	
					$***.**	
			60% ad valorem duty		***.**	$***.**

2. Find the total cost of the following invoice:

Antwerp, Belgium, Apr. 2, *19*

Messrs. A. T. Summers & Co.
New York City

Bought of SCHMIDT & WESTERFELDT

Terms 30 da.

S W	6	pc. Black Silk				
		39.00, 40.50, 39.00,				
		40.00, 41.00, 40.50	240	5 fr.	***	
		Insurance and freight			86	
		Cartage			6	fr. ***
						$***.**
		50% ad valorem duty				**.**
						$***.**

Do not compute duty on insurance and freight, nor on cartage.
39.00, 40.50, etc., above, equal the number of meters in each piece.

3. Copy the following invoice, supplying the missing terms:

Bradford, England, Dec. 5, 19

Invoice of Woolen Goods

Shipped by RADCLIFFE & SON

In the Steamship Winifredian To R. H. Stearns & Co.

Terms 30 da. Boston, Mass.

R/317	25	pc. Black Wool Crepon 68 69 69 68 69 60 55 60 56 54 60 60 60 68 68 60 45 65 65 55 60 65 65 60 60 1544 1/9	***	*	*			
		Consul's fee		14	10	***	*	*

4. If the foregoing invoice of goods were entered for immediate consumption, the following is the entry that would be made out. Complete the computation in the entry.

Manifest No. 650 Invoiced at Bradford, England, Dec. 15, 19

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by R. H. Stearns & Co. In the Steamer Winifredian

Fred M. Garson Master From Liverpool Arrived Jan. 5, 19

Mark	No.	Packages and Contents	Quantity	Free List	35% ad valorem	Duty	Total
9/13	318	2 cs. Woolen Cloth £135.1 @ $48665 35 % of $???	1544 yd. 256 lb.		135-2-0 $???.??	???.??	

5. How much will R. H. Stearns & Co. have to receive per yard for the foregoing goods in order to realize a gain of 25%?

6. In a recent year the receipts from customs duties were $318,000,000, and from excise duties, $344,000,000. The customs duties for this year were what per cent less than the excise duties? The excise duties were what per cent greater than the customs duties?

7. Find the dutiable value and compute the duty on the following entries of merchandise:

a.

Manifest No. 370 Invoiced at Paris, France, Dec. 12, 19__

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by John Doe & Son In the Steamer Columbia

F. E. Langdon Master From Havre Arrived Jan. 3, 19__

Mark	No.	Packages and Contents	Quantity	Free List	45% ad valorem	15% ad valorem	Duty	Total
E	18	1 cs. Silk Ribbon			375.75 fr.			
L	49	1 " Sewing Needles		28.00 fr.				
A	73	1 " Flat Steel Wire				275 fr.		
		45% of $???					??? ??	
		15% of $???					?? ??	??? ??

There is no duty charged on the value of the steel wire, nor on the quantity or value of the sewing needles; but the values of both of these quantities is reduced to United States money by the customhouse officials for statistical purposes.

b.

Manifest No. 715 Invoiced at Limoges, France, Dec. 11, 19__

INWARD FOREIGN ENTRY OF MERCHANDISE

Imported by Richard Roe & Co. In the Steamer Silesia

John J. Kennedy Master From Havre Arrived Jan. 3, 19__

Mark	No.	Packages and Contents	Quantity	Free List	55% ad valorem	25% ad valorem	Duty	Total
W.H.P.	21/139	2 cs. Decorated China			845.00 fr.			
C.O.L.	376/921	1 " Celluloid Combs	4 kilos			133.50 fr.		
		55% of $???					??? ??	
		25% of $??					?? ??	??? ??

1 kilogram equals about $2\frac{1}{5}$ avoirdupois pounds.

INTEREST AND BANKING

CHAPTER XXV

INTEREST

ORAL EXERCISE

1. A borrows $100 of B for 1 yr. At the end of the year what will A probably pay B besides the face of the loan?

2. C puts $100 in a savings bank and leaves it for 1 yr. What can he draw out at the end of the year besides the money deposited?

3. If you wished to borrow money of a bank in your town, what rate of interest would you have to pay?

4. If you loaned a man $500 for 1 yr., what would you require him to give you as evidence of the loan and security for its payment?

364. The compensation paid for the use of money is called **interest.** Interest is computed at a certain per cent of the sum borrowed. This per cent of interest is called the **rate,** and the sum upon which it is computed, the **principal.**

The rate of interest allowed by law is called the **legal rate.** Persons may agree to pay less than this rate, but not more, unless a higher rate by special agreement is permitted by statute. When an obligation is interest-bearing and no rate is mentioned, the legal rate will be understood. An agreement for interest greater than that allowed by law is called **usury.**

365. In the **commercial world,** 12 mo. of 30 da. each, or 360 da., are reckoned as 1 yr.

This method is not exact, but it is the most common because the most convenient. It has been legalized by statute in some states and is generally used in all the states.

SIMPLE INTEREST

The Day Method

ORAL EXERCISE

1. How many days in a commercial year?

2. What part of a year is 60 da.? 6 da.? What is the interest on \$1 for 1 yr. at 6%? for 60 da.? for 6 da.?

3. How do you find .01 of a number? .001 of a number? What is the interest on \$120 for 60 da. at 6%? for 6 da.?

4. State a short method for finding the interest on any principal for 60 da. at 6%; for 6 da.

5. 1 da. is what part of 6 da.? What is $\frac{1}{6}$ of .001? What is the interest on \$1200 for 1 da. at 6%? on \$180? on \$1500?

6. State a short method for finding the interest on any principal for 1 da. at 6%.

366. In the foregoing exercise it is clear that *0.001 of any principal is equal to the interest for 6 da. at 6%;* or *0.001 of any principal is equal to 6 times the interest for 1 da. at 6%.*

ORAL EXERCISE

1. Find the interest on each of the following for 6 da. at 6%.

a. \$250.	*e.* \$560.	*i.* \$678.	*m.* \$290.	*q.* \$890.
b. \$870.	*f.* \$435.	*j.* \$320.	*n.* \$150.	*r.* \$750.
c. \$358.	*g.* \$430.	*k.* \$100.	*o.* \$325.	*s.* \$580.
d. \$350.	*h.* \$470.	*l.* \$185.	*p.* \$990.	*t.* \$625.

2. Find the interest on each of the above amounts for 12 da. at 6%; for 18 da.; for 24 da.

3. Find the interest on each of the following for 1 da. at 6%.

a. \$360.	*e.* \$660.	*i.* \$600.	*m.* \$480.	*q.* \$840.
b. \$450.	*f.* \$900.	*j.* \$180.	*n.* \$780.	*r.* \$200.
c. \$300.	*g.* \$540.	*k.* \$720.	*o.* \$400.	*s.* \$330.
d. \$420.	*h.* \$240.	*l.* \$500.	*p.* \$120.	*t.* \$960.

4. Find the interest on each of the above amounts for 3 da. at 6%; for 2 da.

367. Example. Find the interest on \$450 for 54 da. at 6 %.

SOLUTION. Pointing off three places to the left gives \$0.45, or 6 times the interest for 1 da. Multiplying this result by 54 gives \$24.30, or 6 times the interest for 54 da. Dividing this result by 6 gives \$4.05, the required interest.

$54 \times \$0.45 = \$24.30$

$\$24.30 \div 6 = \$4.05$

By arranging the numbers as shown in the margin and canceling the work is greatly shortened.

$$\frac{\overset{9}{\cancel{54}} \times \$0.45}{\cancel{6}} = \$4.05$$

WRITTEN EXERCISE

At 6 % find the interest on each of the following problems. Reduce the time expressed in months and days to days.

	PRINCIPAL	TIME		PRINCIPAL	TIME		PRINCIPAL	TIME
1.	\$620	54 da.	7.	\$900.00	29 da.	13.	\$375.80	2 mo. 15 da.
2.	\$175	84 da.	8.	\$865.45	93 da.	14.	\$300.00	3 mo. 19 da.
3.	\$645	42 da.	9.	\$700.00	96 da.	15.	\$171.15	1 mo. 14 da.
4.	\$300	84 da.	10.	\$974.30	62 da.	16.	\$120.00	4 mo. 14 da.
5.	\$600	72 da.	11.	\$178.45	40 da.	17.	\$211.16	6 mo. 16 da.
6.	\$502	66 da.	12.	\$438.55	50 da.	18.	\$665.65	1 mo. 10 da.

ORAL EXERCISE

1. What is the interest on \$800 for 6 da. at 3 % ?

SOLUTION. 80¢ is the interest for 6 da. at 6 %. 3 % is $\frac{1}{2}$ of 6 %; therefore, $\frac{1}{2}$ of 80¢, or 40¢, is the interest for 6 da. at 3 %.

2. If the interest at 6 % is \$45, what is the interest for the same time at 3 % ? at 12 % ? at 2 % ? at 1 % ? at $1\frac{1}{2}$ % ?

3. Formulate a short method for changing 6 % interest to 8 % interest.

SOLUTION. 8 % is $\frac{1}{3}$ more than 6 %; hence, *the interest at 6 % increased by $\frac{1}{3}$ of itself equals the interest at 8 %.*

4. State a short method for changing 6 % interest to 7 % interest; to 5 % ; to 9 % ; to $7\frac{1}{2}$ % ; to $4\frac{1}{2}$ %.

5. If the interest at 6 % is \$120, what is the interest at 7 % ? at 5 % ? at 8 % ? at 4 % ? at $7\frac{1}{2}$ % ? at $4\frac{1}{2}$ % ?

368. In the foregoing exercise it is clear that *6% interest increased by $\frac{1}{2}$ of itself equals 9% interest; by $\frac{1}{3}$ of itself, 8% interest; by $\frac{1}{4}$ of itself, $7\frac{1}{2}$% interest; by $\frac{1}{6}$ of itself, 7% interest;* also that

6% interest decreased by $\frac{1}{3}$ of itself equals 4% interest; by $\frac{1}{4}$ of itself, $4\frac{1}{2}$% interest; by $\frac{1}{6}$ of itself, 5% interest; also that

6% interest divided by 2 equals 3% interest; by 3, 2% interest; by 6, 1% interest; by 4, $1\frac{1}{2}$% interest.

6% interest multiplied by 2 equals 12% interest.

6% interest is changed to 10% interest by dividing by 6 and removing the decimal point one place to the right; to any other rate by dividing by 6 and multiplying by the given rate.

WRITTEN EXERCISE

Using the exact number of days, find the interest on:

1. \$2500 from Sept. 18, 1915, to Feb. 6, 1916, at 9%; at $3\frac{1}{2}$%; at 4%; at 3%.

2. \$1700 from Nov. 20, 1915, to Jan. 16, 1916, at 8%; at $2\frac{1}{2}$%; at $5\frac{1}{2}$%; at $3\frac{1}{2}$%; at 4%.

3. \$2750 from Dec. 16, 1915, to Jan. 17, 1916, at 7%; at 2%; at 4%; at 5%; at 1%; at 10%.

4. \$6250 from Dec. 18, 1915, to Feb. 6, 1916, at $7\frac{1}{2}$%; at 10%; at $1\frac{1}{2}$%; at $4\frac{1}{2}$%; at 9%; at 8%; at 7%; at 3%.

The Banker's Sixty-Day Method

ORAL EXERCISE

1. 60 da. (2 mo.) is what part of a commercial year?

2. What is the interest on \$1 for 2 mo. at 6%? for 60 da.?

3. How can you find 0.01 of a number? What is the interest on \$50 for 60 da. at 6%? on \$370? on \$590? on \$214.55?

4. What fractional part of 60 da. is 30 da.? 20 da.? 15 da.? 10 da.? What is the interest on \$1680 for 60 da.? for 30 da.? for 20 da.? for 15 da.? for 10 da.?

5. State a simple way to find the interest on any principal for 60 da. at 6%; for 30 da.; for 20 da.; for 15 da.; for 10 da.

6. Read aloud the following, supplying the missing words:

a. 60 da. minus $\frac{1}{12}$ of itself equals 55 da.; 60 da. minus —— of itself equals 50 da.; 60 da. minus —— of itself equals 40 da.; 60 da. minus —— of itself equals 45 da.

b. 60 da. plus $\frac{1}{12}$ of itself equals 65 da.; 60 da. plus —— of itself equals 70 da.; 60 da. plus —— of itself equals 75 da.; 60 da. plus —— of itself equals 80 da.; 60 da. plus —— of itself equals 90 da.

7. What is the interest on \$600 for 60 da. at 6%? for 55 da.? for 50 da.? for 40 da.? for 45 da.?

8. What is the interest on \$1200 for 60 da.? for 65 da.? for 70 da.? for 75 da.? for 80 da.? for 90 da.?

9. State a short way to find the interest at 6% for 80 da.; for 90 da.; for 50 da.; for 65 da.; for 55 da.; for 75 da.; for 70 da.; for 40 da.; for 45 da.

369. In the above exercise it is clear that *removing the decimal point two places to the left in the principal gives the interest for 60 da. at 6%.*

370. Examples. 1. Find the interest on \$1950 for 20 da. at 6%.

SOLUTION. Removing the decimal point two places to the left gives the interest for 60 da. 20 da. is $\frac{1}{3}$ of 60 da. $\frac{1}{3}$ of \$19.50 = \$6.50.

\$19.50
\$6.50

2. What is the interest on \$8400.68 for 75 days?

SOLUTION. Removing the decimal point two places to the left gives the interest for 60 da. 75 da. is 60 da. increased by $\frac{1}{4}$ of itself; therefore, \$84.0068 increased by $\frac{1}{4}$ of itself or \$105.01 is the required interest. In the following exercise determine the separate interest mentally whenever it is possible to do so.

\$84.0068
21.0017
\$105.0085, or \$105.01

WRITTEN EXERCISE

1. Find the total amount of interest at 6% on:

\$8400 for 60 da.	\$8400 for 12 da.	\$7900 for 20 da.
\$8400 for 30 da.	\$8400 for 10 da.	\$7900 for 15 da.
\$8400 for 20 da.	\$7900 for 60 da.	\$7900 for 12 da.
\$8400 for 15 da.	\$7900 for 30 da.	\$7900 for 10 da.

2. Find the total amount of interest at 6% on:

$1600 for 60 da.	$1600 for 40 da.	$2800 for 75 da.
$1600 for 55 da.	$2800 for 60 da.	$2800 for 80 da.
$1600 for 50 da.	$2800 for 65 da.	$2800 for 90 da.
$1600 for 45 da.	$2800 for 70 da.	$7200 for 55 da.

3. Find the total amount of interest at 6% on:

$1500.60 for 30 da.	$832.60 for 90 da.	$8575.65 for 70 da.
$1800.72 for 20 da.	$720.18 for 10 da.	$6282.40 for 15 da.
$1200.64 for 15 da.	$440.70 for 40 da.	$1460.84 for 65 da.
$8400.60 for 10 da.	$479.64 for 50 da.	$1385.62 for 55 da.

4. Find the total amount of interest at 6% on:

$1800.40 for 90 da.	$7500.00 for 55 da.	$216.90 for 20 da.
$9200.50 for 80 da.	$8200.00 for 75 da.	$432.65 for 15 da.
$3240.64 for 70 da.	$6400.00 for 45 da.	$832.30 for 10 da.
$4125.18 for 45 da.	$1200.45 for 30 da.	$926.17 for 20 da.

ORAL EXERCISE

1. What is the interest on $215 for 6 da. at 6%? on $345? on $415? on $827.50? on $425.90? on $4520.60? State a simple way to find the interest on any principal for 6 da. at 6%.

2. What part of 6 da. is 3 da.? is 2 da.? is 1 da.? What is the interest on $720 for 6 da.? for 3 da.? for 2 da.? for 1 da.? State a brief method of finding the interest on any principal for 3 da. at 6%; for 2 da.; for 1 da.

3. Read aloud the following, supplying the missing words:

a. 6 da. minus $\frac{1}{6}$ of itself equals 5 da.; 6 da. minus —— of itself equals 4 da.

b. 6 da. plus $\frac{1}{6}$ of itself equals 7 da.; 6 da. plus —— of itself equals 8 da.; 6 da. plus —— of itself equals 9 da.

c. State a short method of finding the interest at 6% for 4 da.; for 5 da.; for 7 da.; for 8 da.; for 9 da.

371. In the above exercise it is clear that *removing the decimal point in the principal three places to the left gives the interest for 6 da. at 6%.*

372. Example. What is the interest on \$420 for 8 da. at 6%?

SOLUTION. Removing the decimal point three places to the left gives the interest for 6 da., or \$0.42. Since 8 da. is 6 da. plus $\frac{1}{3}$ of itself, \$0.42 increased by $\frac{1}{3}$ of itself, or \$0.56 is the required interest. In the following exercises determine the separate interests mentally whenever it is possible to do so.

\$.420
.140
\$.56

WRITTEN EXERCISE

1. Find the total amount of interest at 6 % on:

\$800 for 6 da.	\$720 for 6 da.	\$1500 for 6 da.
\$800 for 3 da.	\$720 for 7 da.	\$1500 for 5 da.
\$800 for 2 da.	\$720 for 8 da.	\$1500 for 4 da.
\$800 for 1 da.	\$720 for 9 da.	\$1500 for 9 da.

2. Find the total amount of interest at 6 % on:

\$1168 for 6 da.	\$1600 for 6 da.	\$2400 for 6 da.
\$1168 for 3 da.	\$1600 for 7 da.	\$2400 for 5 da.
\$1168 for 2 da.	\$1600 for 8 da.	\$2400 for 4 da.
\$1168 for 1 da.	\$1600 for 9 da.	\$2400 for 8 da.

3. Find the total amount of interest at 6 % on:

\$640.50 for 8 da.	\$800.10 for 7 da.	\$213.80 for 50 da.
\$920.10 for 20 da.	\$240.80 for 90 da.	\$310.40 for 40 da.
\$280.40 for 15 da.	\$960.70 for 70 da.	\$135.90 for 10 da.
\$390.60 for 50 da.	\$845.60 for 90 da.	\$736.18 for 10 da.

ORAL EXERCISE

1. 600 da. is how many times 60 da.? If the interest on \$1 for 60 da. at 6 % is \$0.01, what is the interest for 600 da.?

2. Give a rapid method for finding 0.1 of a number. What is the interest on \$500 for 600 da. at 6 %? on \$350? on \$214.60? on \$359.80? on \$4500? on \$9243.80? on \$750? on \$2150?

3. What part of 600 da. is 300 da.? 200 da.? 150 da.? 75 da.? 120 da.? 100 da.? 50 da.?

4. What is the interest on \$1400 for 600 da.? for 300 da.? for 200 da.? for 150 da.? for 75 da.? for 120 da.? for 100 da.? for 50 da.?

5. State a brief method of finding the interest for 600 da. at 6%; for 300 da.; for 200 da.; for 75 da.; for 50 da.; for 150 da.; for 100 da.

6. If the interest on \$1 for 600 da. is \$0.10, what is the interest for 6000 da.? In how many days will any principal double itself at 6% interest?

7. What is the interest on \$1 for 6000 da. at 6%? on \$55? on \$75.60? on \$18.90? on \$350? on \$725? on \$9125.70.

8. What is the interest on each of the amounts in problem 7 for 3000 da.? for 2000 da.? for 1000 da? for 1500 da.?

9. What is the interest on \$2500 for 6000 da.? on \$2150? on \$7500? on \$790? on \$155.60?

10. What is the interest on each of the amounts in problem 9 for 6 da.? for 60 da.? for 600 da?

373. In the above exercise it is clear that *removing the decimal point in the principal one place to the left gives the interest for 600 da. at 6%; also that any sum of money will double itself in 6000 da. at 6%.*

WRITTEN EXERCISE

Find the interest at 6% on:

1. \$240 for 3000 da.
2. \$318 for 6000 da.
3. \$912 for 2000 da.
4. \$316 for 1500 da.
5. \$7420.50 for 600 da.
6. \$67218.90 for 30 da.
7. \$8400.50 for 400 da.
8. \$7500.79 for 1500 da.
9. \$1640 for 150 da.
10. \$1260.60 for 1 da.
11. \$17890 for 10 da.
12. \$1696 for 100 da.

ORAL EXERCISE

1. How many times is 6 da. contained in 18 da.? in 24 da.? in 36 da.? in 42 da.? in 54 da.? in 48 da.?

2. What is the interest on \$150 for 6 da.? for 18 da.? for 48 da.? for 54 da.? for 36 da.? for 42 da.? for 12 da.?

3. What is the interest on \$350 for 60 da.? for 180 da.? for 240 da.? for 360 da.? for 420 da.? for 480 da.?

374. **Example.** Find the interest on \$375 for 48 da. at 6%.

SOLUTION. $37\frac{1}{2}$¢ equals the interest for 6 da. 48 da. is 8 times 6 da. Therefore, the interest for 48 da. is 8 times $37\frac{1}{2}$¢, or \$3.

\$0.375
\$3.000

WRITTEN EXERCISE

1. Find the total amount of interest at 6 % on:

$750 for 6 da.	$750 for 36 da.	$750 for 60 da.
$750 for 12 da.	$750 for 42 da.	$750 for 180 da.
$750 for 18 da.	$750 for 48 da.	$750 for 240 da.

2. Find the total amount of interest at 6 % on:

$725 for 18 da.	$690 for 6 da.	$450 for 540 da.
$824 for 36 da.	$129 for 60 da.	$727 for 180 da.
$729 for 42 da.	$475 for 600 da.	$286 for 240 da.
$850 for 54 da.	$8600 for 54 da.	$429 for 420 da.

3. Find the total amount of interest at 6 % on:

$317.40 for 240 da.	$217.18 for 18 da.	$360.40 for 24 da.
$218.60 for 180 da.	$420.50 for 24 da.	$860.50 for 48 da.
$419.80 for 420 da.	$240.70 for 540 da.	$900.60 for 66 da.
$425.60 for 120 da.	$290.60 for 180 da.	$400.80 for 84 da.

375. In some cases it is advisable to find the interest on the principal for 1 da. and then multiply by the number of days.

ORAL EXERCISE

1. What is the interest on $600 for 17 da. at 6 %?

SOLUTION. The interest for one day is .000$\frac{1}{6}$ of the principal, or 10¢. The interest for 17 da. is 17 times 10¢, or $1.70.

2. What is the interest on $6000 for 49 da. at 6%? on $300? on $240? on $3000? on $1800? on $840? on $600?

3. State the interest at 6% on:

a. $600 for 19 da.	*e*. $6000 for 37 da.	*i*. $ 900 for 17 da.
b. $300 for 37 da.	*f*. $3000 for 43 da.	*j*. $1500 for 40 da.
c. $240 for 43 da.	*g*. $2400 for 67 da.	*k*. $ 600 for 139 da.
d. $180 for 27 da.	*h*. $1800 for 89 da.	*l*. $ 300 for 179 da.

376. Frequently it is well to mentally divide the days into convenient parts of 6 or 60.

Thus, 97 da. = 60 da. + 30 da. + 6 da. + 1 da.; 71 da. = 60 da. + 10 da. + 1 da.; 49 da. = 8 times 6 da. + 1 da.

ORAL EXERCISE

Separate the days in the following exercise into 6 da. or 60 da., or into convenient parts of 6 da. or 60 da.

1. 8 da.	**7.** 7 da.	**13.** 86 da.	**19.** 17 da.
2. 67 da.	**8.** 22 da.	**14.** 55 da.	**20.** 25 da.
3. 27 da.	**9.** 11 da.	**15.** 84 da.	**21.** 85 da.
4. 13 da.	**10.** 63 da.	**16.** 14 da.	**22.** 89 da.
5. 72 da.	**11.** 37 da.	**17.** 97 da.	**23.** 19 da.
6. 43 da.	**12.** 23 da.	**18.** 99 da.	**24.** 29 da.

377. Examples. **1.** Find the interest on \$840 for 31 da. at 6%.

SOLUTION. 31 da. = 30 da. + 1 da. The interest for 60 da. is \$8.40 and for 30 da. $\frac{1}{2}$ of this sum or \$4.20. The interest for 6 da. is \$0.84 and for 1 da. $\frac{1}{6}$ of this sum or \$0.14. Adding \$4.20 and \$0.14 the result is the required interest, or \$4.34.

\$8.40
\$4.20
.14
\$4.34

2. What is the interest on \$2500 for 121 da. at 6%?

SOLUTION. 121 da. = $\overline{2 \times 60 \text{ da.}}$ + 1 da. The interest for 60 da. is \$25 and for 120 da. twice this sum, or \$50. The interest for 6 da. is \$2.50 and for 1 da. $\frac{1}{6}$ of this sum, or \$0.42. Adding \$50 and \$0.42 the result is \$50.42, the required interest.

\$25.00
\$50.00
.42
\$50.42

WRITTEN EXERCISE

Find the interest:

	Principal	Time	Rate		Principal	Time	Rate
1.	\$420	3 mo.	6%	**11.**	\$450	4 mo.	$4\frac{1}{2}$%
2.	\$650	4 mo.	5%	**12.**	\$600	2 mo.	5%
3.	\$360	92 da.	4%	**13.**	\$720	8 mo.	3%
4.	\$250	30 da.	3%	**14.**	\$840	2 mo.	$1\frac{1}{2}$%
5.	\$380	24 da.	7%	**15.**	\$120	7 mo.	6%
6.	\$900	55 da.	6%	**16.**	\$280	9 mo.	$3\frac{1}{2}$%
7.	\$550	47 da.	3%	**17.**	\$885.90	20 da.	3%
8.	\$800	29 da.	5%	**18.**	\$240.00	21 da.	6%
9.	\$400	90 da.	4%	**19.**	\$420.18	25 da.	$2\frac{1}{2}$%
10.	\$270	11 da.	1%	**20.**	\$560.17	27 da.	6%

378. It has been observed that 6 times \$800 = 800 times \$6; that 0.01 of \$715 = 715 times \$0.01; etc. Hence,

379. *The principal in dollars and the time in days may be interchanged without affecting the amount of interest.*

380. Example. Find the interest on \$600 for 179 da. at 6 %.

SOLUTION. \$600 for 179 da. = \$179 for 600 da.; $\frac{1}{10}$ of the principal equals the interest for 600 da.; $\frac{1}{10}$ of \$179 = \$17.90, the required interest.

ORAL EXERCISE

State the interest at 6 % on:

1. \$60 for 27 da.
2. \$30 for 13 da.
3. \$20 for 171 da.
4. \$10 for 186 da.
5. \$15 for 145 da.
6. \$12 for 179 da.
7. \$10 for 131 da.
8. \$100 for 120 da.
9. \$200 for 189 da.
10. \$150 for 192 da.
11. \$360 for 91 da.
12. \$420 for 87 da.
13. \$540 for 21 da.
14. \$660 for 37 da.
15. \$750 for 56 da.
16. \$3600 for 218 da.
17. \$2000 for 183 da.
18. \$1200 for 155 da.
19. \$1800 for 181 da.
20. \$2400 for 218 da.

381. \$1500 on interest for 24 da. at 8 % = \$2000 (\$1500 + $\frac{1}{3}$ of itself) on interest for 24 da. at 6 %, or \$1500 on interest for 32 da. (24 da. + $\frac{1}{3}$ of itself) at 6 %. Hence,

382. *If either the principal or the time is increased or decreased by any fraction of itself, the interest is increased or decreased by the same fraction.*

383. Examples. 1. Find the interest on \$480 for 279 da. at $7\frac{1}{2}$ %.

SOLUTION. $7\frac{1}{2}$ % is $\frac{1}{4}$ more than 6 %. Increase the principal by $\frac{1}{4}$ of itself, and the result is \$600. Interchanging dollars and days, the problem is "Find the interest on \$279 for 600 da." Pointing off one place in the new principal, the result is \$27.90, the required interest.

2. Find the interest on \$2795.84 for 80 da. at $4\frac{1}{2}$ %.

SOLUTION. $4\frac{1}{2}$% is $\frac{1}{4}$ less than 6% interest. 80 da. decreased by $\frac{1}{4}$ of itself equals 60 da. The interest on \$2795.84 for 60 da. = \$27.96, the required result.

ORAL EXERCISE

State the interest on:

1. \$279.86 for 45 da. at 4 %.
2. \$478.65 for 45 da. at 4 %.
3. \$769.64 for 48 da. at $7\frac{1}{2}$ %.
4. \$217.49 for 80 da. at $4\frac{1}{2}$ %.
5. \$767.53 for 80 da. at $4\frac{1}{2}$ %.
6. \$2400 for 39 da. at 5 %.
7. \$2700 for 37 da. at 4 %.
8. \$2400 for 87 da. at $4\frac{1}{2}$ %.
9. \$1600 for 95 da. at $4\frac{1}{2}$ %.
10. \$3200 for 59 da. at $4\frac{1}{2}$ %.

The Six Per Cent Method

384. This method is best adapted to finding the interest when the time is *one year*, or *more than one year*.

ORAL EXERCISE

1. If the interest on \$1 for 1 yr. at 6 % is \$0.06, what is the interest on \$1 for 2 yr.? for 3 yr.? for 4 yr.? for 6 yr.? for 8 yr.? for 10 yr.?

2. If the interest on \$1 for 1 yr. at 6 % is \$0.06, what is the interest on \$1 for 1 mo.? for 2 mo.? for 3 mo.? for 6 mo.? for 10 mo.? for 7 mo.? for 8 mo.?

3. What is the interest on \$1 for 1 yr. 6 mo. at 6 %? for 2 yr. 6 mo.? for 3 yr. 4 mo.? for 3 yr. 6 mo.? for 4 yr. 8 mo.? for 1 yr. 10 mo.? for 5 yr. 6 mo.? for 2 yr. 9 mo.?

4. What is the interest on \$50 for 1 yr. at 6 %? for 1 yr. 6 mo.? for 2 yr.? for 3 yr. 6 mo.? for 2 yr. 8 mo.? for 1 yr. 10 mo.? for 2 yr. 6 mo.? for 4 yr. 6 mo.? for 1 yr. 9 mo.?

5. If the interest on \$1 for 1 mo. at 6 % is \$0.005 (5 mills), what is the interest for 1 da.? for 2 da.? for 3 da.? for 4 da.? for 6 da.? for 12 da.? for 18 da.? for 28 da.? for 24 da.?

6. What is the interest on \$1 for 1 yr. 1 mo. 1 da. at 6 %? for 2 yr. 3 mo. 3 da.? for 1 yr. 10 mo. 6 da.? for 4 yr. 4 mo. 24 da.? for 1 yr. 5 mo. 12 da.? for 2 yr. 1 mo. 1 da.?

385. In the above exercise it is clear that:

\$0.06 = *interest on* \$1 *for* 1 *yr. at* 6 %.
\$0.005 = *interest on* \$1 *for* 1 *mo. at* 6 %.
\$0.000$\frac{1}{6}$ = *interest on* \$1 *for* 1 *da. at* 6 %.

ORAL EXERCISE

Find the interest on $1 at 6% for:

1. 1 yr. 4 mo. 12 da.
2. 1 yr. 8 mo. 18 da.
3. 1 yr. 7 mo. 24 da.
4. 1 yr. 9 mo. 27 da.
5. 2 yr. 6 mo. 6 da.
6. 3 yr. 4 mo. 9 da.
7. 5 yr. 3 mo. 3 da.
8. 4 yr. 8 mo. 4 da.

Find the interest at 6% on:

9. $250 for 2 yr.
10. $400 for 5 yr.
11. $700 for 4 yr.
12. $300 for 3 yr. 4 mo.
13. $500 for 4 yr. 2 mo.
14. $350 for 3 yr.
15. $450 for 2 yr. 3 mo.
16. $150 for 1 yr. 6 mo.
17. $50 for 1 yr. 2 mo. 6 da.
18. $10 for 2 yr. 6 mo. 6 da.

386. Example. What is the interest on $600 for 2 yr. 8 mo. 15 da. at 6% ?

Solution. Find the interest on $1 for 2 yr.; on $1 for 8 mo.; on $1 for 15 da. The sum of these interest items equals $0.1625, the interest on $1 for the given time at 6%. Multiplying this interest by the given number of dollars, 600, the product is the required interest, $97.50. Change to any other rate as in § 362.

$0.12 = int. on $1 for 2 yr.
.04 = int. on $1 for 8 mo.
.0025 = int. on $1 for 15 da.
$0.1625 = int. on $1 for the given time.
600 × $0.1625 = $97.50, int. on $600 for 2 yr. 8 mo. 15 da. at 6%.

Sometimes it is shorter to find the interest on $1 for the given time at any given rate, and multiply by the number of dollars in the principal. Thus to find the interest on $400 for 2 yr. 6 mo. at 8%, take 400 times 20¢ ($2\frac{1}{2} \times 8¢$); on $500 for 5 yr. 3 mo. at 4%, take 500 times 21¢ ($5\frac{1}{4} \times 8¢$); on $600 for 1 yr. 9 mo. at 4% take 600 times 7¢; etc.

ORAL EXERCISE

Find the interest:

	Principal	Time	Rate		Principal	Time	Rate
1.	$400	1 yr. 2 mo.	6%	7.	$840	1 yr. 6 mo.	6%
2.	$500	2 yr. 4 mo.	6%	8.	$100	3 yr. 6 mo.	5%
3.	$300	4 yr. 6 mo.	6%	9.	$960	4 yr. 2 mo.	6%
4.	$250	1 yr. 8 mo.	6%	10.	$300	3 yr. 4 mo.	3%
5.	$200	2 yr. 10 mo.	3%	11.	$240	2 yr. 6 mo.	4%
6.	$300	1 yr. 11 mo.	6%	12.	$180	1 yr. 8 mo.	6%

WRITTEN EXERCISE

Find the interest:

1. Principal, $74.90; time, 6 mo. 15 da.; rate, 4%.
2. Principal, $986.00; time, 11 mo. 28 da.; rate, $4\frac{1}{2}$%.
3. Principal, $1900.00; time, 10 mo. 28 da.; rate, 6%.
4. Principal, $87.55; time, 4 yr. 5 mo. 6 da.; rate, 6%.
5. Principal, $735.00; time, 4 yr. 4 mo. 4 da.; rate, 6%.
6. Principal, $609.50; time, 2 yr. 7 mo. 6 da.; rate, 6%.
7. Principal, $875.40; time, 1 yr. 2 mo. 21 da.; rate, $4\frac{1}{2}$%.
8. Principal, $1124.75; time, 1 yr. 5 mo. 14 da.; rate, 6%.
9. Principal, $1245.00; time, 3 yr. 6 mo. 23 da.; rate, 6%.
10. Principal, $1570.00; time, 1 yr. 9 mo. 25 da.; rate, 5%.

The Reference Method

387. This method employs a series of tables in which interest computations are already worked out, and by the use of which the interest may be found on any sum, at given rates, for any time.

This method is used in banks, insurance offices, and kindred institutions, and it greatly lessens the work of computing interest. Many different systems are published, but the section of an interest table given on page 314 will illustrate the general plan followed.

ORAL EXERCISE

1. What is the interest (use the table, page 314) on $8 for 5 da.? on $80? ($10 \times \$8$); on $800? on $8000?
2. What is the interest on $10 for 7 da.? on $100? on $1000? on $10,000? on $70 for 5 da.? on $700? on $7000?
3. What is the interest on $4 for 11 mo.? on $40 for the same time? on $400? on $4000? on $50,000 for 7 mo.?

388. Example. Find the interest on $9980 for 7 da. at 6%.

SOLUTION: By the table, $10.50 = interest on $9000.
1.05 = interest on $900.
.09 = interest on $80.
$11.64 = interest on $9980.

INTEREST TABLE AT 6%

TIME	$1	$2	$3	$4	$5	$6	$7	$8	$9	$10	TIME
1 da.	.00017	.00033	.0005	.00067	.00083	.001	.00117	.00133	.0015	.00167	1 da.
2 da.	.00033	.00067	.001	.00133	.00167	.002	.00233	.00267	.003	.00333	2 da.
3 da.	.0005	.001	.0015	.002	.0025	.003	.0035	.004	.0045	.005	3 da.
4 da.	.00067	.00133	.002	.00267	.00333	.004	.00467	.00533	.006	.00667	4 da.
5 da.	.00083	.00167	.0025	.00333	.00417	.005	.00583	.00667	.0075	.00833	5 da.
6 da.	.001	.002	.003	.004	.005	.006	.007	.008	.009	.01	6 da.
7 da.	.00117	.00233	.0035	.00467	.00583	.007	.00817	.00933	.0105	.01167	7 da.
8 da.	.00133	.00267	.004	.00533	.00667	.008	.00933	.01067	.012	.0133	8 da.
9 da.	.0015	.003	.0045	.00667	.0075	.009	.0105	.012	.0135	.015	9 da.
10 da.	.00167	.00333	.005	.00667	.00833	.01	.01167	.01333	.015	.01667	10 da.
20 da.	.00333	.00667	.01	.01333	.01667	.02	.02333	.02667	.03	.03333	20 da.
1 mo.	.005	.01	.015	.02	.025	.03	.035	.04	.045	.05	1 mo.
2 mo.	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	2 mo.
3 mo.	.015	.03	.045	.06	.075	.09	.105	.12	.135	.15	3 mo.
4 mo.	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	4 mo.
5 mo.	.025	.05	.075	.10	.125	.15	.175	.20	.225	.25	5 mo.
6 mo.	.03	.06	.09	.12	.15	.18	.21	.24	.27	.30	6 mo.
7 mo.	.035	.07	.105	.14	.175	.21	.245	.28	.315	.35	7 mo.
8 mo.	.04	.08	.12	.16	.20	.24	.28	.32	.36	.40	8 mo.
9 mo.	.045	.09	.135	.18	.225	.27	.315	.36	.405	.45	9 mo.
10 mo.	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	10 mo.
11 mo.	.055	.11	.165	.22	.275	.33	.385	.44	.495	.55	11 mo.
1 yr.	.06	.12	.18	.24	.30	.36	.42	.48	.54	.60	1 yr.
2 yr.	.12	.24	.36	.48	.60	.72	.84	.96	1.08	1.20	2 yr.

WRITTEN EXERCISE

Using the table, find the interest on:

1. $8800 for 4 da.
2. $9600 for 5 da.
3. $7500 for 7 mo.
4. $8500 for 11 mo.
5. $17,000 for 1 da.
6. $29,000 for 1 da.
7. $71,000 for 7 da.
8. $87,000 for 11 da.

PROMISSORY NOTES

389. A written promise to pay a certain sum of money on demand, or at a specified time, is called a **promissory note.**

$ 243.50 New York, Jan. 6, 19

Two months after date I promise to pay to

the order of William B. Harris

Two Hundred Forty-three 50/100 Dollars

at First National Bank with int. at 5%

Value received

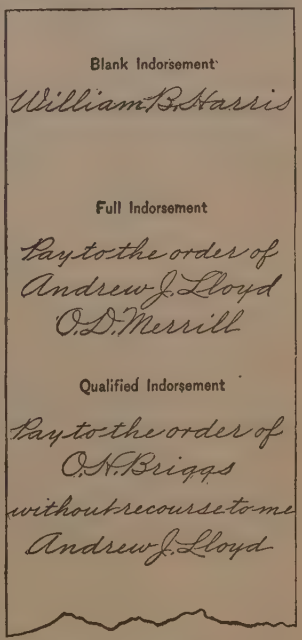
No. 64 Due 3/6/ Ellis B. Pitkin

390. In the foregoing note Ellis B. Pitkin is the **maker**; William B. Harris, the **payee**; and $243.50, the **face**. The note is **negotiable**; that is, it may be transferred by the payee to any other person by indorsement.

If the note were drawn payable to William B. Harris, *or bearer*, it would be transferable by delivery and would be negotiable. If the words *to the order of* were omitted, the note would not be transferable either by indorsement or by delivery; it would be payable to William B. Harris only, and would be called a **non-negotiable** note.

391. If the payee should sell the foregoing note, he would have to indorse it; that is, make it payable to the buyer by a writing on the back of the instrument. This **indorsement** may be made in either of the three ways shown in the margin.

William B. Harris sold the note to O. D. Merrill and effected the transfer by a **blank indorsement**. This is simply William B. Harris's signature. It makes the note payable to bearer. O. D. Merrill sold the note to Andrew J. Lloyd and effected the transfer by a **full indorsement**, an indorsement which specifies the one to whose order the note is made payable. By indorsing the note both William B. Harris and O. D. Merrill make themselves responsible for its payment in case the maker does not pay it. O. H. Briggs was willing to buy the note without Andrew J. Lloyd's guarantee to pay it. The transfer was effected by a **qualified indorsement**. By this indorsement Andrew J. Lloyd avoids the responsibility of an ordinary indorser.



The note just considered is a **time note**; if the words *On demand* were substituted for the words *Two months after date* the form would be called a **demand note**. The note is **interest-bearing** because it contains a clause to that effect; it would draw interest after it became due without any interest clause. A demand note, in which there is no interest clause, draws interest after payment has been demanded.

392. A note in which two or more persons *jointly and severally* promise to pay is called a **joint and several note**; a note in which two or more persons *jointly* promise to pay, a **joint note.**

$ 300.00 Rochester, N.Y., Aug. 5, 19—

Sixty days after date we jointly and severally promise to pay to the order of J. M. Cox & Son

Three Hundred 00/100 Dollars

Payable at Flour City National Bank

Value received

Arthur M. Black

Charles K. Palmer

David B. Small

No. 3 Due Oct. 4, 19

In a joint and several note, the holder may sue and collect of any one signer without proceeding against the others, or he may sue all of them together. In a joint note the signers must be sued jointly. The distinction between a joint and a joint and several note has been abolished by law in many of the states. The above form is a joint and several note. If the words *and severally* were omitted it would be a joint note.

The words *value received* in a note are equivalent to an acknowledgment that there has been a consideration. Their insertion is usual and advisable, but not legally required in all the states.

WRITTEN EXERCISE

Write interest-bearing notes as follows:

1. A demand note; amount, $1283.97; current date; payee, C. H. Good; maker (your name); interest at $5\frac{1}{2}\%$.

2. A time note; amount, $728.79; current date; time, 90 da.; payee, Snow & Co.; maker (your name); interest at $3\frac{1}{2}\%$.

3. A joint note; amount, $1795.73; current date; time, 6 mo.; payee, Ellis & Co.; maker (your name), and Richard Roe; interest at $4\frac{1}{2}\%$. Write a joint note under the same conditions.

4. Find the amount (face plus interest) due 87 da. after date in note No. 1; at the end of the time in note No. 2; at the end of the time in note No. 3.

EXACT INTEREST

393. Exact interest is simple interest for the exact number of days on the basis of 365 da. in a common year, or 366 da. in a leap year.

The United States Government takes exact interest, and its use is growing among business men. In strict justice it is the only correct method of computing interest.

394. The difference between the common year of 365 da. and the commercial year of 360 da. is 5 da., or $\frac{1}{73}$ of the common year.

If any sum were divided into 360 parts, each part would be larger than it would be if the sum were divided into 365 parts. Thus, $\frac{1}{360}$ and $\frac{93}{360}$ are greater than $\frac{1}{365}$ and $\frac{93}{365}$. It is therefore clear that exact interest is less than ordinary interest.

395. To find the exact interest, *compute interest in the usual way for the commercial year, and from the interest thus obtained subtract $\frac{1}{73}$ of itself.*

In many cases the work may be shortened by cancellation.

396. Example. Find the exact interest on \$3285 for 35 da. at 5%.

SOLUTION. $\frac{.05 \times 35 \times \$\cancel{3285}^{9}}{\cancel{365}} = .05 \times 35 \times \$9 = \$15.75.$

WRITTEN EXERCISE

Find the exact interest:

1. \$734.50 for 124 da. at 6%.
2. \$420.60 for 99 da. at 4½%.
3. \$965.50 for 82 da. at 3½%.
4. \$356.40 for 236 da. at 4%.
5. \$672.50 for 53 da. at 5½%.
6. \$546.24 for 38 da. at 4½%.
7. \$1240.35 for 50 da. at 6%.
8. \$1630.25 for 67 da. at 4%.
9. \$150,000 for 28 da. at 6%.
10. \$100,000 for 135 da. at 5%.
11. \$4653.28 for 182 da. at 4%.
12. \$45,000 for 42 da. at 2½%.
13. \$3500 from July 17, 1916, to Nov. 26, 1916, at 3%; at 4½%.
14. \$2315.89 from Mar. 11, 1916, to Sept. 1, 1916, at 6%; at 2%.
15. \$872.54 from Oct. 18, 1915, to Jan. 16, 1916, at 5%; at 7½%.
16. £1006 6*s.* from Apr. 1, 1916, to Feb. 19, 1917, at 3%; at 2%.

PROBLEMS IN INTEREST

ORAL EXERCISE

	Principal	Interest	Time	Rate	Amount
1.	$200	$24	2 yr.	?	?
2.	$250	$30	?	3%	?
3.	$240	$30	?	5%	?
4.	$320	?	3 yr.	5%	?
5.	?	$54	3 yr.	6%	?
6.	$450	$45	2 yr.	?	?
7.	$525	?	4 yr.	2%	?
8.	?	$84	$3\frac{1}{2}$ yr.	6%	?
9.	?	?	3 yr.	4%	$112
10.	$225	$36	4 yr.	?	?
11.	$625	?	$2\frac{1}{2}$ yr.	4%	?
12.	?	$52.50	?	3%	$402.50

13. If the cash price of an article is $125, what will be the sixty-day credit price if money is worth 6%?

Suggestion. The cash price plus the interest for the given time at the given rate equals the credit price.

14. If the thirty-day credit price of an article is $50.25, what will be the cash price if money is worth 6%?

Suggestion. The credit price divided by the amount of one dollar for the given time at the given rate equals the cash price.

15. If the cash price of an article is $240.50, what will be the sixty-day credit price if money is worth 6%?

16. If the four months credit price of an article is $163.20, what will be the cash price if money is worth 6%?

17. If the cash price of an article is $265.50, what will be the sixty-day credit price if money is worth 6%?

18. If the cash price of an article is $210, what will be the sixty-day credit price if money is worth 4%?

19. One contractor offers to do a certain piece of work for $425.50 cash; another offers to do the same work for $441, payable in 1 yr. If money is worth 5%, which is the better offer?

WRITTEN EXERCISE

1. Which is the better for a tailor, to sell a suit for $65 cash, or for $73.15 on 9 mo. time, money being worth 6%?

2. Which is the better, to sell carpet at $1.50 per yard cash, or at $1.68 per yard on 1 yr. time, money being worth 5%?

3. Which is the more advantageous, to buy an article for $58.50 cash or for $61.80 on 6 mo. time, money being worth 6%?

4. A merchant paid $160 cash for 4 sewing machines. After keeping them in stock 1 yr. 6 mo. he sold them for $190.80, on one year's time without interest. If money is worth 6% what was his gain or loss at the time of the sale?

5. An invoice of merchandise listed at $2500, on which trade discounts of 20% and 10% were allowed, was purchased at 90 da. What was the actual cash value of the debt on the day of the purchase, money being worth 5%?

6. A merchant bought 600 bbl. of flour at $7.50 per barrel. Terms: one half on account, 3 mo.; one half on account, 6 mo. At the end of 1 mo. he paid the cash value of the entire bill. How much did he gain, money being worth 6%?

7. Sept. 8 you purchased of Edward Sprague & Son, at trade discounts of 20% and 25%, an invoice of coffee listed at $2006. Terms: 30 da. Sept. 20 you sent Edward Sprague & Son a check for the actual cash value of the bill. What was the amount of the check, money being worth 6%?

PERIODIC INTEREST

397. Periodic interest is simple interest on the principal increased by the simple interest on each installment of interest that was not paid when due.

As periodic interest can be legally enforced in only a few states, special contracts should be made if it is to be collected. Where technically illegal, periodic interest is often collected; as, when a series of notes is given for the interest on a note secured by a real-estate mortgage, such notes to draw interest if not paid when due.

398. Example. If payments of interest are due semiannually, what is the interest on $1000 for 3 yr. at 6%?

SOLUTION

$180 = interest on $1000 for 3 yr. at 6%.

$30 is the interest on $1000 for one semiannual period, 6 mo.

1st installment of interest, $30, was unpaid for	2 yr. 6 mo.
2d installment of interest, $30, was unpaid for	2 yr.
3d installment of interest, $30, was unpaid for	1 yr. 6 mo.
4th installment of interest, $30, was unpaid for	1 yr.
5th installment of interest, $30, was unpaid for	6 mo.
The sum of the periods for which interest was unpaid is	7 yr. 6 mo.

The interest on each $30 for the period it was unpaid is the same as the interest on $30 for the sum of the periods.

13.50 = interest on $30 for 7 yr. 6 mo., at 6%.

$193.50 = the total interest due.

WRITTEN EXERCISE

1. If payments of interest are due annually, what is the interest on $850 for 5 yr., at 8%?

2. If payments of interest are due quarterly, what is the interest on $1380 for 2 yr. 6 mo., at 4%?

3. What is the difference between the simple interest and periodic interest (payable annually) on $1800 for 6 yr. at 4%?

4. If payments of interest are due semiannually, what amount should be paid in settlement of a debt of $1450 which has run 5 yr. at 6%?

5. If payments of interest are due annually, what amount will settle a debt of $1500 for 5 yr., at 6%, if the first installment of interest was paid when due?

COMPOUND INTEREST

399. Compound interest is interest computed, at certain intervals, on the sum of the principal and unpaid interest.

Interest may be compounded annually, semiannually, quarterly, or even monthly. In most states the law does not sanction the collection of compound interest, but if it is agreed upon by the parties, the taking of it does not constitute usury. It is a general custom of savings banks to allow compound interest. Compound interest is also used by life insurance companies.

400. Example. What is the compound interest on $6000 for 4 yr., if the interest is compounded annually at 5%?

SOLUTION.

$6000	= 1st principal.
300	= interest 1st year.
6300	= amount, or the principal the 2d year.
315	= interest 2d year.
6615	= amount, or the principal the 3d year.
330.75	= interest 3d year.
6945.75	= amount, or the principal the 4th year.
347.29	= interest 4th year.
7293.04	= amount due at the end of the 4th year.
$7293.04	− $6000 = $1293.04, compound interest for 4 yr.

WRITTEN EXERCISE

1. If interest is compounded annually, what will be the amount of $600 for 5 yr. at 6%?

2. If interest is compounded semiannually, what will be the compound interest on $1500 for 2 yr. 6 mo. at 4%?

3. A placed $750 in a savings bank Jan. 1, 1915, and interest was added thereto every 6 mo. at the rate of 4%. No withdrawals having been made, what was the balance due Jan. 1, 1917?

TABLE SHOWING THE AMOUNTS OF $1 AT COMPOUND INTEREST COMPOUNDED ANNUALLY

YR.	2%	2½%	3%	3½%	4%	4½%	5%	YR.
1	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1
2	1.04040	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250	2
3	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	3
4	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	4
5	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	5
6	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	6
7	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	7
8	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	8
9	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	9
10	1.21899	1.28009	1.34392	1.41060	1.48024	1.55297	1.62889	10
11	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	11
12	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	12
13	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	13
14	1.31948	1.41297	1.51259	1.61870	1.73168	1.85194	1.97993	14
15	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	15
16	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	16
17	1.40024	1.52162	1.65285	1.79468	1.94790	2.11338	2.29202	17
18	1.42825	1.55966	1.70243	1.85749	2.02582	2.20848	2.40662	18
19	1.45681	1.59865	1.75351	1.92250	2.10685	2.30786	2.52695	19
20	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	20

ORAL EXERCISE

Refer to the table, page 321, *and give rapid answers to the following:*

1. What is the amount of \$1 for 12 yr. at 4%? at 3%? at 5%? at $4\frac{1}{2}$%? at $2\frac{1}{2}$%?

2. What is the amount of \$1 for 18 yr. at $4\frac{1}{2}$%? at $3\frac{1}{2}$%? at 2%? at 3%? at $2\frac{1}{2}$%?

3. What is the amount of \$1 for 9 yr. at 5%? at $4\frac{1}{2}$%? at $2\frac{1}{2}$%? at $3\frac{1}{2}$%? at 3%? at 4%?

4. What is the amount of \$1 for 20 yr. at 2%? at 5%? at $4\frac{1}{2}$%? at $3\frac{1}{2}$%? at $2\frac{1}{2}$%? at 3%?

5. What is the amount of \$10 for 10 yr. at 4%? for 20 yr. at 2%? for 5 yr. at 5%?

6. What is the amount of \$100 for 5 yr. at 2%? for 11 yr. at $3\frac{1}{2}$%? for 19 yr. at 5%?

401. Example. What is the compound interest on \$8000 for 10 yr., if interest is compounded annually at 5%?

SOLUTION. \$1.62889 = amount of \$1 for 10 yr. at 5%.
8000 × \$1.62889 = \$13031.12, amount due in 10 yr. at 5%.
\$13031.12 − \$8000 = \$5031.12, the compound interest.

If interest is compounded semiannually, take one half the rate for twice the time; if quarterly, take one fourth the rate for four times the time.

The table may be used for periods longer than 20 yr. For 40 yr. multiply the amount of \$1 for 20 yr. by itself, and the product will be the amount for 40 yr.; for 35 yr. multiply the amount of \$1 for 20 yr. by the amount of \$1 for 15 yr.; the result will be the amount for 35 yr.

WRITTEN EXERCISE

Find the compound interest:

	PRINCIPAL	RATE	TIME	INTEREST PAYABLE
1.	\$7500	4%	5 yr.	Annually
2.	\$2500.	2%	12 yr.	Annually
3.	\$5600	$3\frac{1}{2}$%	20 yr.	Annually
4.	\$3350	5%	10 yr.	Semiannually
5.	\$2875	3%	17 yr.	Annually
6.	\$4600	4%	15 yr.	Semiannually

Sinking Funds

402. A **sinking fund** is a sum of money set aside at regular intervals for the purpose of paying off an existing or anticipated indebtedness, or of replacing a value which will disappear by depreciation, exhaustion, or termination.

The payment of a public or a corporation debt and the replacing of certain public and corporate values are sometimes facilitated by regularly investing a certain sum in some form of security. The interest and principal from these investments from year to year constitute a *sinking fund*, which, it is planned, shall accumulate to an amount sufficient to redeem the debt when it falls due or replace the value when it disappears.

ORAL EXERCISE

1. In what time will any sum of money double itself at 4% simple interest? at 3%? at 6%? at $4\frac{1}{2}$%?

2. How long (approximately) will it take \$1 to double itself at $3\frac{1}{2}$%? compound interest, compounded annually? (See table, page 321.)

3. How long (approximately) will it take any sum to double itself at $4\frac{1}{2}$% compound interest, compounded annually? at 5% compound interest, compounded annually?

4. If you put \$1 at compound interest to-day, \$1 one year from to-day, and so on for 20 yr., how much would you have at the end of the twentieth year, interest being compounded annually at $4\frac{1}{2}$%? (See table below.)

403. In the following table is shown the sum to which \$1, paid at the beginning of each year, will increase at certain rates of compound interest in any number of years not exceeding twenty.

Yr.	2%	4%	$4\frac{1}{2}$%	Yr.	2%	4%	$4\frac{1}{2}$%
1	1.020000	1.040000	1.045000	11	12.412089	14.025805	14.464031
2	2.060400	2.121600	2.137025	12	13.680331	15.626837	16.159913
3	3.121608	3.246464	3.278191	13	14.973938	17.291911	17.932109
4	4.204040	4.416322	4.470709	14	16.293416	19.023587	19.784054
5	5.308120	5.632975	5.716891	15	17.639285	20.824531	21.719336
6	6.434283	6.898294	7.019151	16	19.012070	22.697512	23.741706
7	7.582969	8.214226	8.380013	17	20.412312	24.645412	25.855083
8	8.754628	9.582795	9.802114	18	21.840558	26.671229	28.063562
9	9.949721	11.006107	11.288209	19	23.297369	28.778078	30.371432
10	11.168715	12.486351	12.841178	20	24.783317	30.969201	32.783136

WRITTEN EXERCISE

1. At the beginning of each year for 10 yr. a certain railroad company put aside out of the profits of the previous year $50,000 as a sinking fund. If this sum was invested at 4% compound interest, compounded annually, what did it amount to at the end of the tenth year?

2. Jan 1, 1915, a certain city borrowed $500,000 and agreed to pay the same on Jan. 1, 1925. What sum must be invested on Jan. 1, 1915, and annually for 10 yr., in securities, paying $4\frac{1}{2}$% compound interest, compounded annually, in order to pay the loan when it becomes due?

3. On Dec. 31, 1915, a certain town borrowed $40,000 with which to build a new high school. It was agreed that this amount should be paid on Dec. 31, 1920. What sum must the town set aside and invest at $4\frac{1}{2}$% compound interest, compounded annually, on Jan. 1, 1913, and each year thereafter for 5 yr., in order to pay the debt when it becomes due?

4. What sum must a town set aside and invest annually to rebuild a bridge costing $30,969.20, estimated to last 20 yr., allowing 4% compound interest, compounded annually?

WRITTEN REVIEW EXERCISE

1. What amount of interest (in United States money) at 6% will accrue on a debt of £84 12*s.* in 5 mo. 24 da.?

2. The yearly taxes on a house and lot which cost $12,500 are $162. How much should the house rent for per month to clear 6% on the investment?

3. A bought 16,000 bu. of wheat at 85¢, and paid for it in 10 da. 46 da. from the date of purchase he sold the wheat for 92¢ per bushel, cash. If money was worth 4%, what did he gain?

4. A savings bank account was opened July 1, 1914, with a deposit of $800. Interest was credited every 6 mo. at 4%. No withdrawals or subsequent deposits having been made, what was the balance of the account Jan. 1, 1920?

5. The note on page 314 was not paid until May 27. How much was due the holder of the note on that date?

6. Jan. 1, 1915, B invested $24,000 in a manufacturing business. July 1, 1917, he withdrew $33,000, which sum included the original investment and the net gains. What average yearly per cent of simple interest did the investment yield?

7. Derby & Co. offer B the following terms: $^{2}/_{10}, ^{N}/_{30}$. Jan. 1, B bought a bill of goods amounting to $4000 which he paid Jan. 31. What rate of interest did he practically pay on the net amount of the bill by not taking advantage of the cash offer?

8. In a certain town the taxes are due Sept. 15 of each year, and all taxes unpaid by Oct. 15 are subject to interest from the date they are due, at 6%. The following taxes were paid on the dates named: Oct. 18, $68.40; Oct. 21, $22.50; Oct. 25, $132.75; Oct. 31, $98; Nov. 11, $176.80; Nov. 23, $326.30; Dec. 2, $45; Dec. 16, $13.25; Dec. 29, $21. How much interest was paid, the time being the exact number of days?

9. Jan. 1, 1915, F bought a piece of city property for $20,000, paid cash $4000, and gave a note and mortgage for 5 yr. without interest, to secure the balance. To cover the interest, which it was agreed should be met quarterly, he gave twenty notes for $240 each, one maturing every three months. The first five installments of interest were paid when due, and the balance of the mortgage and the interest were paid Jan. 1, 1920. Find the final payment.

CHAPTER XXVI

BANK DISCOUNT

ORAL EXERCISE

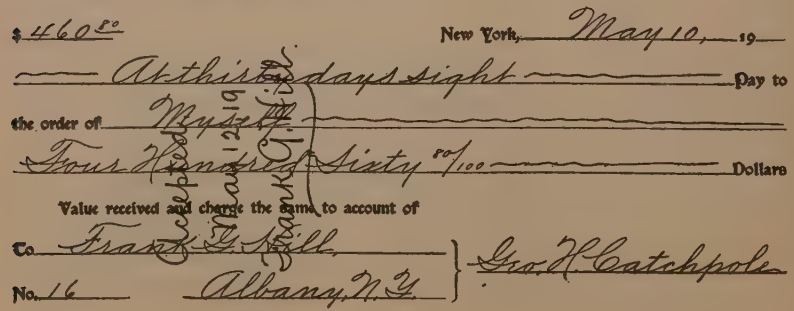
1. What is meant by a promissory note? by the face of a note? by the time? by the maker? by the payee?

2. How would you word a promissory note for $600, dated at your place to-day, payable in 60 da. at a bank in your place, with interest at 5%, to C. B. Powell, signed by yourself?

3. What is meant by negotiable? by indorsing a note? Illustrate a blank indorsement; an indorsement in full; a qualified indorsement.

404. A **commercial bank** is an institution chartered by law to receive and loan money, to facilitate the transmission of money and the collection of negotiable paper, and, in some cases, to furnish a circulating medium.

405. If the **holder** (owner) of a promissory note wishes to use the money promised before it becomes due, a commercial bank will usually buy the note, provided the holder can show that it will be paid at **maturity**, that is, when it becomes due. This is called **discounting** the note.



406. A **commercial draft** is now frequently used, instead of the promissory note, as security for the payment of goods sold on credit. Such a draft may be defined as a written order in which one person directs another to pay a specified sum of money to the order of himself or to a third person.

The circumstances under which the foregoing draft was drawn are as follows: Geo. H. Catchpole sold Frank G. Hill goods amounting to $460.80. Terms: 30-da. draft. The draft and an invoice were made out and sent to Frank G. Hill by mail. Frank G. Hill accepted the draft, that is, signified his intention to pay it by writing the word *accepted*, the date, and his name across the face. The draft was then returned to Geo. H. Catchpole, who may discount it the same as he would an ordinary promissory note.

The parties to a draft are the **drawer**, the **drawee**, and the **payee**. In the foregoing draft, George H. Catchpole is both the drawer and the payee, and Frank G. Hill is the drawee.

A draft payable after sight begins to mature from the date on which it is accepted. An acceptance *must*, therefore, be dated in a draft payable after sight, but it may or may not be dated in a draft payable after date.

$796.40 Buffalo, N. Y., Apr. 6, 19—

Thirty days after date Pay to

the order of Myself

Seven Hundred Ninety-six 40/100 Dollars

Value received and charge the same to account of

To F. L. & T. Easthy, } Frank M. Bennett

No. 21 Lockport, N. Y. }

Some states allow three **days of grace** for the payment of notes and other negotiable paper. Days of grace are obsolete in so many of the states that they are not considered in the exercises in this book. Some states provide that when paper matures on Sunday or a legal holiday it must be paid the day preceding such Sunday or legal holiday; others provide that it must be paid on the day following. To hold all interested parties, the laws of any given state should always be observed. When the time of negotiable paper is expressed in months, **calendar months** are used to determine the date of maturity; but when the time is expressed in days, the **exact number of days** is used. Thus, a note payable 2 mo. after July 15 is due Sept. 15; but a note payable 60 da. after July 15 is due Sept. 13. Paper payable 1 mo. from May 31, Aug. 31, etc., is due June 30, Sept. 30, etc.

407. The time from the date of discount to the maturity of paper is called the **term of discount**; the whole sum specified to be paid at maturity, the **value**, or **amount**, of the paper.

The term of discount is usually the *exact number of days* from the date of discount to the date of maturity. Some banks, however, find the term of discount by compound subtraction, and then reduce the time to days; *e.g.* the term of discount on a note due May 6 and discounted Mar. 1 is counted as 2 mo. 5 da., or 65 da. *In this text the term of discount is the exact number of days from the date of discount to the maturity of the paper.*

408. The reduction made by a bank for advancing money on negotiable paper not due is called **bank discount**. The value of negotiable paper at maturity, minus the bank discount, is called the **proceeds**.

Bank discount is always the simple interest for the term of discount on the whole sum specified to be paid at maturity.

409. The accompanying **maturity table** is sometimes used by bankers in finding the maturity of notes and drafts. The following examples illustrate its use.

MATURITY TABLE

1	Jan. − 1	13
2	Feb. + 2	14
3	Mar. − 1	15
4	Apr.	16
5	May − 1	17
6	June	18
7	July − 1	19
8	Aug. − 1	20
9	Sept.	21
10	Oct. − 1	22
11	Nov.	23
12	Dec. − 1	24

410. Examples. 1. Find the maturity of a note payable (*a*) 6 mo. from Apr. 27, 1915; (*b*) 6 mo. from Sept. 25, 1915.

SOLUTIONS. (*a*) Referring to the table, observe that April is the 4th month; adding 4 and 6, the result is 10, and the 10th month (see number on left) is October. The note is therefore due Oct. 27, 1915.

(*b*) September is the 9th month. $9 + 6 = 15$, and the 15th month (see number on right) is March of the next year. The note is therefore due Mar. 25, 1916.

2. Find the maturity of a note payable 90 da. from Jan. 18, 1916.

SOLUTION. $1 + 3 = 4$, and the 4th month is April. If the note were payable in 3 mo., it would be due Apr. 18. Referring to the table, note that 2 da. (1 da. + 1 da.) must be subtracted for January and March, and 2 da. added for February. The note is therefore due Apr. 18.

After the student has become familiar with the principles of the table it will not be found necessary to consult it.

ORAL EXERCISE

Find the maturity of each of the following notes:

	Date	Time		Date	Time
1.	Apr. 6, 1915	30 da.	6.	Jan. 30, 1916	30 da.
2.	Oct. 6, 1916	3 mo.	7.	Jan. 31, 1915	30 da.
3.	Nov. 9, 1915	60 da.	8.	May 10, 1916	90 da.
4.	Jan. 31, 1916	1 mo.	9.	June 19, 1916	60 da.
5.	Sept. 18, 1915	90 da.	10.	Nov. 15, 1916	30 da.

Find the maturity of each of the following acceptances:

	Date	Time after Date		Date	Time after Date
11.	Apr. 3	30 da.	14.	Dec. 31	2 mo.
12.	May 5	60 da.	15.	Jan. 12	1 mo.
13.	Jan. 29	1 mo.	16.	Feb. 18	3 mo.

Find the maturity of each of the following acceptances:

	Date Accepted	Time after Sight		Date Accepted	Time after Sight
17.	Aug. 12	3 mo.	20.	Apr. 25	60 da.
18.	Sept. 18	2 mo.	21.	May 17	3 mo.
19.	Oct. 30	4 mo.	22.	June 18	30 da.

WRITTEN EXERCISE

Find the maturity and the term of discount:

	Date	Time	Discounted
1.	Jan. 16, 1916	3 mo.	Mar. 1
2.	Jan. 31, 1916	1 mo.	Feb. 3
3.	Feb. 12, 1916	90 da.	Mar. 2
4.	Feb. 24, 1916	60 da.	Apr. 1
5.	Mar. 31, 1916	90 da.	May 13

	Date of Draft	Time after Date	Date Accepted	Date Discounted
6.	Feb. 7	60 da.	Feb. 8	Feb. 9
7.	Mar. 12	30 da.	Mar. 12	Mar. 15

	Date of Draft	Time after Sight	Date Accepted	Date Discounted
8.	May 31	60 da.	May 31	June 3
9.	Mar. 17	90 da.	Mar. 20	Mar. 21

411. The following **time table** is frequently used by bankers in finding the exact number of days between any two dates:

TABLE OF TIME

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
JANUARY	365	31	59	90	120	151	181	212	243	273	304	334
FEBRUARY	334	365	28	59	89	120	150	181	212	242	273	303
MARCH	306	337	365	31	61	92	122	153	184	214	245	275
APRIL	275	306	334	365	30	61	91	122	153	183	214	244
MAY	245	276	304	335	365	31	61	92	123	153	184	214
JUNE	214	245	273	304	334	365	30	61	92	122	153	183
JULY	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST	153	184	212	243	273	304	334	365	31	61	92	122
SEPTEMBER	122	153	181	212	242	273	303	334	365	30	61	91
OCTOBER	92	123	151	182	212	243	273	304	335	365	31	61
NOVEMBER	61	92	120	151	181	212	242	273	304	334	365	30
DECEMBER	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any other month, within a year, is found in the column of the last month directly opposite the line of the first month. Thus, from June 6 to Sept. 6 is 92 da.; from Apr. 1 to Oct. 1 is 183 da.; from Aug. 26 to Dec. 26 is 122 da. The exact number of days between *any two dates*, leap years excepted, is found as in the following illustrations:

412. Examples. 1. How many days from Mar. 1 to May 11?

SOLUTION. From Mar. 1 to May 1 is 61 da. From May 1 to May 11 is 10 da. 61 da. + 10 da. = 71 da., the required result.

2. How many days from July 26 to Oct. 6?

SOLUTION. From July 26 to Oct. 26 is 92 da. From Oct. 26 back to Oct. 6 is 20 da. 92 da. − 20 da. = 72 da., the required result.

ORAL EXERCISE

By the table find the exact number of days from:

1. July 8 to Sept. 8.
2. Jan. 6 to Mar. 6.
3. Jan. 23 to June 23.
4. Feb. 13 to July 13.
5. Mar. 11 to Sept. 11.
6. Mar. 21 to Aug. 21.
7. May 31 to Aug. 1.
8. Feb. 23 to Sept. 23.
9. Mar. 24 to July 12.
10. May 11 to Aug. 31.
11. Aug. 15 to Dec. 10.
12. Nov. 25 to Mar. 25.

413. Examples. 1. Find the proceeds of a note for \$3000, payable in 78 da., discounted at 6 %.

SOLUTION. 78 da. = the term of discount.
\$39 = the bank discount.
\$3000 − \$39 = \$2961, the proceeds.

2. A note for \$6000 payable in 60 da. from May 10, 1915, with interest at 6 %, is discounted May 25, at 6 %. Find the maturity, the term of discount, the bank discount, and the proceeds.

SOLUTION. July 9, 1915 = the maturity.
45 da. = the term of discount.
\$60 = the interest on the note for 60 da.
\$6060 = the value of the note at maturity.
\$45.45 = the bank discount.
\$6014.55 = the proceeds.

414. The accompanying diagram illustrates a convenient outline for learning the proper method of computing bank discount. It will be observed that the first problem is an interest-bearing note, and the second problem a non-interest-bearing note. The items in black ink are taken from the problem, and the items in red ink are found as previously explained.

Memoranda	1	2	3
Date of Paper	Jan. 1	Mar. 1	
Face of Paper	\$300	\$600	
Time	2 mo.	3 mo.	
Rate of Interest	6%		
Rate of Discount	6%	6%	
Date of Discount	Jan. 30	May 1	
Maturity	Mar.	June 4	
Term of Discount	30 da.	34 da.	
Interest	\$3	—	
Value	\$303	\$600	
Discount	\$1.52	\$3.10	
Proceeds	\$301.48	\$596.90	

WRITTEN EXERCISE

1. Assuming that the model note, page 9, was discounted July 2, at 6 %, find the bank discount and the proceeds.

2. Assuming that the model note, page 314, was discounted Jan. 20, at 6 %, find the bank discount and the proceeds.

3. Assuming that the model note, page 316, was discounted Aug. 26, at 6 %, find the bank discount and the proceeds.

4. Assuming that the model draft, page 326, was discounted May 15, at 6 %, find the bank discount and the proceeds.

5. Assuming that the model draft, page 327, was discounted April 12, at 6%, find the bank discount and the proceeds.

6. Find the proceeds of the following **joint note:**

$895.40 BALTIMORE, MD., May 25, 1915.

Six months after date, for value received, we promise to pay to the order of Ralph D. Gibson Eight Hundred Ninety-five $\frac{40}{100}$ Dollars, at Exchange National Bank.

SETH M. BULLARD.

Discounted July 2, 1915, at 5%. ISAAC C. WATKINS.

7. Find the proceeds of the following **joint and several note:**

$1000.00 COLUMBUS, O., May 1, 1915.

Three months after date we jointly and severally promise to pay to the order of Wilson N. Burton One Thousand Dollars, at Second National Bank, Columbus, O., with interest at 6%.

Value received. JOHN M. SELLERS.

Discounted June 2, 1915, at 6%. DANIEL W. SHELDON.

8. Find the proceeds of the following **firm note:**

$1250.00 ST. LOUIS, MO., Aug. 20, 1915.

Ninety days after date we promise to pay to the order of C. M. Courtwright Twelve Hundred Fifty Dollars, at the National Bank of Redemption, with interest at 5%.

Value received. J. M. COX & SON.

Discounted Sept. 1, 1915, at 6%.

9. Sept. 26 you sold R. M. Stein, Portland, Me., a bill of hardware amounting to $2480, less 20%, 25%, and 10%. Terms: $\frac{1}{2}$ by 60-da. note with interest at 6%; $\frac{1}{2}$ on account 60 da. What was the amount of the note which was this day received?

10. Oct. 12 you discounted at Union Bank, at 6%, R. M. Stein's note received Sept. 26, the bank giving you credit for the proceeds. If the bank charges $\frac{1}{10}$% for collecting out-of-town paper, what was the amount of the proceeds credited?

A small fee called *collection and exchange* is sometimes charged on discounted paper payable out of town. The charge is by no means uniform, being controlled largely by the size of the depositor's account and the general custom of the banks in any given locality.

11. The following is a part of a page from a bank's discount register. Copy it, supplying all missing terms. The notes were all discounted June 17.

No.	Date of Paper	Time	When Due	Term of Discount	Rate of Discount	Value of Paper		Disc.		Coll. & Exch.		Proceeds Credited	
20	Apr. 25	3 mo.			6 %	2000	00						
21	May 1	3 mo.			6 %	3500	00			3	50		
22	Apr. 1	90 da.			6 %	1500	00						
23	Apr. 15	90 da.			6 %	900	60						
24	June 15	30 da.			6 %	378	90				38		

12. Sept. 15 your balance in the First National Bank was $1725.90. You immediately offered for discount, at 6%, the following notes, the proceeds of which were to be placed to your credit: E. M. Robinson's 30-day note dated Sept. 1, for $300; C. E. Reardon's note payable 3 mo. from July 25, with interest at 6%, for $427.65; C. W. Allen's 60-day note dated Aug. 1, for $321.17; F. H. Clark's 60-day note dated July 30, for $1500. What was your credit at the bank after discounting the notes?

13. April 6, 1915, Peter W. Berger has on deposit in the First National Bank $523.87. He draws a check for $1176.45, and then discounts the following notes at the bank, at 6%, receiving credit for the proceeds. What was the balance of his account after the notes were discounted and credited?

a.

$346.50 HARTFORD, CONN., Mar. 1, 1915.

Ninety days after date I promise to pay Peter W. Berger, or order, Three Hundred Forty-six $\frac{50}{100}$ Dollars, at First National Bank, Hartford, Conn.

Value received. HENRY S. LANE.

b.

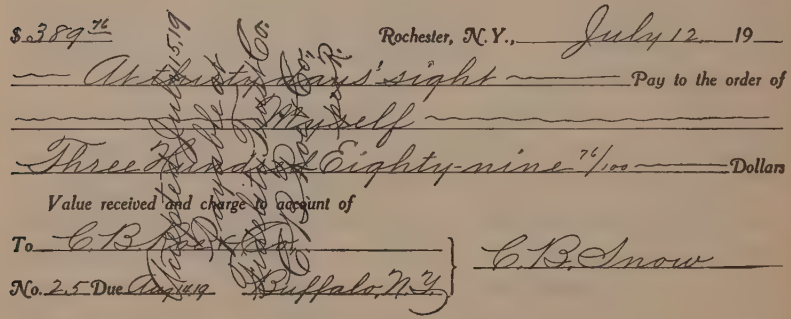
$575.00 HARTFORD, CONN., Feb. 1, 1915.

Aug. 1, 1915, I promise to pay Peter W. Berger, or order, Five Hundred Seventy-five Dollars, at Second National Bank, Hartford, Conn.

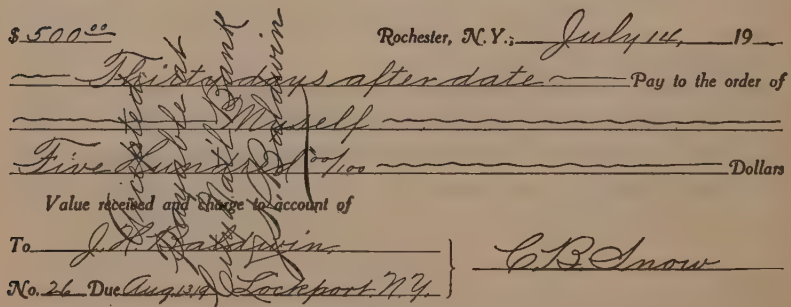
Value received. SAMUEL D. SKIFF.

14. July 18, C. B. Snow's bank balance is $312.90. He discounts at 6% the following drafts, and then issues a check in payment for 5 sewing machines at $75, less 20% and 25%. What is the amount of his balance after issuing the check?

a.



b.



BANK LOANS

415. The foregoing exercises have reference to paper bought or discounted by a bank. Money is frequently loaned upon the notes of the borrower, indorsed by some one of known financial ability, or secured by the deposit of stocks, bonds, warehouse receipts, or other collaterals. These notes, if drawn on time, are not interest-bearing, but the bank discounts them by deducting from their face the interest for the full time.

416. Loans are sometimes made on **call** or **demand notes**; that is, on notes that can be called or demanded at any time after they are made. These notes are interest-bearing and are drawn for the exact sum loaned.

Call or *demand loans* generally bear a lower rate of interest than loans on time. They are made principally to brokers and investors, who use them to pay for stocks; but they are also made to merchants and others to some extent. Business men, however, generally prefer to borrow on time, for they do not wish to be embarrassed by having the loans called in at an unexpected time. *Time loans* are usually drawn for thirty, sixty, or ninety days. If the borrower requires money for a longer period, the bank will usually allow him to renew the note when it falls due.

WRITTEN EXERCISE

1. Jan. 7, 1915, E. L. Jennings & Co. desire to extend their business, and for this purpose borrow money at 6 % of the First National Bank of New York, on the following note. How much will the bank place to the credit of E. L. Jennings & Co.?

$5000.00 New York, Jan. 7, 19—

Ninety days after date we promise to pay to the order of First National Bank

Five Thousand 00/100 Dollars

at their banking house

Value received

No. 15 Due Apr. 7, 19 E. L. Jennings & Co.

2. You gave the Union National Bank, of your city, your note, for $1200, at 60 da., indorsed by Williams & Rogers. How much cash will the bank advance you, if discount is deducted at the rate of 6%?

3. Howe & Rogers, Buffalo, N.Y., borrowed $12,000 of Merchants National Bank on their demand note secured by 300 shares of Missouri Pacific Railway stock, at $50. If the rate of interest was $2\frac{1}{2}$ %, how much was required for settlement 39 da. after the loan was made?

4. Jan. 2, 1915, C. W. Allen & Co., brokers, borrowed of First National Bank, Boston, Mass., $15,000 on the following **collateral note.** How much was required for full settlement of the loan 57 da. after it was made?

$15000.00 Boston, Mass., Jan. 2, 19

On demand for value received, we promise to pay to the order of First National Bank of Boston at their banking house Fifteen Thousand 00/100 Dollars with interest at 3½% per annum

As collateral security for the payment of the note and all other liabilities to said bank, either absolute or contingent, now existing or to be hereafter incurred, we have deposited with it:

100 shares Lake Shore & Michigan Southern Railway Stock

Should the market value of the same decline, we promise to furnish satisfactory additional collateral on demand, which may be made by a notice in writing, sent by mail or otherwise, to our residence or place of business. On the nonperformance of either of the above promises we authorize the holder or holders hereof to sell said collateral and any collaterals added to or substituted for the same, without notice, at public or private sale, and at or before the maturity hereof, he or they giving us credit for any balance of the net proceeds of such sale remaining after paying all sums absolutely or contingently due and then or thereafter payable from us to said holder or holders. And we authorize said holder or holders, or any person in his or their behalf, to purchase at any such sale.

C. W. Allen & Co.

FINDING THE FACE

417. Example. I wish to borrow $1980 of a bank. For what sum must I issue a 60-da. note to obtain the amount, discount being at the rate of 6%?

SOLUTION.

Let the face of the note = $1
Then the bank discount = $0.01
And the proceeds = $0.99
But the proceeds = $1980
$1980 ÷ $0.99 = 2000

∴ the face of the note is 2000 × $1, or $2000.

WRITTEN EXERCISE

1. What must be the face of a 30-da. note in order that when discounted at 6% the proceeds will be $1990? Of a 60-da. note, same conditions?

2. You wish to borrow $3940 cash. What must be the face of a 90-da. note in order that when discounted at 6% the proceeds will be the required sum?

A WRITTEN REVIEW TEST

(Time, approximately, forty minutes)

Copy each of the following problems; complete the work, and check the result.

When a certain number of days is written in the *Discount* column, compute the discount at 6% for the given time. When $\frac{1}{10}\%$ is written in the *Collection and Exchange* column, compute the collection on the face of the paper.

When copying the problem do not copy the number of days in the discount column, nor the $\frac{1}{10}\%$ in the collection and exchange column, but compute the discount or the collection and enter the *result* in the required column.

	Face of Paper	Discount	Coll. & Exch.	Proceeds
1.	$356.40	$3.56	$0.36	?
	442.50	2.21	0.44	?
	365.30	60 da.	0.37	?
	297.45	2.97	$\frac{1}{10}\%$	?
	175.40	1.75	0.18	?
	217.50	2.18	$\frac{1}{10}\%$	?
	246.30	30 da.	———	?
	?	?	?	?

	Face of Paper	Discount	Coll. & Exch.	Proceeds
2.	$325.65	$1.63	$0.33	?
	150.60	90 da.	0.15	?
	327.85	3.28	0.33	?
	180.96	60 da.	$\frac{1}{10}\%$	?
	313.46	3.13	0.31	?
	286.32	1.43	0.29	?
	?	?	?	?

	Face of Paper	Discount	Coll. & Exch.	Proceeds
3.	$422.50	30 da.	$\frac{1}{10}\%$	?
	384.20	$3.84	$0.38	?
	519.40	5.19	0.52	?
	280.50	90 da.	0.28	?
	375.90	1.25	———	?
	245.32	2.45	$\frac{1}{10}\%$	?
	?	?	?	?

CHAPTER XXVII

PARTIAL PAYMENTS

THE UNITED STATES METHOD

ORAL EXERCISE

1. A note for $500 bears interest at 6%. What amount would pay the note and interest at the end of 1 yr.?

2. Suppose that a payment of $130 was made at the end of 1 yr. After the accrued interest has been paid, how much is there left to apply to the face of the note?

3. After the $100 has been applied to the face of the note, what amount does the maker continue to keep? On what sum, therefore, should he pay interest after the first year?

4. The maker kept the remaining $400 another year. How much interest was then due? What was the total amount due?

5. If a payment of $224 was made at this time, what amount still remained unpaid? If the balance of the note was paid three years after it was issued, what was the amount of the payment?

418. Partial payments are payments in part of a note or bond.

Such payments may be made either before or after maturity. They should be acknowledged by indorsement on the back of a note or bond. Current forms for indorsing partial payments on notes are illustrated on page 342.

419. The **United States method** of partial payments (as suggested in problems 1–5 above) has been adopted by the Supreme Court of the United States, and made the *legal method* in nearly all the states.

This is the method ordinarily used by individuals when the time between the date of the note and its payment is *more than one year.*

420. Example. A note for $1200, dated Jan. 1, 1915, bearing interest at 6%, had payments indorsed upon it as follows: Mar. 1, 1915, $212; July 1, 1915, $15; Sept. 1, 1915, $515; Nov. 1, 1915, $175. How much was due upon the note at final settlement, Apr. 1, 1916.

SOLUTION

Face of note		$1200.
Interest from Jan. 1, 1915, to Mar. 1, 1915 (2 mo.)		12.
Amount due Mar. 1, 1915		1212.
Payment Mar. 1, 1915		212.
New principal, or amount to draw interest after Mar. 1, 1915		1000.
Interest from Mar 1, 1915, to July 1, 1915 (4 mo.)	$20.	
Interest exceeds the payment and the principal remains unaltered.		
Interest from July 1, 1915, to Sept. 1, 1915 (2 mo.)	$10.	
Total interest due Sept. 1, 1915		30.
Amount due Sept. 1, 1915		1030.
Sum of the payments since July 1 ($15 + $515)		530.
New principal, or amount to draw interest after Sept. 1, 1915		500.
Interest from Sept. 1, 1915, to Nov. 1, 1915 (2 mo.)		5.
Amount due Nov. 1, 1915		505.
Payment Nov. 1, 1915		175.
New principal, or amount to draw interest after Nov. 1, 1915		330.
Interest from Nov. 1, 1915, to Apr. 1, 1916 (5 mo.)		8.25
Amount due at settlement, Apr. 1, 1916		$338.25

It will be observed in the foregoing example that the United States method provides: (1) that *the payment must first be applied to discharge the accrued interest;* (2) that *the surplus, if any, after paying the interest may be used to diminish the principal;* and (3) that *if any payment is less than the accrued interest, the principal remains unaltered until some payment is made with which the preceding neglected payment or payments is more than sufficient to discharge the accrued interest.*

CONDENSED FORM FOR WRITTEN WORK

DATES			INTEREST PERIODS			PER CENTS OF INTEREST	PRINCIPALS	INTERESTS ON PRINCIPALS	AMOUNTS OF PRINCIPALS	PAYMENTS
Yr.	Mo.	Da.	Yr.	Mo.	Da.					
1915	1	1								
1915	3	1		2	0	$.01	$1200.00	$12.00	$1212.00	$212.00
1915	7	1		4	0	.02	1000.00	20.00		15.00
1915	9	1		2	0	.01	1000.00	10.00	1030.00	515.00
1915	11	1		2	0	.01	500.00	5.00	505.00	175.00
1916	4	1		5	0	.025	330.00	8.25	338.25	
1	3	0	1	3	0	$.075	$338.25, balance due Apr. 1, 1916			

When there are many payments, the work may be simplified as shown in the foregoing outline. First write the date and the face of the note and then the dates and the amounts of the payments. Next find the interest periods and the per cents of interest. Test the accuracy of the work to this point (1) by finding the difference between the date of the note and the date of settlement and comparing it with the sum of the interest periods; and (2) by comparing the sum of the per cents of interest with the interest on $1 for the full time as shown by the sum of the interest periods. Complete the remainder of the work as suggested by the outline.

WRITTEN EXERCISE

1. Jan. 2, 1916, J. E. King & Co. borrowed of E. B. Peterson & Bro. $1000 and gave in payment a note payable in 6 mo., with interest at 5%. July 2, J. E. King & Co. made a payment of $500 and issued a new note at 90 da., with interest at 6% for the balance due. What was the face of the new note?

2. Jan. 30, 1915, you sold Irwin & Co. 5 Eureka Elevator Pumps at $475, less a trade discount of $16\frac{2}{3}\%$. Terms: note at 6 mo. with interest at 6%. What was the face of the note? At the maturity of the note Irwin & Co. paid you cash $1000 and gave you a new note at 6 mo., with interest at 6% for the balance due. What was the face of the new note? Sept. 1, 1915, Irwin & Co. paid you $200, and Dec. 1, $300, on their note of July 30. What was due on the note Feb. 9, 1916?

3. On the note below indorsements were made as follows: May 1, 1915, $75; Sept. 2, 1915, $90; Oct. 2, 1915, $165; Jan. 2, 1916, $125.

$825.40 OMAHA, NEB., Jan. 2, 1915.

Apr. 2, 1916, I promise to pay Wilson & Allen, or order, Eight Hundred Twenty-five $\frac{40}{100}$ Dollars, at their office, with interest at 6%.

Value received. JOHN D. AVERILL.

What was due at the maturity of the note?

4. Find the amount due on each of the following notes July 1, 1916:

a.

$1200.00 Rochester, N. Y., Jan. 15, 1915.

Fifteen months after date I promise to pay to the order of Robert Hebblethwaite & Son Twelve Hundred 00/100 Dollars at Alliance Bank, with interest at 5%.

Value received

No. 12 Due Apr. 15, 1916. W. B. Martin

b.

$2000.00 Philadelphia, Pa., July 1, 1915.

On demand we promise to pay to the order of Richard Roe & Co. Two Thousand 00/100 Dollars at their office, with interest at 6%.

Value received

No. 16 Due — Taylor Bros. & Co.

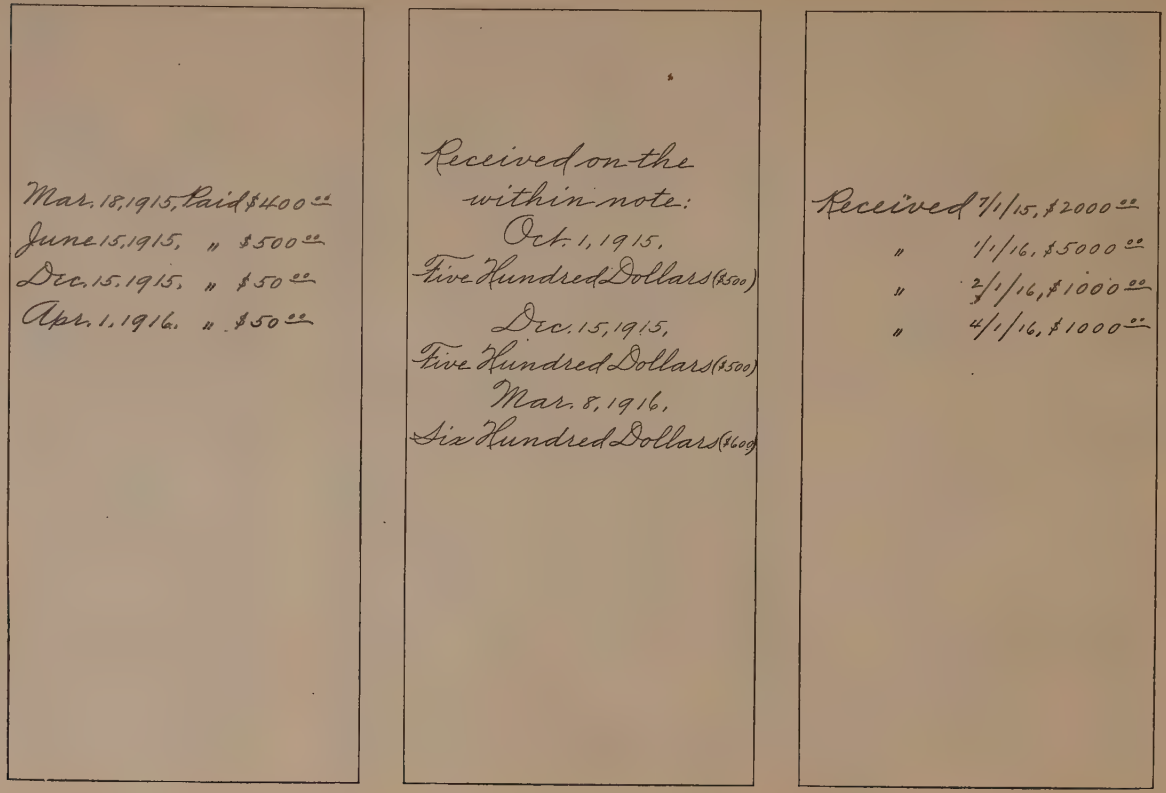
c.

Secured by Mortgage of Real Estate in Boston to be recorded in Suffolk Registry of Deeds.

$12000.00 Boston, Mass., Jan. 1, 1915.

Eighteen months after date, for value received I promise to pay to the order of Charles W. Lawson & Co. Twelve Thousand 00/100 Dollars at their office, with interest at the rate of six per centum per annum during the said term, and for such further time as the said principal sum or any part thereof shall remain unpaid.

Signed in the presence of O. K. Bresee Chas. B. Sherman


Mar. 18, 1915, Paid $400.00
June 15, 1915, " $500.00
Dec. 15, 1915, " $50.00
Apr. 1, 1916, " $50.00
Received on the within note:
Oct. 1, 1915,
Five Hundred Dollars ($500)
Dec. 15, 1915,
Five Hundred Dollars ($500)
Mar. 8, 1916,
Six Hundred Dollars ($600)
Received 7/1/15, $2000.00
" 1/1/16, $5000.00
" 2/1/16, $1000.00
" 4/1/16, $1000.00

THE MERCHANTS' METHOD

ORAL EXERCISE

1. A note for $500 is dated July 1, 1915, payable in 1 yr. with interest at 6%. If no payments have been made, what is due on the note July 1, 1916?

2. A payment of $300 was indorsed on the note Jan. 1, 1916. What was the amount of this payment at the time the note became due?

3. If the value of the note at maturity is $530 and the value of the payment $309, what is the balance due?

4. By the United States method what is the balance due at maturity on the note described in problems 1 and 3? How does this balance compare with the balance in problem 3?

421. The **merchants' method** is based on custom rather than on legal authority. It is used by most banks and business men on *short-time* notes and other obligations.

The principles of the merchants' method are suggested in problems 1–3. This method provides that: (1) *the face of the note shall draw interest to the date of settlement;* (2) *interest shall be allowed on each payment from the time it is made to the date of settlement.*

422. Example. On a note for $600, dated May 13, 1916, payable on demand, with interest at 6%, payments were made as follows: June 28, 1916, $100; Aug. 28, 1916, $200. What was due at settlement, Sept. 28, 1916?

SOLUTION

Face of note May 13, 1916		$600.00
Interest from May 13, 1916, to Sept. 28, 1916 (4 mo. 15 da.)		13.50
Value of note Sept. 28, 1916, the date of settlement		$613.50
Payment June 28, 1916	$100.00	
Interest on this payment from June 28, 1916, to Sept. 28, 1916 (3 mo.)	1.50	
Payment Aug. 28, 1916	200.00	
Interest on this payment from Aug. 28, 1916, to Sept. 28, 1916 (1 mo.)	1.00	
Value of the payments Sept. 28, 1916, the date of settlement		$302.50
Balance due Sept. 28, 1916, the date of settlement		$311.00

Some houses find the time by compound subtraction and some use the exact number of days. In the following exercise find the difference in time by compound subtraction in problems 1–2, and use the exact number of days in problems 3–7.

WRITTEN EXERCISE

1. Solve problem *a*, page 341, by the merchants' method for partial payments. Compare the results by the two methods.

2. On a note for $1200, dated Apr. 16, 1916, payable on demand, with interest at $4\frac{1}{2}\%$, payments were made as follows: June 15, 1916, $500; July 18, 1916, $200. What was due at settlement, Sept. 16, 1916?

3. June 15 you borrowed $25,000 at Traders' National Bank on your demand note secured by a deposit of 300 shares of Illinois Central Railroad Stock at $110. June 27 you paid $5000, July 2, $10,000, and July 30, $5000. Aug. 2 you paid the remainder of the note and interest, and withdrew the collaterals. What was the amount of the last payment, money being loaned at $4\frac{1}{2}\%$?

4. The following is a partial page of the demand and loan register of a large bank. Copy it, supplying the amount of interest due Nov. 15, money being loaned at $4\frac{1}{2}\%$.

CHARLES W. SHERMAN

No.	Date Loaned	Amount Loaned	Date of Payment	Part of Loan Paid	Balance of Loan	Interest	Collateral	Value of Collateral
347	Apr. 1	20,000 00	May 15	5,000 00	15,000 00	? ? ? ??	250 shares	
			July 1	5,000 00	10,000 00	? ? ? ??	Penn. R.R.	
			Sept. 1	6,000 00	4,000 00	? ? ? ??	Stock . .	31,250 00
			Nov. 15	4,000 00		? ? ? ??		
						? ? ? ??		

The balance due by the merchants' method may be found in the manner suggested by the above account. The interest is found on the face of the note to the date of the first payment. The payment is deducted and the interest found on the balance to the date of the second payment, and so on. The results obtained by this process are exactly the same as the results obtained by § 421.

5. Solve problem 4 by the United States method and compare the result with the merchants' method.

6. Assuming that the collateral note, page 336, has the following payments indorsed on its back, find the amount due at final settlement, Feb. 28, 1916. Indorsements: Jan. 15, 1916, $3000; Jan. 31, 1916, $5000; Feb. 5, 1916, $1000.

7. A collateral note dated at Philadelphia, Pa., July 10, 1916, for $20,000 payable at the Quaker City National Bank is indorsed as follows: Aug. 8, 1916, $3500; Sept. 12, 1916, $7500; Nov. 19, 1916, $4000; Dec. 31, 1916, $5000. What was due on the note Dec. 31, 1916, interest being at the rate of 4%?

To solve the problem copy and complete the following interest statement:

Philadelphia, Dec. 29, 19__

M. Roswell Post
721 Chestnut St, City

To THE QUAKER CITY NATIONAL BANK, Dr.

To interest on demand loans, as follows:

$20000.00 from 7/10 to 8/8, 29 days, $??.??
$16500.00 from 8/8 to 9/12, 35 days, $??.??
$9000.00 from 9/12 to 11/19, ?? days, $??.??
$5000.00 from ??/?? to ??/??, ?? days, $??.??
???.??

Please send us the above interest on or before the 31st inst.

Respectfully,
W. D. Brelsford
CASHIER

8. Make an interest statement, similar to the above, for problem 6.

9. Make an interest statement, similar to the above, for problem 3.

10. Bring to the class a canceled note on which partial payments are recorded. Find, by the United States method and by the merchants' method, the amount required to cancel the note. Which method is the better for the debtor? for the creditor?

CHAPTER XXVIII

BANKERS' DAILY BALANCES

423. Some **commercial banks** and **trust companies** pay interest on the daily balances of their depositors.

Whether interest shall be allowed on a depositor's account is usually determined by the size of his daily balances. As a rule, no interest is allowed on small balances subject to check. All balances not subject to check usually draw interest. In an active account, that is, an account in which the balance changes frequently, interest is seldom allowed except on an even number of hundred dollars, and all parts of a hundred dollars are rejected.

The form of the book in which accounts with depositors are recorded varies in different sections. What is known as the Boston individual ledger (see form, page 38) is extensively used. Another form of depositors' ledger is that shown in the example below.

424. Example. Verify the balance due on the following account beginning Mar. 1, 1916, interest settlements being made monthly at 3%.

M. W. FARNHAM

EXPLANATION	DATE		F.	DEBIT		BALANCE		CREDIT		F.	DATE		EXPLANATION
	1916										1915-6		
						1056	25				Dec.	31	
						1656	25	600	00	15	Jan.	7	Currency
						2556	25	900	00	15		11	N. Y. draft
Check	Jan.	15	14	510	00	2046	25						
						3746	25	1700	00	17	Jan.	22	N. Y. draft
Note	Jan.	25	16	210	00	3536	25						
Check		28	16	500	00	3036	25						
						3042	08	5	83	17	Jan.	31	Interest
						4042	08	1000	00	21	Feb.	8	N. Y. draft
Check	Feb.	15	20	500	00	3542	08						
Check		22	22	1340	00	2202	08						
						2209	49	7	41	23	Feb.	28	Interest

SOLUTION. The credit slip on page 347 shows a form used for recording the daily balances. Only two money columns are used, one for hundreds and the other for thousands. No interest is computed except on an even number of hundred dollars, and all parts of a hundred dollars are rejected.

Beginning Jan. 1 the daily balance of M. W. Farnham's account for 6 da. was $1056.25; record $1000 on the credit slip as shown in the margin. A deposit of $600 was made Jan. 7, making the balance $1656.25 for the next 4 da.; record $1600 on the credit slip as shown in the margin. A deposit of $900 on Jan. 11 made the balance $2556.25 for the next 4 da.; record $2500 on the credit slip as shown in the margin. A withdrawal of $510 on Jan. 15 left a balance of $2046.25 for the next 7 da.; record $2000 on the credit slip as shown in the margin. A deposit of $1700 on Jan. 22 made the balance $3746.25 for the next 3 da.; record $3700 on the credit slip as shown in the margin. A withdrawal of $210 on Jan. 25 left a balance of $3536.25 for the next 3 da.; record $3500 on the credit slip. A withdrawal of $500 on Jan. 28 left a balance of $3036.25 for the next 4 da. This records the balance for each day in January. Adding these balances the result is $70,000, and the interest on this sum for 1 da. at 3% is $5.83. Adding $5.83 to $3036.25 gives the balance to the credit of the depositor Feb. 1 as $3042.08.

DAILY CREDIT BALANCES

M. W. Farnham

1907	Jan.		Feb.	
1	1		3	
2	1			
3	1			
4	1			
5	1			
6	1			
7	1	6		
8	1	6	4	
9	1	6		
10	1	6		
11	2	5		
12	2	5		
13	2	5		
14	2	5		
15	2		3	5
16	2			
17	2			
18	2			
19	2			
20	2			
21	2			
22	3	7	2	2
23	3	7		
24	3	7		
25	3	5		
26	3	5		
27	3	5		
28	3			
29	3			
30	3			
31	3			
Total	70	0	88	9
Interest	5	83	7	41

Enter the daily balances for February as shown in the margin. The result is found to be $88,900, and the interest on this sum for 1 da. at 3% is $7.41. $7.41 added to the balance of the depositor's account Feb. 28 gives $2209.49 as the balance to his credit beginning Mar. 1.

In practice the daily balances are usually written as shown in the February column of the accompanying credit slip. The total is then found by multiplication and addition. Thus, the total of the February column is $7 \times \$3000 + 7 \times \$4000 + 7 \times \$3500 + 7 \times \$2200$, or $88,900.

Some accountants also use the pure interest method in finding the amount due. Thus, the interest on $3000 for 7 da., plus the interest on $4000 for 7 da., plus the interest on $3500 for 7 da., plus the interest on $2200 for 7 da. equals $7.41, the same as by the first method.

In the examples which follow the student may use either of the three methods suggested.

WRITTEN EXERCISE

1. The Rochester Trust and Safe Deposit Co. allows interest to its depositors on daily balances at 3% per annum, payable quarterly. Find the cash balance of the following account with Chas. M. Sherman, Apr. 1, 1916. Jan. 1, 1916, deposited $1200; Jan. 12 drew out $400; Jan 30 deposited $800; Jan. 31 drew out $400; Feb. 10 deposited $800; Feb. 25 drew out $100; Mar. 10 deposited $800; Mar. 20 drew out $900; Mar. 25 deposited $300.

2. At the close of an interest period, Feb. 28, 1916, Harvey & Smith's balance with the Fidelity Trust Co. was $2246. During the month of March they made the following deposits: Mar. 3, $2500; Mar. 9, $1750; Mar. 24, $2645.75; Mar. 28, $1310.50; Mar. 30, $500. They also drew out by check as follows: Mar. 4, $1050; Mar. 6, $2000; Mar. 8, $720; Mar. 12, $840.50; Mar. 16, $450; Mar. 19, $430; Mar. 23, $1000; Mar. 26, $150; Mar. 29, $267. How much interest should be credited Mar. 31, the rate being 3% per annum? What was the balance of the account after the interest was credited?

3. Find the cash balance of the following account May 31, 1916, assuming that interest is allowed on daily balances at 3% and added to the account monthly.

A. S. OSBORN

EXPLANATION	DATE		F.	DEBIT		BALANCE		CREDIT		F.	DATE		EXPLANATION
	1916										1916		
						1200	00				Feb.	28	
Check	Mar.	12		100	00	1500	00	400	00		Mar.	12	Currency
						2000	00	500	00			25	Currency
Check		31		100	00	2400		500	00			31	N. Y. draft
						[illegible]		[illegible]				31	Interest
						[illegible]		700	00		Apr.	15	N. Y. draft
Note	Apr.	20		50	00	[illegible]		200	00			20	N. Y. draft
Check		30		1200	00	[illegible]		[illegible]				30	Interest
						[illegible]		250	00		May	10	Currency
Check	May	31		500	00	[illegible]		[illegible]				31	Interest

CHAPTER XXIX

SAVINGS-BANK ACCOUNTS

425. A **savings bank** is an institution, chartered by the state, in which savings or earnings are deposited and put to interest.

The deposits in a savings bank are practically payable on demand. Most banks reserve the right to require notice of withdrawal from 30 to 60 da. in advance; but this right is seldom exercised.

The period of time which must elapse before dividends of interest are declared is called the **interest term.** Dividends of interest are usually declared semiannually; but in some sections they are declared quarterly. The stated days on which balances begin to draw interest are called **interest days.** In some savings banks deposits begin to draw interest from the first of each quarter; in others, from the first of each month.

In nearly all savings banks, only such sums as have been on deposit for the *full time between the interest days* draw interest. Thus, if the interest days begin on the first of each quarter, only those sums that have been on deposit for the full quarter draw interest.

426. Interest is computed on an even number of dollars, and all fractions of a dollar are rejected. When interest is not withdrawn it is placed to the credit of the depositor and draws interest the same as any regular deposit. Savings banks therefore allow **compound interest.**

427. Examples. 1. In the Walker Institution for Savings the interest term is 6 mo. and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1. Verify the balance due on the following account Jan. 1, 1916, at 4 %.

SOLUTION. The account was opened July 1, 1915, by a deposit of $500. July 10 this sum was increased by a deposit of $10, making the balance $510; Aug. 14 this sum was diminished by a withdrawal of $20, making the balance $490; Oct. 4 this sum was diminished by a withdrawal of $200, making the balance $290. The account was similarly increased and diminished until Dec. 31, when there was a balance of $300.75 due the depositor.

The Walker Institution for Savings

in account with

Mrs. Robert M. Sanborn

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1915 July	1	500	—					500	—
	10	10	—					510	—
Aug.	14					20	—	490	—
Oct.	4					200	—	290	—
	10	70	75					360	75
Dec.	10	40	—					400	75
	31					100		300	75
1916 Jan.	1			7	80			308	55

The smallest balance for the first interest period, July 1 to Oct. 1, was $490. The interest on $490 for 3 mo. at 4% is $4.90. The smallest balance for the second interest period, Oct. 1 to Jan. 1, was $290. The interest on $290 for 3 mo. at 4% is $2.90. $4.90 plus $2.90 equals $7.80, the dividend of interest due the depositor Jan. 1. Since this sum is not withdrawn, it is placed to the credit of the depositor, making his balance $308.55.

2. In the Warren Institution for Savings interest dividends are declared semiannually and the interest days are Jan. 1, Apr. 1, July 1, and Oct. 1. Verify the balance due on the following account Jan. 1, 1916, at 4%.

The Warren Institution for Savings

in account with

Miss Mabel C. Courtright

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1915 Jan.	1	500	—					500	—
Mar.	1	300	—					800	—
May	10	100	—					900	—
July	1			13	—			913	—
1916 Jan.	1			18	26			931	26

SOLUTION. The smallest balance for the first interest period was \$500; the interest on \$500 for 3 mo. at 4% is \$5. The smallest balance for the second interest period was \$800; the interest on \$800 for 3 mo. at 4% is \$8. \$5 + \$8 = \$13, the total interest due the depositor July 1. \$900 + \$13 = \$913. This balance remained unchanged for the next 6 mo. The interest on \$913 for 6 mo. at 4% is \$18.26. \$913 + \$18.26 = \$931.26, the amount due the depositor Jan. 1, 1916.

WRITTEN EXERCISE

1. Solve example 1 above, assuming that the interest days are the first day of each month; also example 2.

2. Copy the following account, supplying the missing amounts. Interest at $4\frac{1}{2}$ %; interest days, Jan. 1, Apr. 1, July 1, and Oct. 1.

MANHATTAN SAVINGS BANK

IN ACCOUNT WITH *Mr. Chas. B. Sherman*

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1915									
Jan.	1	600	00					* * *	* *
Jan.	31					100	00	* * *	* *
Mar.	1	250	00					* * *	* *
May	6					50	00	* * *	* *
May	31	100	00					* * *	* *
July	1			* *	* *			* * *	* *

3. Copy and complete the following account. Interest at 4%; interest days, Jan. 1, Apr. 1, July 1, and Oct. 1.

FIDELITY SAVINGS BANK

IN ACCOUNT WITH *Mr. Frank M. Ellery*

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1915									
Jan.	1	300	00					* * *	* *
Mar.	6	200	00					* * *	* *
Mar.	30					125	00	* * *	* *
Apr.	17	165	50					* * *	* *
July	1	100	00	* *	* *			* * *	* *
Aug.	15					75	00	* * *	* *
Aug.	31	58	40					* * *	* *
Oct.	1	250	00					* * *	* *
Dec.	1					110	50	* * *	* *
1916									
Jan.	1			* *	* *			* * *	* *

POSTAL SAVINGS BANKS

428. The law establishing postal savings banks became effective in the United States, January 3, 1911. Herewith are some of the leading provisions of this law.

429. An account may be opened and deposits made by any person of the age of 10 yr. or over.

430. Deposits will be accepted only from individuals, and no account will be opened in the name of a corporation, association, society, firm, or partnership, or in the name of two or more persons jointly.

431. Deposits may be accepted without regard to the residence of the depositor or the post office of which he is a patron, but a person can have but one postal savings account, either at the same office or at different offices.

432. No account may be opened for less than $1, nor will fractions of a dollar be accepted for deposit at any time.

433. Postal savings deposits will be evidenced by certificates in fixed denominations of $1, $2, $5, $10, $20, $50, issued in the name of the depositor. Such certificates are non-transferable and non-negotiable. Each certificate must bear the depositor's name, the number of his account, the date of issue, the name of the office receiving the deposit, and the date on which interest will begin. Interest will begin on each deposit the first day of the month following the deposit.

A deposit on March 1 will not begin to draw interest until April 1.

434. Interest at the rate of 2% per annum shall be allowed and paid on the amount represented by each postal savings certificate for each full year that it remains on deposit. No interest will be allowed for a fractional portion of a year. Compound interest is not allowed, but a depositor may withdraw interest accrued and make a new deposit of it which will then draw interest.

435. A postal savings card with nine postal savings stamps affixed may be presented and accepted as a deposit for $1, either in opening an account or in adding to an existing account, or it may be redeemed in cash.

436. Postal savings cards and stamps are transferable and need not be presented for deposit by the original purchaser. They may be sold to any person in any quantity desired.

437. A depositor may surrender his deposits in whole or in part, in the sum of $20 or any multiple thereof up to $500, and receive in lieu of such surrendered deposits postal savings bonds in appropriate denominations. Such exchange may be made by a depositor as of January 1 and July 1 of each year, but applications therefor shall be made at least 1 mo. previously. These bonds bear interest at the rate of $2\frac{1}{2}\%$ per annum, payable semi-annually.

WRITTEN EXERCISE

1. If John A. Sellman deposited $7 on the tenth day of each month for the year 1911, beginning Jan. 10 and making his last deposit on Dec. 10, what sum would be to his credit on March 1, 1912? on Sept. 1, 1912?

2. If John A. Gilson deposited $12 on the first day of each alternate month for the year 1913, beginning Jan. 1, what sum would be to his credit on Jan. 1, 1914? on April 1, 1914?

3. If no additional deposits were added to the account in problem 2, what sum would be to the credit of Mr. Gilson on Jan. 1, 1915?

4. Ralph C. Varner made the following deposits and received postal savings certificates therefor: Nov. 4, 1911, $15; Mar. 12, 1912, $25; July 11, 1912, $30; Oct. 2, 1912, $23. What was due on this account July 1, 1915?

5. Henry M. Werner made the following deposits and received postal savings certificates therefor: Mar. 1, 1914, $11; June 4, 1914, $19; Sept. 8, 1914, $10; Dec. 23, 1914, $10. What was due on this account Jan. 1, 1915? on Apr. 1, 1915?

CHAPTER XXX

EXCHANGE

DOMESTIC EXCHANGE

ORAL EXERCISE

1. Mention some objections to sending actual money by express.

2. If $50 sent by mail in a registered letter is lost, to what extent are the postal authorities liable?

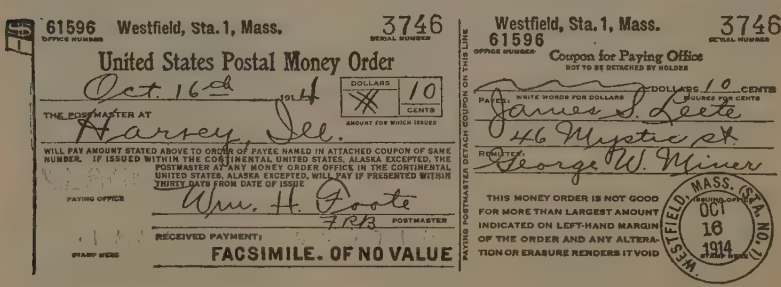
3. In what ways may you pay a debt at any distant point without actually sending the money?

438. The process of settling accounts at distant points without actually sending the money is called **exchange.**

Money Orders

439. Money orders, as issued by post offices, express companies, and banks are frequently used in making payments at a distance.

440. A **postal money order** is a government order for the payment of money, issued at one office and payable at another.

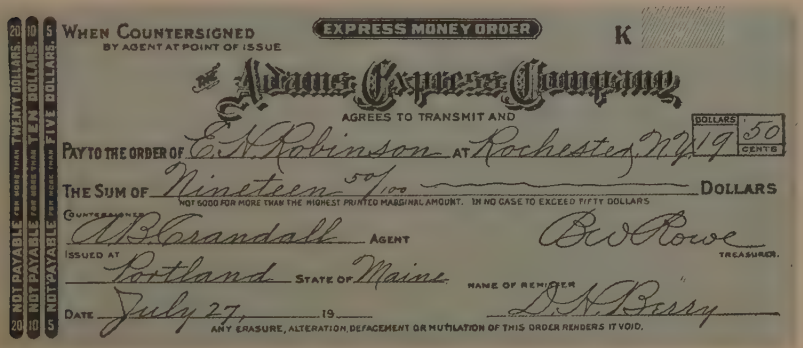

61596 Westfield, Sta. 1, Mass. 3746
United States Postal Money Order
Oct. 16th 1914
DOLLARS 10 CENTS
THE POSTMASTER AT Harvey, Ill.
WILL PAY AMOUNT STATED ABOVE TO ORDER OF PAYEE NAMED IN ATTACHED COUPON OF SAME NUMBER. IF ISSUED WITHIN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, THE POSTMASTER AT ANY MONEY ORDER OFFICE IN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, WILL PAY IF PRESENTED WITHIN THIRTY DAYS FROM DATE OF ISSUE
PAYING OFFICE
Wm. H. Foote
POSTMASTER
RECEIVED PAYMENT:
FACSIMILE. OF NO VALUE
Westfield, Sta. 1, Mass. 3746
61596
Coupon for Paying Office
DOLLARS 10 CENTS
PAYEE: James S. Leete
46 Mystic St.
REMITTER: George W. Miner
THIS MONEY ORDER IS NOT GOOD FOR MORE THAN LARGEST AMOUNT INDICATED ON LEFT-HAND MARGIN OF THE ORDER AND ANY ALTERATION OR ERASURE RENDERS IT VOID
WESTFIELD, MASS. (STA. NO. 1)
OCT 16 1914

The fees (**rate of exchange**) charged for postal money orders are:
For orders for sums not exceeding

$2.50	3¢	Over $30.00 to $ 40.00	15¢
Over 2.50 to $ 5.00	5¢	Over 40.00 to 50.00	18¢
Over 5.00 to 10.00	8¢	Over 50.00 to 60.00	20¢
Over 10.00 to 20.00	10¢	Over 60.00 to 75.00	25¢
Over 20.00 to 30.00	12¢	Over 75.00 to 100.00	30¢

The maximum amount for which a single postal money order may be issued is $100. When a larger sum is to be sent, additional orders must be obtained. When an order is issued, the money is not sent from one post office to another. The transfer is merely a matter of bookkeeping, the money being received by the government at one office and paid out at another. If a postal money order is lost, a duplicate may be obtained from the Post Office Department at Washington.

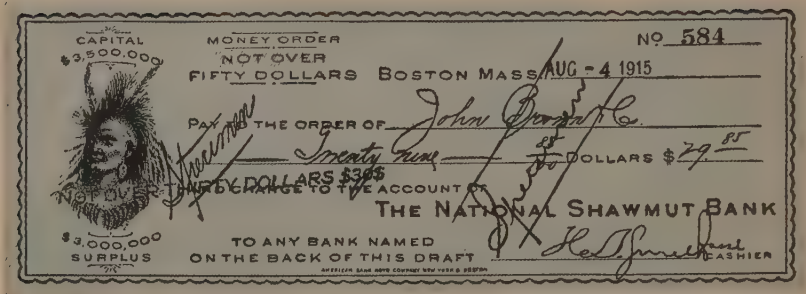
441. An **express money order** is an order for the payment of money, issued by an express company and payable at any of its agencies.


WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE
EXPRESS MONEY ORDER
K
Adams Express Company
AGREES TO TRANSMIT AND
PAY TO THE ORDER OF E. A. Robinson AT Rochester, N.Y.
DOLLARS 19 CENTS 50
THE SUM OF Nineteen 50/100 DOLLARS
NOT GOOD FOR MORE THAN THE HIGHEST PRINTED MARGINAL AMOUNT. IN NO CASE TO EXCEED FIFTY DOLLARS
COUNTERSIGNED A. B. Crandall AGENT
B. V. Rowe TREASURER.
ISSUED AT Portland STATE OF Maine
NAME OF REMITTER D. N. Berry
DATE July 27, 19
ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID.
NOT PAYABLE FOR MORE THAN TWENTY DOLLARS
NOT PAYABLE FOR MORE THAN TEN DOLLARS
NOT PAYABLE FOR MORE THAN FIVE DOLLARS

The fees charged for express money orders are the same as those for postal money orders. The maximum amount for which a single express money order may be issued is $50. A postal money order must not bear more than one indorsement; but an express money order may bear any number of indorsements.

442. A **bank money order** (see form, page 356) is an order for the payment of money issued by a bank and payable at certain other banks in different parts of the country.

The charge for a bank money order is usually the same as that for a postal money order.



443. A telegraphic money order is a telegram of an express or telegraph company, at any given place, ordering the payment of money at another designated place.

THE UNION TELEGRAPH CO.

INCORPORATED

23,000 OFFICES IN AMERICA CABLE SERVICE TO ALL THE WORLD

ROBERT C. CLOWRY, President and General Manager

SEND the following message subject to the terms on back hereof, which are hereby agreed to. } Boston, Mass., July 27, 19

To The Union Telegraph Co.

Rochester, N.Y.

Findable Charles Osgood ten East Avenue Fichant Findelkind

The Union Telegraph Co.

These telegrams are usually in cipher; that is, in a language not understood by those who are unfamiliar with the system of abbreviations (code) used. The sender and the receiver must each have a code. The following code will illustrate the principle of telegraphing in cipher:

Code Word	Meaning
Fichant	One hundred dollars
Ficheron	One thousand dollars
Findable	Please pay —— of —— your city $——.
Findelkind	On production by him of positive evidence of his personal identity.

The principle of a telegraphic money order is the same as that of a postal money order; no money is transferred from one place to another. The charge for an order is usually 1% of the amount to be transmitted plus twice the rate for a single ten-word message.

The following are the rates for a ten-word message from Boston to the places named:

New York	$0.30	Chicago	$0.50	Galveston	$0.75
Philadelphia	$0.35	San Francisco	$1.00	Rochester	$0.40

ORAL EXERCISE

1. What was the total cost to the sender of the postal money order, page 354? the express money order, page 355? the telegraphic money order, page 356? the bank money order, page 356?

2. What will be the total cost of a postal money order for 27¢? $2.19? $5.28? $10.40? $18.90? $45.10? $35.89? $125 ($100 + $25)? $75.29? $49.82? $127.16?

3. What will be the total cost of an express money order for $6.20? $28.80? $19.50? $27.95? $48.90? $65 ($50 + $15)? $111? $37.59? $41.72? $65.59? $114?

4. What will be the total cost of a telegraphic money order from Boston to New York for $50? $75? $100? $125? $150? $200? $300? $400? $450? $500? from Boston to Philadelphia? from Boston to San Francisco? from Boston to Chicago?

5. Translate the following telegraphic money order: *Findable F. J. Reed, 20 Park St. ficheron findelkind.* How much will it cost for such an order from Boston to Galveston? from Boston to Chicago? from Rochester to Boston?

WRITTEN EXERCISE

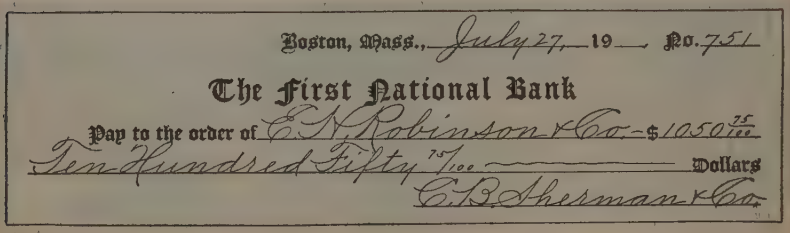
1. Find the total cost of 5 postal money orders for the following amounts: $3.10; $8.19; $25.06; $18.50; $20.

2. Find the total cost of six express money orders for the following amounts: $1.25; $10; $6.80; $16.25; $80; $19.50.

3. Find the total cost of the following telegraphic money orders: one from Boston to New York for $50; one from Boston to Philadelphia for $500; one from Boston to San Francisco for $175; one from Boston to Galveston for $300; one from Boston to Rochester for $250.

Checks and Bank Drafts

444. Business men usually keep their money on deposit with a commercial bank or trust company and make most payments, at home and at a distance, by **check**; that is, an order on a bank from one of its depositors for the payment of money.


Boston, Mass., July 27, 19—. No. 751

The First National Bank

Pay to the order of E. H. Robinson & Co. $1050 75/100

Ten Hundred Fifty 75/100 Dollars

C. B. Sherman & Co.

A check may be drawn for any amount so long as it does not exceed the balance on deposit to the credit of the drawer. It may be drawn payable to: (1) the order of a designated payee, in which case the payee must indorse it before the money will be paid over; (2) the payee, or bearer, in which case any one can collect it; (3) "Cash," in which case any one can collect it.

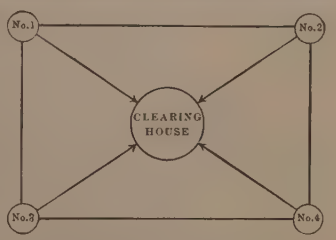
Interior View of a Clearing House.

C. B. Sherman & Co. and E. H. Robinson & Co. in the foregoing check both reside in Boston. On receiving the check, E. H. Robinson & Co. indorse it and deposit it for credit with their bank, say the National Shawmut Bank. The First National Bank and the National Shawmut Bank, as well as each of the other banks in the city, has many depositors who draw checks upon it which are deposited by the payees in various other city banks, and it also receives daily for credit from its own depositors checks drawn upon various other city banks.

Each bank therefore has a daily balance to settle or to be settled with each of the other banks. To some it has payments to make and from others it has payments to receive. If these balances were adjusted in money, each bank would have to send a messenger to each of the debtor

banks to present accounts and receive balances. This would be a risky and laborious task. To facilitate the daily exchanges of items and settlements of balances resulting from such exchanges there has been established in every large city a central agency, called a **clearing house**. This agency is an association of banks which pay the expense of conducting it in proportion to the average amount of their clearings.

Suppose, for example, that the banks constituting a clearing house are Nos. 1, 2, 3, and 4. No. 1 presents at the clearing house items against Nos. 2, 3, and 4, and Nos. 2, 3, and 4 present items against No. 1. So, likewise, with No. 2 and each of the other banks. In the clearing house there are usually two longitudinal columns containing as many desks as there are banks in the association. At a given time a settling clerk from each bank takes his place at his desk inside of one of the columns and a delivery clerk from each bank takes his place outside the column. Each delivery clerk advances, one desk at a time, and hands over to each settling clerk his exchange items against that bank. After the circuit of the desks has been completed each delivery clerk is at the point from which he started, and each settling clerk has on his desk the claims of all of the other banks against his bank. Each settling clerk then compares his claims against other banks with those of other banks against him and strikes a balance. This balance may be in favor of or against the clearing house. If No. 1 brought claims against Nos. 2, 3, and 4 aggregating $211,000 and Nos. 2, 3, and 4 brought claims against No. 1 aggregating $200,000, there is $11,000 due No. 1 from the clearing house. But if No. 1 brought to the clearing house exchange items aggregating $200,000 and took away exchange items aggregating $211,000, there is $11,000 due the clearing house from No. 1. So, likewise, with No. 2 and each of the other banks. When all of the exchanges have been completed, the clearing house will have paid out the same amount that it has received.



But all checks received by banks are not payable in the city. Suppose that A. W. Palmer, of Chicago, Ill., owes C. B. Andrews, of Westfield, Mass., $500 and that the amount is paid by a check on the City National Bank of Chicago. C. B. Andrews receives the check and offers it for credit at the Farmers and Traders Bank of Westfield, Mass. The Westfield Bank has no account with any Chicago bank, but it has with the First National Bank of Boston, and the check is sent to that bank for credit. The First National Bank wishes to increase its New York balance and the check is forwarded to Chemical National Bank of New York for credit. Chemical National Bank next mails the check to Commercial National Bank of Chicago, the

bank with which it has regular dealings in that city. Commercial National Bank sends the check to the clearing house and it is carried to the City National Bank by a messenger from that bank. Thus, all of a depositor's checks will in time be presented to the bank on which they are drawn. When presented, they will be charged to the depositor, cancelled, and later returned to him to be filed as receipts.

Banks frequently charge their depositors a small fee (**rate of exchange**) for collecting out-of-town checks. This fee is rarely over $\frac{1}{10}\%$, but there is no uniformity in the matter. Sometimes when a customer keeps a large bank account, no charge whatever is made for the collection.

445. It often happens that a person will find it necessary to make payment to one who does not care to take the risk of a private check or to one who should not be called upon to pay the cost of cashing a check. In such cases some other form of instrument of transfer must be used. A very common and convenient method of making a remittance is by means of a check of one banking institution upon another called a **bank draft.**

Boston, Mass., July 16, 19__ No. 920

Traders National Bank

Pay to the order of C. C. Canan — $400.60

Four Hundred 60/100 — Dollars

To Chemical National Bank, New York

W. A. Kinman, Cashier

Banks in the different cities frequently keep running accounts with each other and make periodical settlements. At the time of drawing the above draft Traders National Bank of Boston very likely has checks and drafts drawn upon New York banks which it has received from its depositors. These it sends to Chemical National Bank to cover the amount of the draft. Corresponding transactions may also take place in New York. Chemical National Bank may sell its draft on Traders National Bank and, to cover the amount, remit checks and drafts on Boston banks which it has received from its depositors. What is occurring between these two places is also occurring between all manner of places; but drafts upon New York banks and other financial centers are the most used in making remittances.

A bank draft is sometimes drawn payable to the one to whom it is to be sent. It is better, however, to have it drawn payable to the purchaser who may indorse it over to the person to whom it is to be sent. In this way the name of the sender appears on the draft, and when canceled, the draft will serve the purpose of a receipt. Banks usually sell drafts at a slight premium on the face. This premium is called **exchange**. It varies somewhat (see page 366), but is seldom more than $\frac{1}{10}$%.

446. There are still other methods of transmitting funds through the instrumentality of a bank. A depositor may exchange his own check for that of a **cashier's check**. The latter, being a check of the cashier on his own bank, would pass among strangers better than a depositor's check.

Cashier's Check

Boston, Mass., July 17, 19— No. 35431

NATIONAL SHAWMUT BANK

Pay to the order of Edgar L. Marston $2449 50/100 Dollars

Twenty-four Hundred Forty-nine 50/100 Dollars

Henry D. Sharpe
Cashier

In New York City these checks are occasionally used instead of the New York draft. As New York exchange is in demand in all parts of the country, the expediency of the course is apparent.

447. By depositing a sum of money in a bank a person may receive a certificate, called a **certificate of deposit**. This will direct the payment of the sum deposited to any person whom the depositor may name.

$5000.00 Boston, Mass., July 17, 19— No. 2499

National Shawmut Bank

Charles W. Browne has deposited

Five Thousand 00/100 Dollars

payable to the order of E. B. Stanton

on the return of this certificate properly indorsed.

Henry T. Fowler
Cashier

The payee in a certificate of deposit will have no difficulty in getting the certificate cashed or the amount credited to him by his bank.

ORAL EXERCISE

1. Assuming that the bank which cashed the check on page 5 charged $\frac{1}{8}\%$ collection, what was the amount credited to the depositor?

2. Silas Long of New York deposited the following check. The bank deducted $\frac{1}{10}\%$ for collection. How much was placed to Silas Long's credit?

Boston, Mass., May 4, 19__ No. 1

The Union Bank

Pay to the order of Silas Long $112.45

One Hundred Twelve 45/100 Dollars

E. C. Dunn

3. B deposited three out-of-town checks in his bank as follows: \$300; \$700; \$750. If the bank charged $\frac{1}{10}\%$ collection, what amount was placed to B's credit?

4. Bring to the class a number of canceled checks and take several of them and trace them from the time they were issued until they were filed as receipts by the drawer. Show why a canceled check is the best kind of a receipt for the payment of money?

5. How much did the bank draft on page 360 cost the purchaser if the exchange was at $\frac{1}{10}\%$ premium?

WRITTEN EXERCISE

1. Find the cost of a bank draft for \$3958.75 at $\frac{1}{10}\%$ premium; of a bank draft for \$679.80 at $\frac{1}{20}\%$ premium; of a bank draft for \$768.54 at 50¢ per \$1000 premium.

2. To cover the cost of a bank draft bought at $\frac{1}{10}\%$ premium, I gave my bank a check for \$250.25. What was the face of the draft? What was the rate of premium per \$1000?

3. How large a bank draft can be bought for $850.85, exchange being at $\frac{1}{10}\%$ premium?

4. Find the proceeds of the accompanying deposit, $\frac{1}{10}\%$ collection and exchange being charged on the out-of-town checks.

When the receiving teller takes a deposit from a customer, he classifies the items on the deposit ticket, as shown in the accompanying illustration. If the coin and bills passed in count right, these items are checked (√) on the deposit slip; if a check on a clearing house bank is received, it is marked with the number of that bank in the clearing house; if a check on the teller's bank is received, it is marked "B"; if a check on an out-of-town bank is received, it is marked "X."

THE UNION NATIONAL BANK

DEPOSITED BY

M. D. Knowlton & Co

Boston, Aug. 7, 19

Specie	√	90	00
Bills	√	500	00
Checks . . Traders	#5	275	00
Albany	X	860	00
Central	#42	127	90
Union	B	214	10
Springfield	X	720	50
Westfield	X	120	90
		????	??
Coll & Exch		?	??
		????	??

5. Write a bank draft using the following data: your address and the current date; drawer, Central National Bank; drawee, Chemical National Bank, New York; amount, $711.94; payee, C. E. Denison; cashier, your name. How large a check will pay for the draft at $\frac{1}{10}\%$ premium? Write the check.

6. Suppose that the members of the class whose surnames begin with the letters from A to G inclusive have a deposit with Traders National Bank; that the members whose surnames begin with the letters from H to N inclusive have a deposit with City National Bank; that the members whose surnames begin with O to S inclusive have a deposit with First National Bank; and that the members whose surnames begin with T to Z inclusive have a deposit with Central Bank. Let each student write a check on his bank in favor of one of his classmates, and let this classmate indorse the check and deposit it with his bank. Then form a clearing house, strike a balance between the different banks, and have these balances adjusted by the payment of school money.

Commercial Drafts

448. Business men frequently employ the **commercial draft** as an aid in the collection of accounts that are past due.

$260.50 Springfield, Mass., July 3, 19__

At sight Pay to the order of

Ourselves

Two Hundred Sixty 50/100 Dollars

Value received and charge to account of

To Edgar McMickle

No. 132 Due Paterson, N.J. } Wilbert, Closs & Co.

The above is a common form of draft used for collection purposes. Edgar McMickle owes Wilbert, Closs & Co. $260.50. The amount is due, and Wilbert, Closs & Co. draw a draft on Edgar McMickle and leave it with their Springfield bank for collection. The Springfield bank forwards it to its correspondent in Paterson and this bank sends it by messenger to Edgar McMickle. When he pays the draft, the Paterson bank notifies the Springfield bank, and that bank deducts a small fee (**collection and exchange**) for collecting the draft, and credits Wilbert, Closs & Co. for the proceeds.

449. It has been seen (page 327) that the **time draft** is frequently used in connection with sales of merchandise.

$500 Boston, Mass., July 15, 19__

Thirty days after date Pay to the order of

Ourselves

Five Hundred 00/100 Dollars

Value received and charge to account of

To L. B. Wade & Co.

No. 72 Due Westfield, Mass. } Quincy, Bradley & Co.

per W. O. W.

Suppose Quincy, Bradley & Co. sell L. B. Wade & Co. a bill of merchandise amounting to $500. Terms: 30-da. draft for the amount of the bill. The draft, as above, and the bill in regular form would be drawn up and

sent to L. B. Wade & Co. for acceptance. The object of drawing a time draft in connection with sales of merchandise is twofold: (1) when accepted, the draft serves as a written contract; (2) since an acceptance is negotiable, it may be discounted and cash realized upon it before maturity. Such a draft is frequently left with a bank for collection instead of being remitted with the bill. The bank will then first present the draft for acceptance and later for payment.

ORAL EXERCISE

1. If you exchange your check for a cashier's check, is there any charge for the accommodation?

2. If the sight draft on page 364 was collected by a bank which charged $\frac{1}{8}\%$ collection, how much was placed to the credit of Wilbert, Closs & Co.?

3. You deposited in Shawmut National Bank $5000, received the certificate of deposit shown on page 361, and remitted it to E. B. Stanton on account. Would there be any exchange?

WRITTEN EXERCISE

1. The draft on page 364 was accepted July 17, and discounted July 25. If the bank charged $\frac{1}{10}\%$ collection and 6% interest, how much was placed to the credit of the drawers?

2. Mar. 27 Wilson Bros., Chicago, Ill., drew a 30-da. draft on E. W. King, Toledo, O., in favor of themselves, payable 30 da. after date, for $3500, and mailed it for acceptance. Apr. 1 the draft was received accepted; Apr. 2 it was discounted at City Bank. If the charges were $\frac{1}{20}\%$ collection and 6% interest, what amount was credited to Wilson Bros.?

3. Apr. 17 O. H. Brooks, Buffalo, N.Y., drew a sight draft on Slocum & Co., Hartford, Conn., in favor of himself, for $391, and left it with his bank (First National) for collection. First National Bank sent the draft to its Hartford correspondent (Commercial National), and 5 da. later informed O. H. Brooks that the draft had been collected, and the amount, less $\frac{1}{8}\%$ collection, placed to his credit. If O. H. Brooks's bank balance was $758.62 before the draft was drawn, what was it after the draft was credited? Write the draft and show the indorsements.

4. Aug. 9 you sold C. D. Mead & Co., San Francisco, Cal., 39 mahogany sideboards at $162.50, delivered the goods to the Interstate Transportation Co., and received a through **bill of lading** (receipt for the goods and an agreement to transport and deliver them to the consignee or to his order). You then drew a sight draft on C. D. Mead & Co. in favor of your bank, attached the draft to the bill of lading, and left it with your bank for collection. Your bank indorsed the draft and the bill of lading and sent them to First National Bank of San Francisco for collection and credit. Aug. 23 you received advice that the draft had been collected, and the amount, less $\frac{1}{4}\%$, placed to your credit. What was the amount credited?

When First National Bank of San Francisco received the draft, it notified C. D. Mead & Co. They paid the draft, and the bank gave them the bill of lading. When goods are shipped in this manner, the transportation company will not deliver the goods until the consignee presents the bill of lading.

Fluctuation of Rates of Exchange

450. It has been seen that money orders always sell for more than their face value, and that bank drafts frequently cost a little more than their face value. When exchange costs its face value, it is said to be **at par**; when it costs more than its face value, it is said to be **at a premium**; when it costs less than its face value, it is said to be **at a discount**.

On bank drafts for small sums, say $500 or less, exchange is usually at a uniform premium. This premium is to pay the banks for their trouble and the expense of shipping money to the centers on which the drafts are drawn, when balances at these points become low. But exchange on the trade centers of the country may be at par at one time, at a premium at another, and at a discount at still another. For example, during the late fall months, when the grain crops begin to be sent East, New York is sending a great many checks and drafts to the section of which Chicago is the trade center. Exchange on New York is then very plentiful in Chicago, and if a man in Chicago wished to buy a draft on New York for a large amount, say $10,000 or more, the Chicago banks will sell it to him at a discount. But if a man in New York at that time wished to buy a draft on Chicago for $10,000, he would have to pay a premium, because the New York banks would be anxious not to decrease their Chicago balances.

Early in the spring, when New York importers and jobbers are sending foreign and domestic manufactured goods for distribution in the West, a great many checks and drafts are being sent from the West to New York, and exchange is at a discount in New York and at a premium in Chicago. This principle applies at all trade centers between which exchange operations go on. Smaller places make their settlements in or through larger places, and the main exchange transactions go on between the few leading cities, with converging lines on New York.

The rate of exchange between two cities will never exceed the cost of shipping actual money from one of the cities to the other, except in time of panic or a financial unrest. Thus when the cost of sending money by express from New York to Chicago is $5 per $10,000, the discount in New York or the premium in Chicago will not greatly exceed $\frac{1}{20}\%$ ($5 per $10,000). To prevent the rates from going any higher the banks will arrange for the shipment of actual money from New York to Chicago.

As a rule no charge is made for cashing bank drafts on the trade centers of the country, like New York, Chicago, and Philadelphia.

451. It has been seen that banks frequently charge a small fee for collecting paper payable out of town.

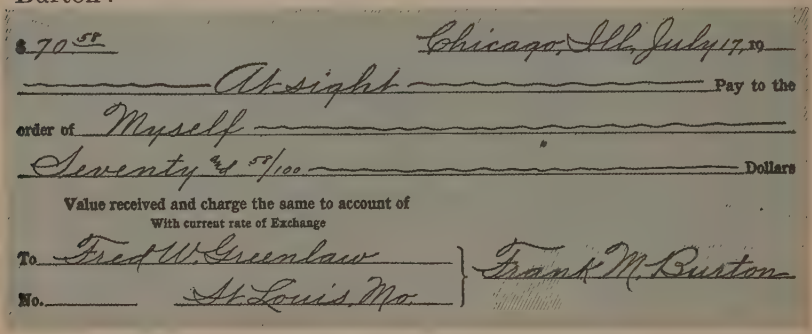
In some cases the rates of collection are more or less arbitrary; in others they are governed by trade movements, the same as rates of exchange. In still others the clearing house association fixes the rate.

ORAL EXERCISE

Find the cost of the following bank drafts:

1. $18,500 at $\frac{1}{20}\%$ discount; at 40¢ per $1000 premium.
2. $516.90 at $\frac{1}{10}\%$ premium; at 50¢ per $1000 discount.
3. $1600.80 at 75¢ per $1000 premium; at $\frac{1}{10}\%$ discount.
4. A draft for $4000 was bought for $3998. Was exchange at a premium or at a discount, and what rate?
5. J. E. Smith & Co. drew at sight on E. M. Barrows for $250 and made collection through their bank. If the bank charged $\frac{1}{10}\%$ for collection, for what amount did J. E. Smith & Co. receive credit?
6. During the late fall many checks and drafts are being sent to the southern cities in payment for shipments of cotton. At such times is exchange on New York likely to be at a discount or at a premium in New Orleans? in New York?

7. Frank M. Burton wishes to collect an account of $70.58 and for this purpose draws the following draft and leaves it with the National Express Co. for collection. If the express company charges 25¢ for collection, how much will it collect of Fred W. Greeniaw? how much will it pay Frank W. Burton?


$70.58 Chicago, Ills. July 17, 19

At sight Pay to the

order of Myself

Seventy & 58/100 Dollars

Value received and charge the same to account of
With current rate of Exchange

To Fred W. Greenlaw
No. St. Louis, Mo.

Frank M. Burton

Note that the draft contains the clause "With current rate of Exchange." This means that the drawee is requested to pay the face of the draft plus the cost of exchange. Nearly all express companies have arrangements by which they undertake the collection of notes and accounts. The process of collecting is simple. The note or draft covering the amount of the account is placed in a collection envelope furnished by the express company, and sent to its destination. If collection cannot be made, notice is given with reasons for refusal; if collection is made, the money is sent back in the collection envelope, and the amount, less collection charges, paid to the one for whom the collection was undertaken. The charge varies with the distance.

WRITTEN EXERCISE

1. A bank draft for $15,000 was bought for $14,992.50. Was exchange at a premium or at a discount, and what rate? At this rate find the cost of a draft for $17,121.98; a draft for $12,929.75; a draft for $127,162.89.

2. I gave the American Express Co. an account of $178.50 for collection. If the collection charges were $2.50 per $1000, how much did I receive from the company? At this rate what should be the proceeds from the collection of three drafts with amounts as follows: $125.60; $218.90; and $134.50?

3. An agent sold for me 1000 T. hay at \$17.50 per ton. He paid \$125 for cartage, \$75 for storage, charged $2\frac{1}{2}\%$ commission, and remitted the proceeds by a bank draft bought at $\frac{1}{8}\%$ premium. What was the face of the draft?

4. A Boston commission merchant sold for his principal in Chicago 27,518 lb. leather at $25\frac{7}{8}¢$ per pound. If he charged a commission of $4\frac{1}{2}\%$, how large a bank draft, bought at \$1.50 per \$1000 premium, should he remit to his principal?

5. Mar. 8 Edward Whitman & Co. drew a draft payable 30 da. after date on Elwood & Spears for \$375.98 and had it discounted at City Bank. If the rate of collection was $\frac{1}{8}\%$ and interest 5%, what were the proceeds of the draft?

6. Copy and complete the following letter of advice, assuming that the rate of collection is $\frac{1}{8}\%$ on Nos. 720 and 716, and $\frac{1}{10}\%$ on Nos. 692 and 710. Check the results.

NATIONAL EXCHANGE BANK

ALBANY, N.Y., *Feb. 12,* 19

MR. *Chas. W. Hamilton,* Cashier
Merchants National Bank
Boston, Mass.

DEAR SIR, — We credit your account this day for the proceeds of collections as stated below.

Respectfully yours,
L. H. PIERSON, Cashier

YOUR NO.	PAYER	AMOUNT		CHARGES		PROCEEDS	
720	*E. C. Mills*	*800*	*00*	*1*	*00*	*799*	*00*
716	*W. E. Frey*	*900*	*75*	*	**	***	**
692	*C. O. Bond & Co.*	*3750*	*50*	*	**	****	**
710	*W. D. Long & Son*	*37500*	*00*	**	**	*****	**
		42951	*25*	**	**	*****	**

FOREIGN EXCHANGE

Foreign Money

ORAL EXERCISE

1. Repeat the table for English money; for French money; for German money. (See Appendix B, page 451.)

2. What is the value of a pound sterling in United States money? of a franc? of a mark?

3. Express $4866.50 in English money; £100 in United States money. Express $1930 in French money; 1000 fr. in United States money. Express $238 in German money; 10000 M. in United States money.

A pound sterling is commonly thought of as about $5; a shilling or a mark as about 25¢; a penny as about 2¢; a franc or lira as about 20¢; a guilder as about 40¢. In problems 4–6 use these approximations.

4. Express $100 as English money; as German money; as French money; 1500 guilders in United States money.

5. Express as United States money: £15; £8 5*s.*; £25 10*s.*; 100 M.; 1500 M.; 1750 M.; 75 fr.; 350 fr.; 200 fr.

6. A and B while abroad spent 3 wk. in Naples, Italy. If their expenses here averaged 25 lire apiece per day, how much was this in United States money for the 3 wk.?

WRITTEN EXERCISE

1. Express as pounds and decimals of a pound: £25 6*s.*; £150 15*s.*; £200 10*s.* 6*d.*; £300 12*s.* 9*d.*

2. Reduce to United States money: £25 10*s.*; £120 9*s.*

3. Reduce to United States money: 275 M.; 1500 M. 75 pf.; 315 fr.; 725 fr.; £115 10*s.* 6*d.* Reduce $1250 to English money; to French money; to German money.

4. In a recent year the funded debt of the German Empire amounted to 2,733,500,000 M., of which 1,240,000,000 M. bore interest at $3\frac{1}{2}\%$ and 1,493,500,000 M. at 3%. Express in United States money the interest on the funded debt for 1 yr.

The Metric System

452. The **metric system** is a system of measures having a decimal scale of relation. It was invented by France, and is now used in practical business in a large part of the civilized world. It has been authorized by law in Great Britain and the United States, but is not generally used in these countries except in foreign trade and in scientific investigations.

The **principal units** of the system are the **meter** for length, the **liter** for capacity, and the **gram** for weight. Submultiples and multiples of these units are easily learned when the meaning of the prefixes is known. The Latin prefixes, **deci**, **centi**, and **milli** mean respectively 0.1, 0.01, and 0.001 of the unit. The Greek prefixes **deca**, **hekto**, **kilo**, and **myria** mean respectively, 10, 100, 1000, and 10,000 times the unit.

Table of Length

10 **millimeters (mm.)**	= 1 **centimeter (cm.)**	=	.01 meter.
10 **centimeters**	= 1 decimeter (dm.)	=	.1 meter.
10 decimeters	= 1 **meter (m.)**	=	1. meter.
10 **meters**	= 1 dekameter (Dm.)	=	10. meters.
10 dekameters	= 1 hektometer (Hm.)	=	100. meters.
10 hektometers	= 1 **kilometer (Km.)**	=	1000. meters.
10 **kilometers**	= 1 myriameter (Mm.)	=	10,000. meters.

The units in common use are indicated by black-faced type.

Table of Square Measure

100 sq. millimeters	= 1 **sq. centimeter (sq. cm.)**	=	.0001 sq. meter.
100 **sq. centimeters**	= 1 sq. decimeter (sq. dm.)	=	.01 sq. meter.
100 sq. decimeters	= 1 **sq. meter (sq. m.)**	=	1. sq. meter = 1 centare.
100 **sq. meters**	= 1 sq. dekameter (sq. Dm.)	=	100. sq. meters = 1 are.
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)	=	10,000. sq. meters = 1 hectare.
100 sq. hektometers	= 1 **sq. kilometer (sq. Km.)**	=	1,000,000. sq. meters.
100 **sq. kilometers**	= 1 sq. myriameter (sq. Mm.)	=	100,000,000. sq. meters.

The **centare**, **are** (a.), and **hektare** are common terms in land measurements.

Table of Cubic Measure

1000 cu. millimeters	= **cu. centimeter (cu. cm.)**	=	.000001 cu. m.
1000 **cu. centimeters**	= 1 cu. decimeter (cu. dm.)	=	.001 cu. m.
1000 cu. decimeters	= 1 **cu. meter (cu. m.)**	=	1. cu. m.
1000 **cu. meters**	= 1 cu. dekameter (cu. Dm.)	=	1000. cu. m.
1000 cu. dekameters	= 1 cu. hektometer (cu. Hm.)	=	1,000,000. cu. m.
1000 cu. hektometers	= 1 **cu. kilometer (cu. Km.)**	=	1,000,000,000. cu. m.
1000 **cu. kilometers**	= 1 cu. myriameter (cu. Mm.)	=	1,000,000,000,000. cu. m.

The cubic meter is also called a **stere**, a unit used in measuring wood.

TABLE OF CAPACITY

10 milliliters (ml.)	= 1 **centiliter** (cl.)	=	.01 liter.
10 **centiliters**	= 1 deciliter (dl.)	=	.1 liter.
10 deciliters	= 1 **liter** (l.)	=	1. liter.
10 **liters**	= 1 dekaliter (Dl.)	=	10. liters.
10 dekaliters	= 1 **hektoliter** (Hl.)	=	100. liters.
10 **hektoliters**	= 1 kiloliter (Kl.)	=	1000. liters.

A liter is the same as a cubic decimeter.

TABLE OF WEIGHT

10 **milligrams** (mg.)	= 1 **centigram** (cg.)	=	.01 gram.
10 **centigrams**	= 1 decigram (dg.)	=	.1 gram.
10 decigrams	= 1 **gram** (g.)	=	1. gram.
10 **grams**	= 1 dekagram (Dg.)	=	10. grams.
10 dekagrams	= 1 hektogram (Hg.)	=	100. grams.
10 hektograms	= 1 **kilogram** (Kg.)	=	1000. grams.
10 **kilograms**	= 1 myriagram (Mg.)	=	10,000. grams.
10 myriagrams	= 1 quintal (Q.)	=	100,000. grams.
10 quintals	= 1 **tonneau** (T.)	=	1,000,000. grams.

The tonneau is usually called a **metric ton**.

TABLE OF APPROXIMATE VALUES

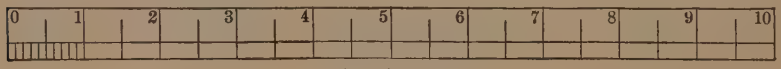
A meter	= $3\frac{1}{4}$ ft. or 1.1 yd.	A stere	= $\frac{8}{11}$ cd.
A kilometer	= $\frac{5}{8}$ mi.	A gram	= $15\frac{1}{2}$ gr.
A square meter	= $1\frac{1}{5}$ sq. rd.	A kilogram	= $2\frac{1}{5}$ lb. av.
An are	= 4 sq. rd.	A liter	= 1 qt.
An hectare (Ha)	= $2\frac{1}{2}$ A.	An hektoliter	= $2\frac{1}{5}$ bu.
A cubic meter	= 1.3 cu. yd.	A metric ton	= 2200 lb.

ORAL EXERCISE

1. Name the prefix which means 10,000; 0.001; 100; 0.01; 10; 0.1; 1000.

2. Read the following: 2.5 m.; 72 mm.; 95.5 cm.; 302.05 km. Express 475.125 m. in millimeters; in hektometers.

3. Which of the divisions of the following scale are millimeters? centimeters?



1 decimeter

4. A certain tower is 200 m. high; this is approximately how many feet?

5. How many square meters in 1 a.? how many ares in 5 Ha.? in 25 Ha.?

6. How many liters in 1 cu. m.? in 5 cu. m.? Find the cost of 5 Kl. of milk at 5¢ a liter; at 4¢ a liter.

7. Find the length of your schoolroom in meters; the weight of any familiar object in kilograms.

8. Bought 1000 m. of cloth. How many yards was this?

9. An importer bought 1000 l. of liquors at 80¢ a liter. If he sold it at $3.50 per gallon, did he gain or lose, and how much?

10. The distance from Paris to Cologne is 510 Km.; from Cologne to Mainz 150 Km. Express these distances in miles.

WRITTEN EXERCISE

1. At $75 an acre find the cost of 75 Ha. of land.

2. Find the cost of 175.75 m. of lace at 65¢ a meter.

3. How many steres of wood in a pile 12 m. long, 1.5 m. wide, and 3 m. high? How many cords?

4. A merchant bought cloth at $1.14 per meter, including duties. For how much must he sell it per yard to gain 33⅓%?

5. I imported 1000 m. of silk dress goods (see duties, page 294) at 10 fr. per meter and sold it at $3 per yard. Did I gain or lose and how much, the silk being 1 yd. wide?

6. The distance between two places on a map is 15.5 cm.; this is $\frac{1}{10{,}000}$ of the actual distance. What is the actual distance in miles?

7. C bought cloth at $2 per meter, including duties, and sold it by the yard at a gain of 25%. What was the selling price per yard?

8. The speed rate of a certain express train is 64 Km. an hour; of a certain mail train, 48 Km. an hour. In a journey of 384 Km. what time will be saved by taking the express instead of the mail train.

Foreign Money Orders

453. Small sums are frequently sent from one country to another by means of foreign money orders.

The **international postal money order** and the **foreign express money order** or check are both extensively used for this purpose. These orders are usually drawn payable in the money of the country on which they are issued. They are similar in form to domestic money orders, but are issued on practically the same principle as the ordinary bank draft.

ORAL EXERCISE

1. D in Chicago wishes to send E in Havre, France, 780 fr. At 19.5¢ to the franc, how large an express money order (in U.S. money) can he buy? 132.10

2. B in New York wishes to send $120 to C in Leipzig, Germany. At 24¢ to the mark, how large an express money order (in marks) can he buy? 500 marks

3. At 1% premium find the cost of an international money order, payable in Great Britain, for each of the following amounts: $40; $50; $75; $100; $150; $200.

4. A in Boston bought an international money order for $20 and sent it to a friend in Liverpool, England. At 1% premium, what did the order cost? For how many pounds sterling (approximately) was it issued?

WRITTEN EXERCISE

1. I wish to send $100 to G in Holland. At 40½¢ to the guilder, how large an express money order can I buy?

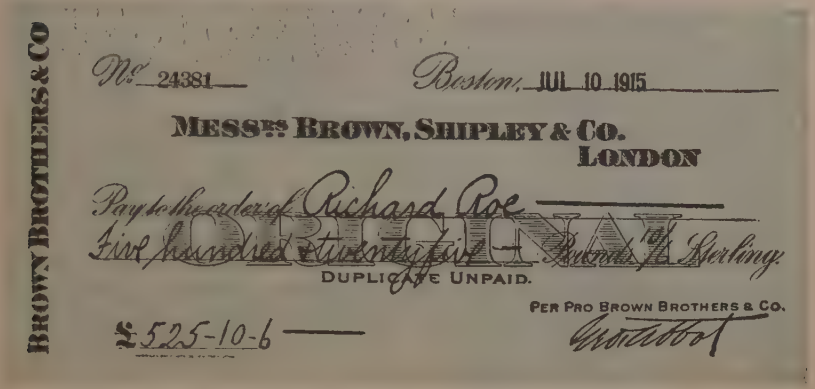
2. I wish to send $50 to a friend in Scotland. At $4.87 to the pound, how large an express money order can I buy?

3. C in Chicago sent D in Geneva an express money order for 256.41 fr. At 19.5¢ to the franc, how much did the order cost C?

4. E in Philadelphia sent F in Naples an international postal money order for 128.21 lira. At 19.5¢ to the lira, how much did the order cost E?

Bills of Exchange

454. Drafts of a person or a bank in one country on a person or a bank in another country are usually called **bills of exchange.**


BROWN BROTHERS & CO

No. 24381 Boston, JUL 10 1915

MESSRS BROWN, SHIPLEY & CO.
LONDON

Pay to the order of Richard Roe

Five hundred & twenty five — Pounds 10/6 Sterling.

ORIGINAL

DUPLICATE UNPAID.

PER PRO BROWN BROTHERS & CO.

£525-10-6

455. Bills of exchange may be divided into three classes: (1) **bankers' bills**, which are drawn by one banker upon another; (2) **commercial bills**, which are drawn by one merchant upon another; (3) **documentary bills**, which are drawn by one merchant upon another and secured by the assignment and transfer of a bill of lading and policy of insurance covering merchandise on its way to the market.

The foregoing form is a bankers' demand draft or check.

Bankers' bills of exchange are frequently issued in duplicate; that is, in sets of two of like tenor and amount. These bills are sometimes sent by different mails; but more frequently the original is sent and the duplicate is placed on file to be sent in case of necessity. Duplicate bills are so conditioned that the payment of one of them cancels the other. The bankers' sole bill of exchange is also used. This is preferred by many, inasmuch as it may be more easily negotiated by the payee when he resides in a city other than the one drawn upon. Commercial and documentary bills of exchange are usually issued in duplicate.

456. The **mint par of exchange** is the actual value of the pure metal in the monetary unit of one country expressed in terms of another.

The mint par of exchange is determined by dividing the weight of pure gold in the monetary unit of one country by the weight of pure gold in the monetary unit of another. Thus, the United States gold dollar contains 23.22 troy grains of pure gold and the English pound sterling, 113.0016 troy grains. $113.0016 \div 23.22 = 4.8665$. Since there is 4.8665 times as much pure gold in the pound sterling as in the gold dollar, the pound sterling is worth 4.8665 times \$1, or \$4.8665. The mint par of exchange is used mainly in determining the values on which to compute customs duties.

457. The **rate of exchange** is the market value in one country of the bills of exchange on another.

The price paid for bills of exchange fluctuates. When the United States owes Great Britain exactly the same amount that Great Britain owes the United States, the debts between these countries can be paid without the transmission of money, and exchange is at par. But when Great Britain owes the United States a greater amount than the United States owes Great Britain, exchange in the United States is at a discount and in Great Britain at a premium, and *vice versa*. The rates of premium or discount are limited by the cost of shipping gold bullion from one country to another. The cost of shipping gold from New York to London is about $\frac{5}{8}\%$. Therefore, when A in New York owes B in London, and A cannot buy a bill of exchange on London for less than \$4.88$\frac{1}{2}$ to \$4.89, it is cheaper for him to export gold. On the other hand, if D in London owes C in New York and C cannot sell a draft on D for more than \$4.83$\frac{3}{4}$ to \$4.84, it is cheaper for him to import gold. The greater part of exchange is drawn on Great Britain, France, Germany, Holland, Belgium, and Switzerland. Because London is the financial center of the world, probably more foreign exchange is drawn on Great Britain than on all the other countries combined.

458. Exchange on Great Britain is usually quoted at the number of dollars to the pound sterling; **exchange on France, Belgium**, and **Switzerland**, at the number of francs to the dollar; **exchange on Germany**, at the number of cents to each four marks; **exchange on Holland**, at the number of cents to each guilder.

The accompanying foreign exchange rates were quoted recently.

	60 Days	Demand
Sterling	4.85$\frac{1}{2}$	4.87$\frac{1}{2}$
Germany, reichsmarks	94$\frac{7}{8}$	95$\frac{3}{8}$
France, francs	5.16$\frac{7}{8}$	5.15
Belgium	5.18$\frac{3}{4}$	5.15$\frac{5}{8}$
Switzerland, francs	5.18$\frac{3}{4}$	5.15$\frac{5}{8}$
Holland, guilders	40	40$\frac{3}{8}$

In Great Britain 3 da. of grace are allowed on all bills drawn payable after sight, but drafts on Great Britain payable at sight or on demand have no grace. There are no days of grace allowed on any drafts drawn on Germany, and nearly all Europe, excepting Holland, where 1 da. of grace is allowed.

WRITTEN EXERCISE

1. Using the foregoing table of quotations, or current quotations clipped from any daily newspaper, find the cost of demand drafts for each of the following amounts:

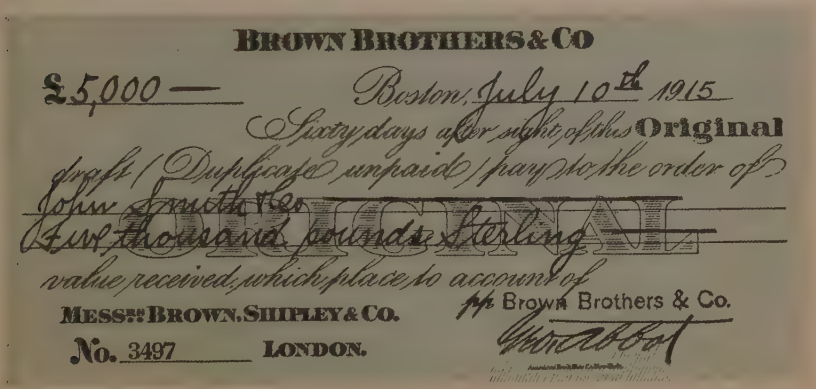
a.	£ 100.	*d.*	160 guilders.	*g.*	200 M.	*j.*	6000 M.
b.	£ 1200.	*e.*	240 guilders.	*h.*	160 M.	*k.*	4000 M.
c.	£ 1800.	*f.*	1200 guilders.	*i.*	2000 M.	*l.*	12000 M.

2. Find the cost of a 60-da. draft for each of the amounts in problem 1.

WRITTEN EXERCISE

1. F. M. Cole & Co., importers, Boston, owe Richard Roe, London, £ 525 10*s*., 6*d*., buy by check the draft illustrated on page 375, and remit it in full of account. If exchange on London is $\$4.87\frac{1}{2}$, what was the amount of the check?

2. Jordan, Marsh & Co. wish to import a quantity of woolen goods from Bradford, England. They make up an order and inclose in payment the following draft which they buy by check, at $\$4.85\frac{1}{2}$. What was the amount of the check?


BROWN BROTHERS & CO

£5,000 — Boston, July 10th 1915

Sixty days after sight of this Original

draft (Duplicate unpaid) pay to the order of

John Smith & Co.

Five thousand pounds Sterling

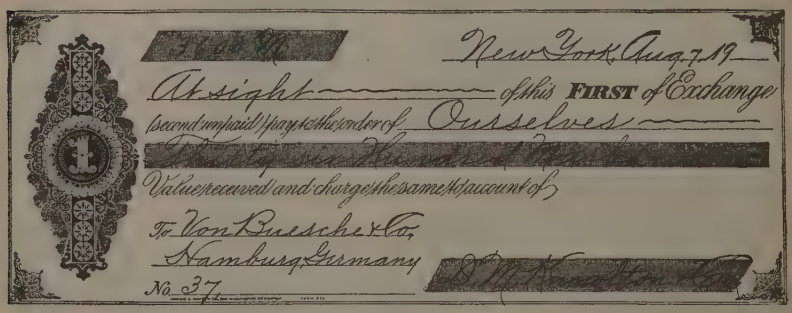
value received, which place to account of

MESSRS BROWN, SHIPLEY & CO. for Brown Brothers & Co.

No. 3497 LONDON.

3. 45 da. before the draft was due (problem 2) John Smith & Co. sold it to Baring Bros. at 2% discount. How much (in English money) did they receive? Write the indorsements which would appear on the back of the draft.

4. D. M. Knowlton & Co. drew the following **commercial bill of exchange** and sold it to Kidder, Peabody & Co. at $96\frac{3}{8}$. How much was received for it?


New York, Aug. 7, 19—

At sight ——— of this FIRST of Exchange

(second unpaid) Pay to the order of Ourselves ———

Dollars

Value received and charge the same to account of

To Von Buesche & Co.,

Hamburg, Germany

No. 37.

Commercial bills of exchange are usually drawn by exporters against funds abroad which have accumulated to their credit from sales previously made. The exporter generally waits until the rates of exchange are high and then draws the draft as above.

5. Aug. 1 T. H. Reed & Co., exporters, Minneapolis, Minn., bought through their broker, 24,000 bu. No. 1 wheat at 84¢ per bushel and paid for same by check. What was the amount of the check, the broker's commission being $\frac{1}{8}$¢ per bushel?

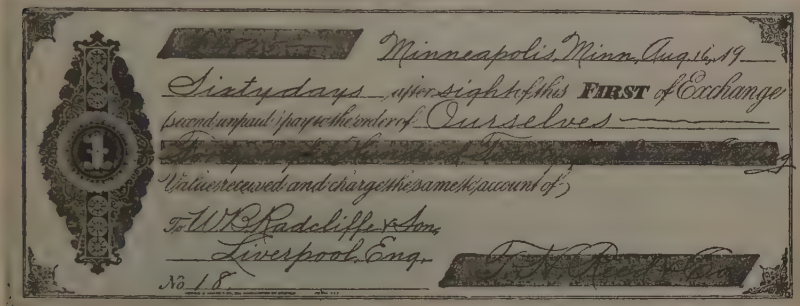
6. Aug. 2 the wheat was delivered and placed with City Elevator for storage. The storage rates were $\frac{3}{4}$¢ per bushel for the first 10 da. or fraction thereof, and $\frac{1}{40}$¢ per bushel for each additional day thereafter. On Aug. 15 the wheat was withdrawn from the City Elevator and delivered to the Soo Freight Line for shipment to W. B. Radcliffe & Son, Liverpool. What was the amount of the storage bill?

7. The wheat was sold to W. B. Radcliffe & Son at £1 12*s*. 2*d*. per quarter (8 bu. or 480 lb.). Make out the bill under date of Aug. 15.

8. On Aug. 15 a through bill of lading in duplicate was received from the Soo Freight Line. If the through freight rate from Minneapolis to Liverpool was 2*d*. per hundredweight, what was the amount of the freight bill?

9. Aug. 16, upon presentation of the bill of lading to the Western Assurance Co., the goods were insured for 10% more than their billed value and a certificate of insurance issued. What was the amount of the premium, the rate being $1\frac{1}{4}$%?

10. T. H. Reed & Co., drew the following draft on W. B. Radcliffe & Son and attached it to the bill of lading and certificate of insurance. These documents, which constitute what is called a **documentary bill of exchange**, were then offered for sale and later sold to Kidder, Peabody & Co., at the rate of \$4.84$\frac{1}{8}$ per pound. How much was received for the bill?


Minneapolis, Minn. Aug. 16, 19—

Sixty days after sight of this FIRST of Exchange

(Second unpaid) pay to the order of Ourselves

Value received and charge the same to account of

To W. B. Radcliffe & Son

Liverpool, Eng.

No. 18

11. Aug. 17 Kidder, Peabody & Co. sold the draft to American Express Co. at \$4.84$\frac{1}{4}$. If the American Express Co. paid by check, what was the amount of the check?

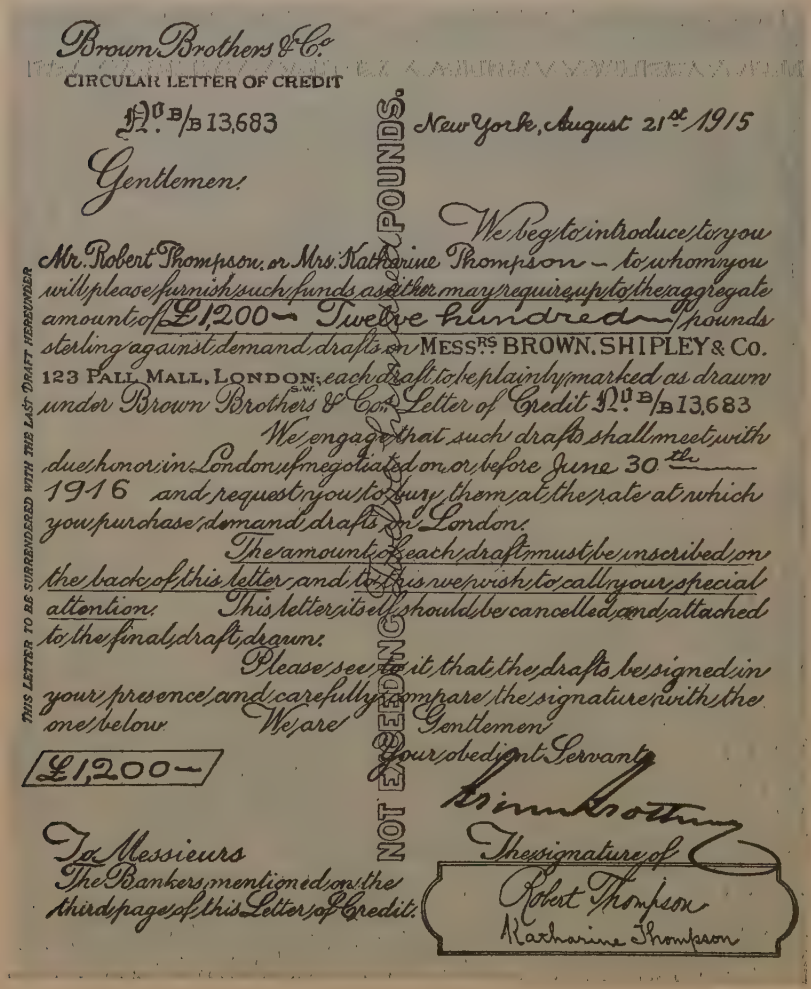
12. American Express Co. forwarded the bill to Provincial Bank, Liverpool, for collection, and this bank presented the draft to W. B. Radcliffe & Son for acceptance. Sept. 1 the wheat arrived by steamer and as the draft was stamped "Surrender documents only upon payment of draft" W. B. Radcliffe & Son had to pay the draft before they could get the documents or the goods. As the draft has 47 da. yet to run, the bank allowed W. B. Radcliffe & Son 1% discount. What was the amount paid by W. B. Radcliffe & Son?

Such drafts are frequently stamped "Surrender documents upon acceptance of the draft." In such cases the documents would be delivered to the consignee upon the acceptance of the draft, and he could then obtain possession of the goods.

13. What was T. H. Reed & Co.'s net gain or loss on the transactions in problems 5–10?

Letters of Credit and Traveler's Checks

459. A **traveler's letter of credit** is an instrument issued by a banker instructing his correspondents in specified places to pay the holder funds in any amount not exceeding a specified sum.



Brown Brothers & Co

CIRCULAR LETTER OF CREDIT

No. B/B 13,683

New York, August 21st 1915

Gentlemen:

We beg to introduce to you Mr. Robert Thompson, or Mrs. Katharine Thompson — to whom you will please furnish such funds as either may require up to the aggregate amount of £1,200— Twelve hundred — pounds sterling against demand drafts on MESSRS. BROWN, SHIPLEY & Co. 123 Pall Mall, London, S.W. each draft to be plainly marked as drawn under Brown Brothers & Co.'s Letter of Credit No. B/B 13,683

We engage that such drafts shall meet with due honor in London if negotiated on or before June 30th 1916 and request you to buy them at the rate at which you purchase demand drafts on London.

The amount of each draft must be inscribed on the back of this letter and to this we wish to call your special attention. This letter itself should be cancelled and attached to the final draft drawn.

Please see to it that the drafts be signed in your presence and carefully compare the signature with the one below. We are Gentlemen

Your obedient Servants

Brown Brothers

£1,200—

The signature of
Robert Thompson
Katharine Thompson

To Messieurs
The Bankers mentioned on the
third page of this Letter of Credit.

THIS LETTER TO BE SURRENDERED WITH THE LAST DRAFT HEREUNDER

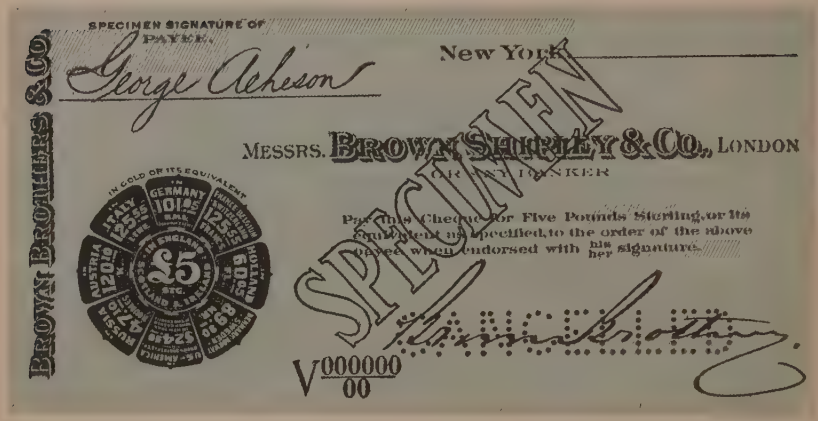
NOT EXCEEDING £ POUNDS.

The purchaser of a letter of credit is required to subscribe his name upon the document as a means of identification later on. Other copies of the signature are left and forwarded to the leading foreign banks drawn upon. When the traveler desires funds, he presents his letter to the proper bank at the place in which he is stopping. The letter itself always specifies the banks that will honor the draft. When the letter is presented to a foreign banker for payment, he draws a sight draft on the London banker, which draft the traveler is required to sign. If the signatures on the letter and on the draft are identical, the amount desired is promptly paid and indorsed on the back of the letter. The indorsements on the back of a letter show at all times the balance available for the traveler. The bank making the last payment retains the letter to send to the drawee in London. Letters of credit are usually drawn payable in pounds sterling, but they are paid in the current money of the country in which they are negotiated. Banks usually charge 1% commission for issuing a letter of credit.

460. Another instrument frequently used by travelers is what is called a **traveler's check.**

When a check is purchased, the buyer signs his name in the upper left-hand corner. When he wishes funds, he presents his check to the correspondent of the express company or bank and signs his name either in the upper left-hand corner or on the back of the check. On the form above, he would sign his name in the lower left-hand corner; but on the form on page 382 he would sign his name on the back. The latter form is considered better because it is more difficult to forge another's signature when there is no signature near at hand from which to copy.

The terms of issue are cash for the face amount plus ½% commission.


BROWN BROTHERS & CO.
SPECIMEN SIGNATURE OF PAYEE.
George Acheson
New York
MESSRS. BROWN SHIPLEY & Co., LONDON
Pay this Cheque for Five Pounds Sterling, or its equivalent as specified, to the order of the above named when endorsed with his her signature.
SPECIMEN
V 000000 00

ORAL EXERCISE

1. At $4.85 to the pound sterling plus 1% commission, what did the letter of credit on page 380 cost?

2. At the same rate, find the cost of a letter of credit for £500; £1000.

3. At $\frac{1}{2}$% commission, what will be the total cost of 10 checks like the sample on page 381? of 20 checks? of 25 checks?

4. At $4.85 to the pound plus $\frac{1}{2}$% commission, what was the cost of a traveler's check on page 382? of a book of 10 checks like the sample on page 382?

WRITTEN EXERCISE

1. On the letter of credit, page 380, the following payments are recorded on the back: Aug. 31, £200; Sept. 9, £400; Oct. 15, £250; Nov. 1, £100; Nov. 12, £200. The holder returns to New York on Nov. 20 and presents the letter to Brown Brothers & Co. for the refund. At $4.85 to the pound, how much will Brown Brothers & Co. pay on the letter?

In this problem it is assumed that Brown Bros. & Co. refund 1% commission on the unused portion of the letter.

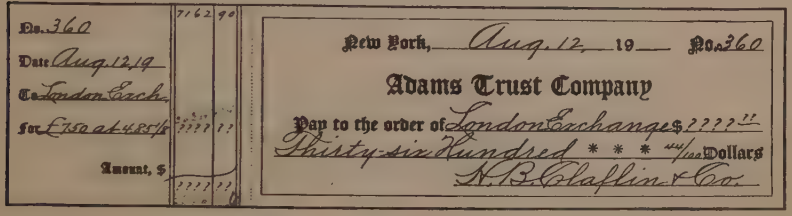
2. At 25¢ per word and 1% of the amount, find the cost of a twenty-one word cable money order from Boston to Paris for 25,000 fr. when exchange is quoted at 5.15⅝.

Money may be cabled from one country to another on the same principle that it is telegraphed from one part of any country to another part. In a cable message a charge is made for each word in the address of the one to whom it is sent.

WRITTEN REVIEW EXERCISE

1. A broker sold for me a bill on Manchester, England, at \$4.84½ and charged ⅛% brokerage. What was the face of the bill, if the proceeds were \$5218.50?

2. How much remains in the bank to the credit of H. B. Claflin & Co. after the following check was issued?


No. 360
Date Aug. 12, 19
To London Exch.
For £750 at 4.85⅝
Amount, $
7162 90
New York, Aug. 12, 19 No. 360
Adams Trust Company
Pay to the order of London Exchange $ 7162.90
Thirty-six Hundred * * * 90/100 Dollars
H. B. Claflin & Co.

3. My agent in Brussels, Belgium, purchased for me carpet amounting to 35,000 fr., and his commission was 5%. I remitted him a draft to cover the cost of the carpet and the commission for buying. If exchange was 5.15⅝, and I paid for the draft by check, what was the amount of the check?

4. My agent in Rotterdam sold for me 525 kegs of tobacco, each containing 50 lb., at ½ guilder per pound, and charged me a commission of 4½%. I drew on him for the proceeds and sold the draft to a broker at 40⅜. If the broker charged ⅛% for his services, what did I receive as proceeds of the draft?

EQUATIONS AND CASH BALANCE

CHAPTER XXXI

EQUATION OF ACCOUNTS

ORAL EXERCISE

1. How long will it take \$5 to produce the same interest as \$10 for 10 da.? The use of \$100 for 1 mo. is equivalent to what sum for 2 mo.?

2. If I have the use of \$50 of A's money for 30 da., how much of my money should he have the use of for 15 da. in return for the accommodation?

3. The interest on \$40 for 2 mo. plus the interest on \$40 for 4 mo. is equal to the interest on \$80 for how many months?

4. D owes E \$100; \$50 is due in 2 mo. and the balance in 4 mo. In how many months may the whole be paid without loss to either party?

5. On Apr. 1 I bought a bill of goods amounting to \$200, payable as follows: \$100 in 3 mo. and the balance in 5 mo. In how many months may the whole sum be equitably paid?

6. A owes B \$400 and pays \$200 30 da. before the account is due. How long after the account is due may B have in which to pay the balance?

461. The process of finding the date on which the settlement of an account may be made without loss of interest to either party is called **equation of accounts.**

Sometimes one or more of the items in a personal account are not paid at maturity and the holder of the account suffers a loss; sometimes one or more of the items are paid before maturity and the holder of the account realizes a gain. To equitably adjust these items of loss and gain, accounts are equated. Retail accounts are not often equated; but wholesale and commission accounts are frequently equated, particularly foreign ones.

462. The time that must elapse before several debts, due at different times, may be equitably paid in one sum is called the **average term of credit**; the date on which payment may be equitably made, the **average date of payment**, the **equated date**, or the **due date.**

463. Any assumed date of settlement with which the several dates in the account are compared for the purpose of determining the actual due date is sometimes called the **focal date.**

The **face value** of each item should always be used in equating accounts. Items not subject to a term of credit and interest-bearing notes are worth their face value on the day they are dated. Items subject to a term of credit and non-interest-bearing notes are not worth their face value until maturity.

SIMPLE ACCOUNTS

ORAL EXERCISE

1. If I owe $200 due Jan. 1 and $400 due Jan. 31, when may both accounts be equitably paid in one sum?

SOLUTION. On Jan. 31, there is legally due $600 + $1 (the interest on $200 for 30 da.). Since more than the face of the account is due, the equitable date of settlement is *before* Jan. 31. It will take $600 one third as long as $200 to produce $1 interest. $\frac{1}{3}$ of 30 da. = 10 da. The whole account may therefore be paid 10 da. before Jan. 31, or Jan. 21, without loss to either party.

2. You sold Baker, Taylor & Co. goods as follows: Apr. 20, $600; Apr. 30, $600. How much is legally due on the account Apr. 30? On what day may the whole account, $1200, be paid without interest?

3. When is the following account due by equation?

A. B. COMER

19—										
Sept.	1	To mdse.	300	00						
	21	To mdse.	300	00						

4. Rowland & Hill bought goods of you as follows: Oct. 16, $400; Oct. 31, $800. How much was legally due on the account Oct. 31? On what date can the whole of the account, $1200, be paid without interest?

464. Example. On what date may the total of the following account be paid without interest?

F. M. Pratt & Co.

19—										
Jan.	1	To mdse.	30	00						
	9	To mdse.	120							
	15	To mdse.	150							
	21	To mdse.	300							
	26	To mdse.	60							

Solution. Take the latest date, Jan. 26, as the focal date. If settlement was made on Jan. 26, the holder of the account might charge interest on each item as shown in the accompanying statement.

The holder loses \$0.11 per day as long as the account remains unsettled. If settlement was made Jan. 26, the loss would be \$0.99, or 9 days' interest; therefore if the account were settled 9 da. before Jan. 26, the holder would lose nothing.

Date		Amount	Days	Interest
Jan.	1	\$30	25	\$.125
	9	120	17	.34
	15	150	11	.275
	21	300	5	.25
	26	60	0	
		\$660		\$.99

The amount of the account = \$660.
The interest on \$660 for 1 da. = \$0.11.
\$0.99 ÷ \$0.11 = 9, or the number of days.
Jan. 26 − 9 da. = Jan. 17, the equated date.

Proof. The proof of the problem must show that the interest on the items *dated before Jan. 17*, the equated date, is offset by the discount on the items *dated after Jan. 17*. The following items are dated before Jan. 17:

Date	Interest Period	Item	Interest
Jan. 1 to 17	16 da.	\$30	\$.08
9 to 17	8	120	.16
15 to 17	2	150	.05
		Total interest,	\$.29

The following items are dated after Jan. 17:

Date	Discount Period	Item	Discount
Jan. 17 to 21	4 da.	\$300	\$.20
17 to 26	9	60	.09
		Total discount,	\$.29

The proof shows that the equated date, Jan. 17, is correct.

Any rate of interest may be used in equating an account. As a matter of convenience, always use 6%. If items are subject to terms of credit, add the time to the date of the items before beginning to equate.

WRITTEN EXERCISE

In each of the following problems find the equated date and prove the work. Assume that all the dates are in 1916.

1. F. M. Drake, Dr.

Mar. 2, To mdse.	$120.
8, To mdse.	180.
11, To mdse.	60.
17, To mdse.	240.
23, To mdse.	150.

2. Louis M. Allen, Dr.

Apr. 3, To mdse.	$160.
9, To mdse.	250.
13, To mdse.	100.
19, To mdse.	280.
23, To mdse.	420.

3. Geo. M. Barton, Dr.

Aug. 3, To mdse., 60 da.	$360.
6, To mdse., 30 da.	240.
11, To mdse., 30 da.	300.
19, To mdse., 30 da.	60.
24, To mdse., 30 da.	180.

4. Leon H. Hazelton, Dr.

June 6, To mdse.	$200.
9, To mdse.	300.
14, To mdse.	400.
24, To mdse.	600.
27, To mdse.	330.

5. Carter & Co., Dr.

May 5, To mdse.	$180.
12, To mdse.	300.
16, To mdse.	230.
20, To mdse.	270.
23, To mdse.	360.

6. Lamson & Roe Co., Dr.

Dec. 1, To mdse., 3 mo.	$294.20.
10, To mdse., 3 mo.	698.40.
20, To mdse., 60 da.	136.60.
24, To mdse., 60 da.	740.60.
28, To mdse., 60 da.	700.40.

7. Brigham & Co., Dr.

Sept. 4, To mdse., 60 da.	$600.
9, To mdse., 60 da.	450.
12, To mdse., 60 da.	350.
17, To mdse., 60 da.	400.
22, To mdse., 30 da.	500.
30, To mdse., net,	150.

8. D. H. Beckwith & Co. Dr.

Nov. 3, To mdse., 2 mo.	$750.50.
8, To mdse., 2 mo.	432.25.
17, To mdse., net,	275.50.
22, To mdse., 2 mo.	210.50.
25, To mdse., 1 mo.	168.30.
28, To mdse., 1 mo.	240.50.

9. Brown, Kerr & Co., Dr.

Oct. 1, To mdse., 3 mo.	$210.
5, To mdse., 60 da.	840.
13, To mdse., 60 da.	720.
21, To mdse., 60 da.	660.
24, To mdse., 60 da.	540.
31, To mdse., net,	300.

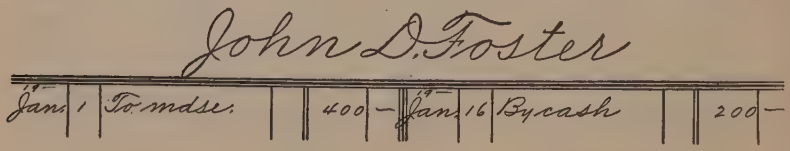
10. D. M. Smith & Co., Dr.

July 3, To mdse.	$420.30.
8, To mdse.	325.70.
11, To mdse.	417.25.
16, To mdse.	186.24.
25, To mdse.	240.60.
29, To mdse.	126.84.

COMPOUND ACCOUNTS

ORAL EXERCISE

1. The following is your account with John D. Foster.


John D. Foster

Jan.	1	To mdse.	400 —	Jan.	16	By cash	200 —

Had no payment been made, when would the account have been due? Since no payment was made until after maturity, you have *lost* the use of $400 for how many days? To offset this loss what should be the date of an interest-bearing note given to cover the balance of the account? Jan. 16 − 30 da. = Dec. ?, the date of an interest-bearing note given to cover the balance of the account.

2. The following is your account with Walter H. Wood.

WALTER H. WOOD

19—					19—				
Apr.	1	To mdse., 30 da.	600	00	Apr.	16	By Cash	300	00

Had no payment been made, when would the account have matured? By the payment recorded you have *gained* the use of $300 for how many days? To offset this gain, you should allow Walter H. Wood to keep the balance of the account how many days after maturity? May 1 + 15 da. = May ?, the date on which the balance is equitably due.

3. May 1 B sold C goods amounting to $500. Terms: 30 da. May 11 C made a payment of $250 on account. On what date is the balance of the account due?

4. Find the date of an interest-bearing note given for the balance of each of the following accounts, assuming that the terms in each case are 30 da.; assuming that the terms are cash.

	NAME	DR.	CR.
a.	H. H. Howard	Jan. 1, $400	Jan. 16, $300
b.	W. H. Lyman & Co.	Jan. 1, $400	Jan. 16, $100
c.	R. H. Delaney & Son	Jan. 1. $400	Jan. 16, $200

465. Examples. 1. Find the equated date for the following :

Irving E. Burdick

Feb. 1	To mdse.	360 —		Feb. 18	By cash	180 —	
14	" "	240 —		24	" "	180 —	

SOLUTION. Take as focal date the *latest* date in the account, Feb. 24.

DEBITS

DATE	ITEMS	INTEREST PERIODS	INTEREST
Feb. 1	$360	23 da.	$1.38
14	240	10	.40
	$600		$1.78

CREDITS

DATE	ITEMS	INTEREST PERIODS	INTEREST
Feb. 18	$180	6 da.	$.18
24	180	0	.00
	$360		$.18

$600 − $360 = $240, the balance of the account. $1.78 − $.18 = $1.60, the interest due the holder of the account on Feb. 24. The interest on $240 for 1 da. = $0.04. $1.60 ÷ $0.04 = 40, the number of days. If the account were settled Feb. 24 there would be *interest for 40 da. due* the holder of it. Therefore the balance of the account is due *40 da. before Feb. 24.* Feb. 24 − 40 da. = Jan. 15, the equated date.

PROOF. To prove the correctness of the above work it is necessary to show that a payment of $240 on Feb. 24 will result in no loss of discount to either party. This may be done by equating the account, using Jan. 15 as the focal date.

DEBITS

DATE	DISCOUNT PERIODS	ITEMS	DISCOUNT
Jan. 15 to Feb. 1	17 da.	$360	$1.02
15 to 14	30	240	1.20
		$600	$2.22

CREDITS

DATE	DISCOUNT PERIODS	ITEMS	DISCOUNT
Jan. 15 to Feb. 18	34 da.	$180	$1.02
15 to 24	40	180	1.20
		$360	$2.22

As there is no difference between the debit discount and the credit discount, the account is proved to be due by equation on Jan. 15, 19—.

2. Find the equated date for the following account:

Watson & Moore

19—					19—			
Apr.	1	To mdse. 60 da.	660 —		May	2	By cash	330 —
	24	" " 30 "	360 —			20	" 30 da. note with int.	300 —
	30	" " 10 "	280 —					

SOLUTION. Assume May 31 to be the date of settlement.

DEBITS

DATE	TERM OF CREDIT	MATURITY	ITEM	INTEREST PERIOD	INTEREST
Apr. 1	60 da.	May 31	$660	0 da.	$.00
24	30	24	360	7	.42
30	10	10	280	21	.98
			$1300		$1.40

CREDITS

DATE	ITEM	INTEREST PERIOD	INTEREST
May 2	$330	29 da.	$1.595
20	300	11	.55
	$630		$2.145

$1300 − $630 = $670, the balance of the account. $2.145 − $1.40 = $0.745, the interest due Watson & Moore on May 31. The interest on $670 for 1 da. = 0.11\frac{1}{6}$. $0.745 ÷ 0.11\frac{1}{6}$ = 6.6 or 7, the number of days. If the account were settled May 31, Watson & Moore might deduct $0.75 from the balance of the account; therefore the balance of the account is not due until 7 da. after May 31, or June 7, 19—.

PROOF. The maturity of each item is used in the proof.

DEBITS

DATE	INTEREST PERIOD	ITEM	INTEREST
May 31 to June 7	7 da.	$660	$ .77
24 to 7	14	360	.84
10 to 7	28	280	1.307
		$1300	$2.917

CREDITS

DATE	INTEREST PERIOD	ITEM	INTEREST
May 2 to June 7	36 da.	$330	$1.98
20 to 7	18	300	.90
		$630	$2.88

$2.917 − $2.88 = $0.037; as this is less than the interest on the balance of the account for $\frac{1}{2}$ da. the solution is probably correct.

WRITTEN EXERCISE

Find the equated date and prove the work:

1. FRED L. UPSON

19—					19—				
Jan.	10	To mdse.	360		Jan.	25	By cash	180	
	30	To mdse.	240		Feb.	12	By cash	120	

2. VINTON L. BROWN & CO.

19—					19—				
Mar.	11	To mdse.	420		Mar.	27	By cash	540	
	23	To mdse.	300			31	By cash	180	
Apr.	6	To mdse.	300		Apr.	24	By cash	300	
	20	To mdse.	120						

3. ANSON L. JAMES

19—					19—				
Mar.	8	To mdse., 10 da.	240	60	Mar.	18	By cash	240	60
	12	To mdse., 10 da.	180	30		24	By 30-da. note with interest	300	
	19	To mdse., 10 da.	246						
	29	To mdse., 10 da.	381	24		31	By cash	257	54

The charge under Mar. 8 was paid when due, Mar. 18. Such items may be omitted in equating the account.

4. MACGREGOR & CO.

19—					19—				
Apr.	7	To mdse., 10 da.	127	54	Apr.	17	By cash	127	54
	25	To mdse.	218	99		30	By cash	100	
May	6	To mdse., 10 da.	87	43	May	16	By cash	206	42
	18	To mdse.	150			24	By mdse.	35	20
	27	To mdse., 10 da.	86	45					

5. DAVID J. UPHAM

19—					19—				
June	7	To mdse.	128	50	June	14	By cash	332	50
	10	To mdse.	432	75		25	By mdse.	67	40
	15	To mdse.	78	55		30	By cash	248	60
	21	To mdse.	246	80	July	15	By cash	500	
	29	To mdse.	312	30		28	By mdse.	88	54
July	3	To mdse.	186	40					
	14	To mdse.	66	36					

ACCOUNT SALES

466. The method of averaging an **account sales** is practically the same as the method of averaging an ordinary ledger account. The charges for freight, commission, guaranty, etc., constitute the **debits** and the sales the **credits** of the account.

Commission and guaranty are sometimes considered due on the date of the last sale, and sometimes on the average date of the sales. When goods are sold promptly, commission and guaranty are generally considered due on the date of the last sale; when the sales are large and there are long intervals between them, commission and guaranty are generally considered due on the average due date of the sales. When goods are sold for cash, the account sales is seldom averaged.

WRITTEN EXERCISE

1. Equate the account sales on page 271, assuming that both sales were made on 30 days' time, and that the commission is due on the date of the last sale.

2. Copy and complete the following account sales. Consider the commission as due on the date of the last sale.

Buffalo, N.Y., July 3, 19

Sale for the account of Wentworth, Stratton & Co.
Indianapolis, Ind.

By C. M. Ettenheimer & Sons
Commission Merchants

		Sales		
June	8	295 bbl. Roller Process Flour, 60 da. $5.75		**** **
	12	315 " Old Grist Mill Flour, Cash 5.45		**** **
July	1	305 " Roller Process Flour, 60 da. 5.67 1/2		**** **
	3	285 " Old Grist Mill Flour, 30 da. 5.75		**** **
		Charges		
June	12	Freight and cartage	112 50	
	9	Insurance	60	
July	3	Storage	30	
	3	Commission, 5% of sales	*** **	
	*	Net proceeds due by equation	**** **	
			**** **	**** **

CHAPTER XXXII

CASH BALANCE

ORAL EXERCISE

1. When is the balance of the following account due?

JAMES B. SWEENEY

19—					19—				
Jan.	1	To mdse., 30 da.	600	00	Jan.	31	By cash	300	00

2. If no interest is charged on overdue balances, how much will settle the account Feb. 28?

3. If interest at 6% is charged on all amounts not paid at maturity, what is the cash balance of the above account Feb. 28?

4. Assuming that interest is charged on amounts not paid at maturity, find the cash balance of the above account March 30, at 6%.

467. The amount due upon an account at any given time is called the **cash balance** of an account.

When interest is not charged and discount is not allowed, the cash balance is the difference between the sides of an account. When interest is charged and discount is allowed, the cash balance is the difference between the sides of an account after interest has been added to overdue items and discount deducted from items not yet due.

Whether or not interest or discount is charged or allowed on ledger accounts is determined by custom or agreement. It is customary for wholesalers to charge interest on all overdue accounts. As a rule, retailers do not charge interest on the items of an overdue account, but they frequently close personal accounts at the end of the year and charge interest on the balances brought down from the date of closing to the date of settlement.

468. Example. What is the cash balance of the following account Aug. 1, 19—, interest being charged on overdue amounts at the rate of 6%?

George L. Stevens

19—					19—			
June	1	To mdse. 30 da.	900 —		June	30	By cash	600 —
	9	" " 10 "	450 —		July	10	" "	300 —
	20	" " 10 "	300 —			18	" "	150 —

SOLUTION.

DEBITS

DATE	TERM OF CREDIT	MATURITY	ITEM	INTEREST PERIOD	INTEREST
June 1	30 da.	July 1	$900	31 da.	$4.65
9	10	June 19	450	43	3.23
20	10	30	300	32	1.60
			$1650		$9.48

CREDITS

DATE	ITEM	INTEREST PERIOD	INTEREST
June 30	$600	32 da.	$3.20
July 10	300	22	1.10
18	150	14	.35
	$1050		$4.65

The debit footing and interest: $1650 + $9.48 = $1659.48
The credit footing and interest: $1050 + $4.65 = $1054.65
The balance due Aug. 1, 1907 = $ 604.83

WRITTEN EXERCISE

1. Find the cash balance due June 1, 19—, on problem 4, page 391, money being worth 5%.

2. Equate the following account and find the cash balance due Aug. 1, 19—, money being worth $4\frac{1}{2}$%.

FREDERICK T. LAWRENCE

19—				19—			
May	4	To mdse., 60 da.	1360	May	14	By cash	360
	17	To mdse., 30 da.	720	June	10	By cash	300
	26	To mdse., 60 da.	1080		21	By cash	420

To find the cash balance of an equated account: *Equate the account. Compute the interest on the balance of the account from the equated date to the date of settlement. Add the interest to the balance of the account and the result is the cash balance due.*

3–6. The following is a page from a sales ledger. Find the cash balance due on each account Aug. 1, money being worth 6 %.

O. N. Brown & Co.

19—					19—					
May	4	To mdse. 10 da.	$ 12	360 —	May	14	By cash	C 63	360 —
	17	" " " "	$ 16	720 —		19	" "	C 89	300 —
	26	" " " "	$ 29	1080 —		21	" "	C 103	420 —

F. M. Breton

19—					19—					
June	10	To mdse. 30 da.	$ 39	800 —	July	10	By cash	C 117	900 —
	30	" " 10 "	$ 20	750 —		15	" "	C 131	650 —
July	2	" " net	$ 35	1295 —					

D. N. Trenholm & Son

19—					19—					
May	23	To mdse. net	$ 31	180 —	May	25	By cash	C 111	180 —
	30	" " 30 da.	$ 42	900 —	June	29	" "	C 115	450 —
June	6	" " " "	$ 36	1000 —	July	5	" "	C 117	450 —
	22	" " " "	$ 48	540 —		6	" 30 da. note with int.	J 16	1000 —
	30	" " " "	$ 54	200 —					

Morey & Son

19—					19—					
May	11	To mdse. 2 mo.	$ 12	1200 —	June	25	By cash	C 110	780 —
	25	" " 30 da.	$ 27	780 —	July	11	" "	C 120	1200 —
June	5	" " " "	$ 32	1066 —		20	" 30 da. note with int.	J 20	1066 —
	19	" " " "	$ 42	630 —					
	30	" " " "	$ 52	420 —					

DIVIDENDS AND INVESTMENTS

CHAPTER XXXIII

STOCKS AND BONDS

STOCKS

469. A **corporation,** or **stock company,** is an association of individuals organized under the laws of a particular state into a body which is by law given the rights and powers and civil liabilities of a person.

Being a mere creature of law a corporation possesses only those properties which its **charter** (the instrument which defines its rights and duties) confers upon it. These are such as are best calculated to effect the object for which it was created. Among the most important is legal continuous existence, irrespective of that of the individuals composing it.

470. The **capital stock** of a corporation represents the interest of the individuals who compose the corporate body in the property and profits of the corporation.

The capital stock is divided into a definite number of shares. It is not necessary to give these shares a par value, but it is customary to do so. The usual par value of a share is $100. However, any amount may be the par value.

471. A **stock certificate** is a legal instrument which evidences that the holder owns a certain number of shares of stock in the corporation or association issuing the certificates. A **stockholder** is a person who owns one or more shares of stock.

The certificate bears the seal of the corporation and is signed by officers (usually the president and the treasurer or their representatives) authorized by the by-laws to sign stock certificates.

Stockholders elect a few of their number to have general control of the company. The stockholders thus elected constitute the board of directors. This board may delegate its powers to several of its members called an executive committee. The control of a corporation is vested in its board of directors.

472. A **dividend** is a sum of money paid by a corporation to its stockholders, each share participating equally. An **assessment** is an amount per share which stockholders are called upon to pay to make up losses or deficiencies.

The board of directors decide upon the rate of dividend, which usually means the rate *per annum on the face value of the stock*, but it may also be expressed as a certain amount in dollars and cents per share.

Shares of stock may be non-assessable.

473. A company may divide its stock into two or more classes; the usual division is **preferred** and **common.**

INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS.

SHARES, $100 EACH

No. 00 Shares

New England Coal & Coke Co.

CAPITAL STOCK, $200,000

$125,000 Common $75,000 Preferred

This Certifies that William M. Stewart is the owner of One Hundred Shares of the Capital Stock of the New England Coal & Coke Co.

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

PREFERRED

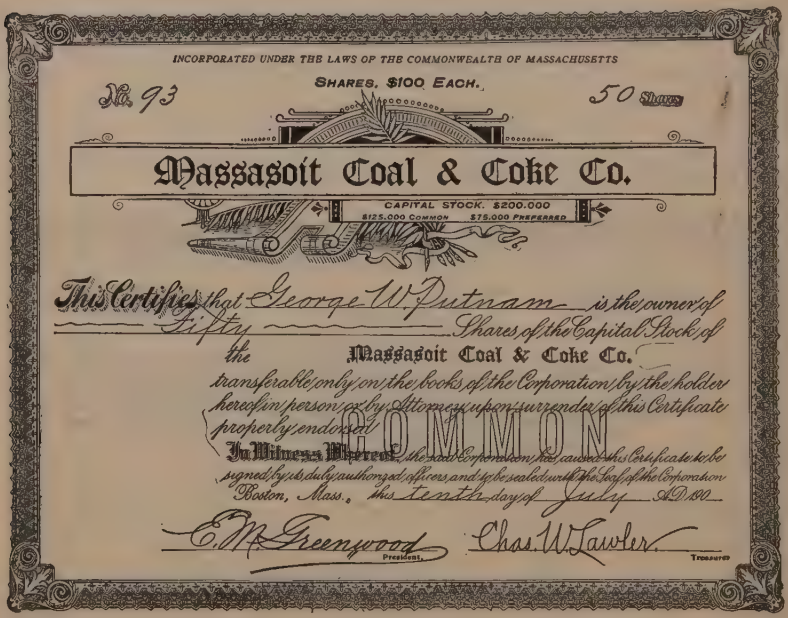
In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation Boston, Mass., this seventeenth day of July A.D. 19—

E. M. Greenwood, President. Chas. W. Lawler, Treasurer.

474. Preferred stock is stock which is entitled, out of profits, to a preferential dividend at a fixed rate. In case a company is dissolved, its preferred stock usually has a preferential claim against the assets of the company up to the par value of the preferred stock outstanding.

The prior rights of preferred stocks over common usually make them safer investments, but stocks are not necessarily safe because preferred.

475. Common Stock is the stock which has the right to the equity in the earnings and the income of the liquidation in the assets of a corporation after the claims of all prior security issues (bonds, notes, preferred stock, etc.) have been satisfied.


INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS

No. 93 SHARES. $100 EACH. 50 Shares

Massasoit Coal & Coke Co.

CAPITAL STOCK. $200.000
$125.000 COMMON $75.000 PREFERRED

This Certifies that George W. Putnam is the owner of Fifty Shares of the Capital Stock of the Massasoit Coal & Coke Co.

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation Boston, Mass. this tenth day of July A.D. 190

E. M. Greenwood President. Chas. W. Lawler Treasurer

476. The **par value** is the face value of stocks; the **market value** is the sum for which the stocks can be sold in the market.

477. If a company is prosperous and pays a higher rate of dividend than the money could earn with no greater risk in the general money market, a share may sell for more than its face value; it is then said to be *above par*, or *at a premium*. If the company pays a lower rate of dividend than could be obtained at no greater risk in the general money market, a share may sell for less than its face value; it is then *below par*, or *at a discount*.

478. A **stock broker** is a person who buys and sells stocks for other persons on commission.

Stocks are usually bought and sold through stock brokers. The broker's commission is usually $\frac{1}{8}\%$ of the par value of the stock, or $12\frac{1}{2}¢$ per share. A charge is made for either buying or selling.

479. When the price of stock is quoted at 97, $118\frac{3}{4}$, $160\frac{1}{2}$, it means that a share whose par value is \$100 can be bought for \$97, \$118.75, \$160.50. If a person buys stock through a broker at $160\frac{1}{2}$, it will cost him \$160.50 + \$0.12$\frac{1}{2}$ brokerage, or \$160.62$\frac{1}{2}$; if he sells stock through a broker for $160\frac{1}{2}$, he will receive as proceeds \$160.50 − \$0.12$\frac{1}{2}$, or \$160.37$\frac{1}{2}$, per share.

On the New York Stock Exchange quotations are expressed in dollars and eighths of a dollar.

In some other cities quotations are expressed in dollars, in eighths of a dollar, and in sixteenths of a dollar. When stocks are quoted at less than \$1 per share, fluctuations are sometimes expressed in cents.

The bulk of transactions in the stock exchanges is in round lots. In New York *round lots* are 100 shares or some multiple thereof; in Boston, 50 shares or some multiple thereof. In these cities any other number of shares than the ones named for each would be called *odd lots*. In Chicago no distinction is made between round and odd lots. Fractional shares of stock which are occasionally in the market are called *scrip*.

ORAL EXERCISE

1. Examine the certificate of stock, page 397. What is the name of the company? From whom did the company get its right to carry on business as a corporation?

2. What is the entire capital stock of the company? Into how many shares is this divided? What per cent of the entire stock of the company does the holder of the certificate own?

3. What kind of stock is represented by the certificate? What is the difference between common and preferred stock?

4. What is the par value of each share? If the market value of each share is \$160, what is the certificate worth?

5. What sum must be laid aside to provide for the dividends on the preferred stock of the company, the rate being 6 %? How much of this sum will the holder of the certificate receive?

6. Examine the stock certificate, page 398. What part of the stock of the company is common stock?

7. A 5 % dividend on the common stock would require how much money from the treasury of the company? Of this sum how much would George W. Putnam receive?

Dividends and Assessments

WRITTEN EXERCISE

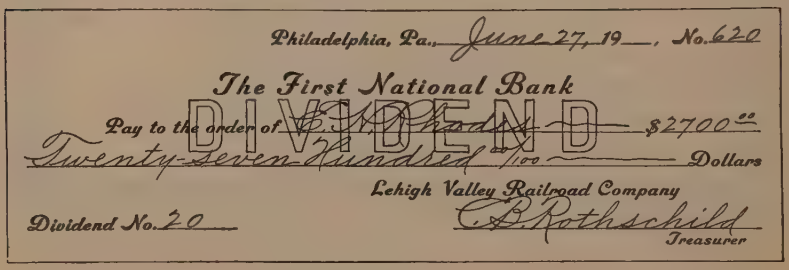
Unless otherwise specified the par value of a share will be understood to be $100.

1. A company with $3,500,000 capital declares an 8 % dividend. What does the holder of 250 shares receive?

2. B holds 450 shares of Lehigh Valley Railroad stock. When the company declares a dividend $7\frac{1}{2}$ %, how much will he receive?

3. What annual income is derived from investing $48,000 in Union Pacific Railroad stock at 120, if $2\frac{3}{4}$ % semiannual dividends are declared?

4. E. H. Rhodes holds 600 shares of Lehigh Valley Railroad stock. If he received the following check as his annual dividend, what was the rate?


Philadelphia, Pa., June 27, 19__, No. 620

The First National Bank

Pay to the order of E. H. Rhodes $2700.00

DIVIDEND

Twenty-seven Hundred 00 Dollars

Lehigh Valley Railroad Company

Dividend No. 20

C. A. Rothschild

Treasurer

5. A company with $1,000,000 capital declares quarterly dividends of $1\frac{1}{4}$ %. What are the annual dividends? What is the amount received annually by D, who owns 475 shares?

6. A corporation with a capital of $125,000 loses $2500. What per cent of his stock must each stockholder be assessed to meet this loss? How much will it cost A, who owns 150 shares?

7. A company with a capital of $750,000 declares a semiannual dividend of $3\frac{1}{4}$ %. How much money does it distribute among its stockholders annually? What is the annual income of a man who owns 200 shares?

8. If the Pennsylvania Railroad declares a semiannual dividend of $2\frac{1}{2}\%$ on a capital stock of $500,000,000, what amount is annually distributed among the stockholders? What is the annual income to J. P. Morgan from this stock if he owns 7,500,000 shares having a par value of $50 each?

9. During a certain year a manufacturing concern with a capital of $750,000 earns $75,500 above all expenses. It decides to save $15,500 of this for emergencies and to divide the remainder in dividends. What is the rate? What would be the amount of A's dividend check if he owns 125 shares?

10. The capital stock of the Gramercy Finance Company is $1,500,000. The gross earnings of the company for a year are $375,000 and the expenses $215,000. What even per cent of dividend may be declared and what would be the amount of undivided profits if 10% of the net earnings are first set aside as a surplus fund? (An even per cent is a per cent without a fraction.)

11. A railway company has a capital of $3,500,000 and declares dividends semiannually. During the period from Jan. 1 to July 1 of a certain year the net earnings of the company were $191,000. Of this amount 10% is carried to surplus fund. What even rate per cent of dividend may be declared on the balance and how much will be carried to undivided profits?

12. A company with a capital stock of $500,000 gains during a certain year $38,750. It decides to carry $5000 of the profits to surplus fund and to declare an even per cent of dividends on the remainder. What sum was divided among the stockholders, and what sum was carried to undivided profits account? What was the annual income to F from this stock if he owned 500 shares?

13. During a certain year the gross earnings of a railroad having a capital stock of $100,000,000 were $65,150,000, and the operating expenses $45,150,000. If the company declared a semiannual dividend of $3\frac{1}{4}\%$ and carried the balance of the net earnings to undivided profits account, how much was divided among the stockholders? How much was the working capital of the company increased?

14. The capital stock of the First National Bank is $3,000,000, and dividends are declared semiannually. The profits of the bank for a certain six months are $185,750. Of this sum 10% is carried to a surplus fund. The directors then vote to declare a semiannual dividend of $3\frac{1}{2}\%$ and carry the balance of the profits to undivided profits account. What amount was carried to surplus fund account? to dividend account? to undivided profits account?

Buying and Selling Stock

480. The following is an abbreviated form of the stock quotations for a certain day on the New York Stock Exchange:

Table of Sales and Range of Prices

Sales	Stocks	Open.	High.	Low.	Clos.	Net Change
2,600	Am. Sugar	$100\frac{1}{2}$	$100\frac{1}{2}$	$99\frac{1}{2}$	$99\frac{3}{4}$	$-\frac{3}{4}$
200	Am. Sugar (pfd.)	110	$110\frac{1}{2}$	$109\frac{1}{2}$	$109\frac{1}{2}$	$-\frac{7}{8}$
10,200	Atchison	$95\frac{1}{8}$	$95\frac{1}{2}$	$91\frac{3}{4}$	92	$-3\frac{1}{8}$
300	Atchison (pfd.)	100	100	100	100	
900	At. Coast Line	$121\frac{1}{8}$	120	116	116	$-4\frac{1}{8}$
13,600	Baltimore & O.	$88\frac{1}{8}$	$88\frac{5}{8}$	$87\frac{1}{4}$	88	$-\frac{1}{8}$
600	Baltimore & O. (pfd.)	$80\frac{1}{2}$	81	$80\frac{1}{2}$	$80\frac{1}{2}$	$-\frac{1}{2}$
147,100	Canadian Pacific	$193\frac{5}{8}$	$200\frac{3}{8}$	$188\frac{1}{2}$	$189\frac{7}{8}$	$-3\frac{3}{4}$
20,200	Chic. M. & St. P.	$98\frac{3}{8}$	$98\frac{1}{2}$	$94\frac{1}{8}$	95	$-3\frac{3}{8}$
300	Chic. M. & St. P. (pfd.)	$137\frac{3}{4}$	135	$134\frac{7}{8}$	$134\frac{7}{8}$	-2
200	General Chem. (pfd.)	108	109	$108\frac{1}{2}$	109	$+1$
2,500	General Electric	144	144	141	141	-3
15,600	Gt. Northern (pfd.)	122	$121\frac{5}{8}$	119	$119\frac{1}{4}$	$-2\frac{1}{2}$
1,600	Illinois Central	110	110	$107\frac{1}{2}$	$107\frac{5}{8}$	$-2\frac{3}{8}$
59,800	Lehigh Valley	$136\frac{1}{2}$	$136\frac{1}{2}$	$132\frac{1}{2}$	$134\frac{3}{4}$	$-1\frac{3}{4}$
650	Louisville & Nash.	$135\frac{3}{4}$	$135\frac{1}{4}$	$131\frac{1}{4}$	$131\frac{1}{4}$	$-4\frac{1}{8}$
2,100	Nat'l Biscuit	131	$130\frac{1}{4}$	125	125	-5
100	Nat'l Biscuit (pfd.)	123	$123\frac{7}{8}$	$123\frac{7}{8}$	$123\frac{7}{8}$	$+\frac{7}{8}$
69,200	South. Pacific	$92\frac{1}{2}$	$91\frac{1}{2}$	$86\frac{1}{2}$	$87\frac{1}{2}$	$-5\frac{1}{2}$
300	South. Pacific (pfd.)	$100\frac{1}{4}$	$97\frac{1}{2}$	$97\frac{1}{2}$	$97\frac{1}{2}$	$-2\frac{1}{4}$
210,100	Union Pacific	$154\frac{1}{2}$	$154\frac{1}{2}$	$148\frac{7}{8}$	$149\frac{7}{8}$	-4
400	Union Pacific (pfd.)	$82\frac{1}{2}$	$82\frac{1}{2}$	82	82	$-\frac{1}{2}$
390,100	U.S. Steel	$58\frac{1}{8}$	$58\frac{3}{8}$	56	$56\frac{3}{8}$	$-2\frac{1}{4}$
4,200	U.S. Steel (pfd.)	109	$109\frac{1}{4}$	$107\frac{1}{4}$	$107\frac{1}{4}$	$-1\frac{1}{2}$

In the first column is shown the number of shares of stock sold; in the second, the name of the stock; in the third, fourth, fifth, and sixth, respectively, the opening, the highest, the lowest, and the closing prices of the day; in the last, the net changes between the closing price of yesterday and to-day. Thus, 2600 shares of American Sugar stock were sold. The opening price was $100.50 per share; the highest price $100.50; the lowest, $99.25; the closing, $99.75, which shows a decline of 75¢ from the preceding day.

ORAL EXERCISE

1. Find in the table (page 402) three cases where a quotation both for common stock and for preferred (*pfd.* stands for *preferred*) stock of the same company is given. Which is worth the more in each case? The par value of all shares is $100.

If the profits of a concern are so great that a large per cent may be paid on the common stock, after paying the fixed rate on the preferred stock, then the common stock may sell for a higher figure than the preferred.

2. What would 100 shares of American Sugar (common) cost if bought through a broker at the lowest price for the day, brokerage being $\frac{1}{8}\%$.

3. What would the seller of the stock realize on the sale?

SUGGESTION. The seller would receive the price for which it was sold minus the brokerage, $\frac{1}{8}\%$.

4. State the cost, at the opening price in the table, of 100 shares each of the following stocks, assuming that the transactions take place through a broker who charges $\frac{1}{8}\%$ commission: Baltimore & Ohio; Canadian Pacific; General Electric; Lehigh Valley. (Base the calculations on the common stock.)

5. At the highest price in the table, state the amount that would be received from the sale of 100 shares of each of the following stocks, assuming that they were sold through a broker who charged $\frac{1}{8}\%$ commission: Southern Pacific; U.S. Steel (preferred); Great Northern (preferred); National Biscuit; American Sugar (preferred); Atchison; General Chemical (preferred); Illinois Central; Union Pacific. (If *preferred* is not named, *common* stock is referred to.)

WRITTEN EXERCISE

Find the cost, at the closing price in the table, of 2500 shares of the following stocks, including brokerage:

1. Canadian Pacific.
2. American Sugar (pfd.).
3. National Biscuit (pfd.).
4. Baltimore & Ohio (pfd.).
5. Atlantic Coast Line.
6. United States Steel (pfd.).

At the closing price for the day find the amount received from the sale of 3500 shares of the following stocks sold through a broker:

7. Illinois Central.
8. Louisville & Nashville.
9. Southern Pacific.
10. Lehigh Valley.
11. Atchison (pfd.).
12. General Electric.
13. Southern Pacific (pfd.).
14. Great Northern (pfd.).

481. Example. I bought 1000 shares Chicago, Milwaukee, & St. Paul preferred stock, at the lowest price in the table, and sold the same at $140\frac{1}{2}$. Allowing for brokerage both for buying and selling, did I gain or lose, and how much?

SOLUTION. Since I bought through a broker, each share cost me $\$134.87\frac{1}{2} + \$0.12\frac{1}{2}$, or \$135; and since I sold through a broker, the proceeds of each share sold was $\$140.50 - \$0.12\frac{1}{2}$, or $\$140.37\frac{1}{2}$. $\$140.37\frac{1}{2} - \$135.00 = \$5.37\frac{1}{2}$, gain on each share. Since $\$5.37\frac{1}{2}$ is gained on each share, 1000 times $\$5.37\frac{1}{2}$, or \$5375, is gained on 1000 shares.

$$\begin{array}{r} \$140.37\frac{1}{2} \\ 135.00 \\ \hline \$5.37\frac{1}{2} \\ 1000 \\ \hline \$5375. \end{array}$$

In the following exercise it is understood that all sales and purchases are made through a broker, who charges a commission of $\frac{1}{8}\%$ both for buying and for selling.

WRITTEN EXERCISE

Find the gain or loss on 500 shares of each of the following stocks bought at the opening price and sold at the price here given:

1. Illinois Central, $108\frac{1}{4}$.
2. General Electric, $147\frac{7}{8}$.
3. Southern Pacific (pfd.), 89.
4. General Chemical (pfd.), 110.
5. American Sugar (pfd.), 103.
6. National Biscuit, $134\frac{1}{2}$.
7. Baltimore & Ohio, $90\frac{3}{8}$.
8. Canadian Pacific, $200\frac{1}{2}$.
9. United States Steel (pfd.), $112\frac{1}{2}$.
10. Atlantic Coast Line, $115\frac{3}{4}$.
11. Great Northern (pfd.), 125.
12. National Biscuit (pfd.), $126\frac{1}{4}$.

13–24. Find the gain or the loss on 1000 shares of each of the above stocks bought at the lowest price and sold at the highest price, in the table.

25. John R. West bought 400 shares of United States Steel, (common) at the opening price in the table and sold it so as to gain \$300. What was the quoted price when he sold it?

26. I bought some United States Steel (preferred) at the opening price in the table and sold it for $112\frac{1}{2}$. If I gained \$650 by the transaction, how many shares did I buy?

27. I bought 2500 shares of General Electric at the lowest price in the table, held it for a year, received 5 % in dividends, and then sold it at $139\frac{1}{8}$. If money was worth $4\frac{1}{2}$ %, did I gain or lose, and how much?

The interest is to be computed on the cost of the stock, the dividend on the par value.

28. I gave my broker orders to buy 1500 shares of Atchison (preferred) and to sell 2000 shares of Canadian Pacific. If he bought at the lowest price in the table and sold at the highest price, what balance will he put to my credit?

BONDS

482. A **bond** is an instrument by which a government, a municipality, or a corporation contracts and agrees to pay a specified sum of money on a given date, at a specified rate of interest. — Rollins.

Bonds are generally issued at a face value of \$1000; less frequently, of \$500; occasionally, of \$100. All bonds of the same issue usually have the same rights and security.

Bonds, the payment of which depends only on the unsecured credit of the issuing company, are called *debenture bonds;* those that have their payment secured by a mortgage on the property of the issuing corporation are called *mortgage bonds;* those that are secured by a deposit with a trustee of collateral are called *collateral trust bonds;* those that provide that the interest on them shall be paid only if earned are called *income bonds.*

Bonds of a national government are called *government bonds;* of a state, a city, a town, or other municipal organization, *municipal bonds.*

The names of the different government bonds are usually derived from the interest they bear and the time when they mature. Thus, "U. S. 2s, 1930," are United States bonds bearing interest at 2 % and maturing in 1930.

From the gross earnings of a company the operating expenses are first deducted; from the net earnings are deducted all fixed charges, such as interest on bonds; then the dividends on preferred stock are paid; and finally out of the remainder dividends on the common stock are paid.

483. With reference to the **form of contract** for the payment of principal and interest there are two kinds of bonds: *coupon* and *registered.*

484. A **coupon bond** is a bond to which are attached interest notes, or coupons, representing the interest due on the bond at stated periods of payment.

The interest notes may be cut off from the bonds at maturity and the amount of interest which they represent collected through a bank. If these notes are not paid when due, they bear interest at the legal rate.

485. A **registered bond** is a bond which has no separate contract for the payment of the interest. Such a bond must be recorded on the books of the corporation in the name of the holder to whom the interest is sent by check.

Coupon bonds may be made payable either to bearer or registered as to principal only (the first custom prevails generally), and may be transferred by delivery or indorsement accordingly. Registered bonds are always drawn payable to some designated person and can be transferred only by assignment and registry on the books of the corporation.

ORAL EXERCISE

1. Examine the bond on page 406. With reference to the form of contract, what kind of a bond is it?

2. How many interest notes (coupons) would be attached to the full bond?

3. When was the bond issued? What date (of maturity) should be written on each interest note?

4. What is the face of the bond? What rate of interest does it bear? What sum should be written on each interest note?

5. How may coupon bonds be transferred? registered bonds?

All bonds are bought and sold "and interest"; that is, interest should be reckoned on the par value from the date of the last interest payment to the date of the purchase or sale, at the rate which the bond pays.

6. If the bond on page 406 was quoted at $105\frac{7}{8}$ when it was purchased, how much did it cost, including $\frac{1}{8}\%$ brokerage? How much did the seller realize on it, if sold Aug. 1. 1915?

7. Has the city or town in which you live any bonded indebtedness (indebtedness secured by bonds)? If so, what are these bonds called, and what rate of interest do they pay?

8. What is the difference in the meaning of *government* bond and *municipal* bond? Upon what authority does the government issue bonds? Upon what authority does a town or a city issue bonds? Must the bond issue be approved by the state in which the town or the city is located?

The Use of Bond Tables

486. The use of tables for finding the interest on notes, bonds, etc., is common among bankers and brokers. No interest tables are illustrated in this connection because they are too extended and complex for a textbook.

Referring to the bond table, page 409, the per cents at the top of the table represent the income on the face of a bond at one of the given rates, and the per cents given in the column at the left represent the income that will be realized when a bond is bought at a certain market price. This table is for a bond maturing 20 yr. from date, with interest payable semiannually.

487. Example. What will be the net income on a 5% bond bought at 97.53?

Solution. In the column headed *5%* find the price named, 97.53, then follow this line to the left and note that in the *Per Cent per Annum* column 5.20 is given; the net income on the price paid for the bond, 97.53, will be 5.2%.

ORAL EXERCISE

Refer to the table and find the cost of:

1. A 6% bond that will net $6\frac{1}{4}$%.
2. A 3% bond that will net 5%.
3. A $4\frac{1}{2}$% bond that will net 4.8%.
4. A 4% bond that will net $5\frac{1}{2}$%.
5. A man purchased 4 bonds, as follows: a 3% bond that would net 4.6%; a $4\frac{1}{2}$% bond that would net 4%; a 5% bond that would net $4\frac{1}{2}$%. What did he pay for each bond?

Refer to the table and find the net income of:

6. A 5% bond that will cost $91.15.
7. A 7% bond that will cost $125.10.
8. A 3% bond that will cost $79.95.
9. A 6% bond that will cost $106.02.
10. A man purchased 5 bonds each of which netted him 5% income. If the bonds which he bought yielded, on the face value, the following rate of income, what did he pay for each one: 4%, $3\frac{1}{2}$%, 5%, 6%, and 7%?

A BOND TABLE

20-YEAR. INTEREST PAYABLE SEMIANNUALLY

PER CENT PER ANNUM	3%	3½%	4%	4½%	5%	6%	7%
3.70	90.17	97.19	104.21	111.24	118.26	132.30	146.35
3⅛	89.51	96.50	103.50	110.49	117.48	131.46	145.44
3.80	88.86	95.82	102.78	109.74	116.70	130.63	144.55
3⅜	87.90	94.81	101.73	108.64	115.56	129.39	143.22
3.90	87.58	94.48	101.38	108.28	115.18	128.98	142.78
4.	86.32	93.16	100.00	106.84	113.68	127.36	141.03
4.10	85.09	91.86	98.64	105.42	112.20	125.76	139.32
4⅛	84.78	91.54	98.31	105.07	111.84	125.37	138.90
4.20	83.87	90.59	97.31	104.03	110.75	124.19	137.63
4¼	83.27	89.96	96.65	103.35	110.04	123.42	136.80
4.30	82.68	89.34	96.00	102.66	109.33	122.65	135.98
4⅜	81.80	88.42	95.04	101.65	108.27	121.51	134.75
4.40	81.51	88.11	94.72	101.32	107.93	121.14	134.35
4½	80.35	86.90	93.45	100.00	106.55	119.65	132.74
4.60	79.22	85.72	92.21	98.70	105.19	118.18	131.16
4⅝	78.94	85.42	91.90	98.38	104.86	117.82	130.77
4.70	78.11	84.55	90.99	97.43	103.86	116.74	129.61
4¾	77.57	83.98	90.39	96.80	103.20	116.02	128.84
4.80	77.02	83.40	89.79	96.17	102.55	115.32	128.08
4⅞	76.22	82.56	88.90	95.24	101.59	114.27	126.95
4.90	75.95	82.28	88.61	94.94	101.27	113.92	126.58
5.	74.90	81.17	87.45	93.72	100.00	112.55	125.10
5.10	73.86	80.09	86.31	92.53	98.76	111.20	123.65
5⅛	73.61	79.82	86.03	92.24	98.45	110.87	123.29
5.20	72.85	79.02	85.19	91.36	97.53	109.87	122.22
5¼	72.34	78.49	84.64	90.78	96.93	109.22	121.51
5.30	71.85	77.97	84.09	90.21	96.33	108.57	120.81
5⅜	71.11	77.19	83.27	89.36	95.44	107.60	119.77
5.40	70.87	76.94	83.01	89.07	95.14	107.28	119.42
5½	69.90	75.92	81.94	87.96	93.98	106.02	118.06
5⅝	68.72	74.68	80.64	86.59	92.55	104.47	116.38
5¾	67.57	73.46	79.36	85.26	91.15	102.95	114.74
5⅞	66.43	72.27	78.11	83.95	89.78	101.46	113.13
6.	65.33	71.11	76.89	82.66	88.44	100.00	111.56
6⅛	64.25	69.97	75.69	81.41	87.13	98.57	110.01
6¼	63.19	68.85	74.51	80.18	85.84	97.17	108.50
6⅜	62.15	67.76	73.36	78.07	84.58	95.79	107.01
6½	61.14	66.69	72.24	77.79	83.34	94.45	105.55
6⅝	60.14	65.64	71.14	76.64	82.13	93.13	104.12
6¾	59.17	64.62	70.06	75.50	80.95	91.83	102.72
6⅞	58.22	63.61	69.00	74.39	79.78	90.57	101.35
7.	57.29	62.63	67.97	73.31	78.64	89.32	100.00

Buying and Selling Bonds

488. Bonds are generally bought and sold through investment bankers or private bankers.

The commission for buying and selling bonds is the same as for buying and selling stocks.

489. The following table is an abbreviated form of the sales, and the opening, highest, lowest, and closing prices of bonds traded in on the New York Exchange on a recent date.

Table of Sales and Range of Prices

Sales	Bonds	Open.	High.	Low.	Clos.	Net Change
5,000	Am. Hide & Leather 6s	103½	103½	103	104	+ ¾
8,000	Brooklyn Rapid Transit con. 5s	103	103	102⅝	103	
6,000	Chesapeake & Ohio 5s	106⅞	107¼	106¾	107¼	+ ⅜
81,000	Chicago, Burlington & Quincy 4s	93⅞	93⅞	92¾	92¾	− 1
15,000	Erie 1st con. 4s	85⅜	85¾	85	85	− ⅜
1,000	Illinois Central 4s	93½	93½	93½	93½	
11,000	Lehigh Valley con. 4½s	99⅝	99⅝	99⅛	99⅛	− ⅝
1,000	Louisville & Nashville gold 5s	108	110	110	110	+ 2
2,000	Manhattan Ry. con. 4s	92	92½	91⅜	91½	− ½
8,000	Missouri Pacific 4s	59	57	56	56	− 2
24,000	N.Y. Central & Hudson River 4s 1934	91⅞	91¾	89½	90	− 1⅞
35,000	Reading general 4s	94¾	95	94½	94½	− ¼
1,000	Standard Gas 6s	89⅝	89⅝	89⅝	89	− ⅝
4,000	Texas Pacific 1st 5s	102	102⅛	102	102⅛	+ ⅛
62,000	Union Pacific 1st 4s	97⅛	97⅞	97⅛	96⅜	− ¾
196,000	United Steel 5s	102¼	102⅝	102¼	101⅞	− ⅜
18,000	Wabash 1st 5s	103⅛	104	103⅛	102¾	− ⅝
8,000	West Shore 4s	93¾	94	93¾	93½	− ¼

In the first column is shown the par value of the bonds sold; in the second, the name of the bonds and the interest they bear; in the third, fourth, fifth, and sixth, respectively, the opening, highest, lowest, and closing prices of the day. In the last column, *Net Change*, the net changes between the closing prices of the given day and the closing prices of the day preceding. Thus, on the day given, $8000 worth of Brooklyn Rapid Transit bonds bearing 5% interest were sold. The opening price was $103 per $100 of par value; the highest price, $103; the lowest price, $102.62½; the closing price, $103; there was no change between the closing price of the day given and the day preceding.

490. Example. What is the cost of $50,000 (par value) Chicago, Burlington & Quincy 4% bonds at the highest price quoted in the table (page 410)?

SOLUTION. $100 of par value cost $93⅞ + $0.12½ brokerage, or $94.
∴ $50,000 of par value will cost 500, *i.e.*, ($50,000 ÷ $100) times $94, or $47,000.

WRITTEN EXERCISE

(Omit the interest in solving these problems)

1. What is the cost of $25,000 American Hide and Leather bonds at the opening price in the table?

2. I gave my broker orders to sell $10,000 Chesapeake & Ohio 5% bonds and buy $10,000 Texas Pacific 1st 5% bonds. If he sold at the highest price in the table and bought at the lowest price, what balance should he place to my credit?

3. Find the proceeds from these sales: $1000 United Steel 5% bonds at the opening price in the table; $5000 Illinois Central 4% bonds at the opening price in the table; $75,000 Chicago, Burlington & Quincy 4% bonds at the closing price in the table; $10,000 Erie 4% bonds at the lowest price in the table.

4. June 1, 1915, a certain city borrowed $250,000 with which to build a new high school, and issued 4½% 10-yr. coupon bonds as security. If these bonds sold (through a broker) at 101⅝, how much was received by the city? If A bought five $1000 bonds, how much did they cost him? If interest is payable semiannually, what date (of maturity) should the last interest note of each bond bear? What will be the amount of each interest note?

5. Find the total cost of the following purchases: $20,000 Erie 4% bonds at the closing price in the table; $2000 Illinois Central 4% bonds at the lowest price in the table; $5000 Louisville & Nashville 5% bonds at the lowest price in the table; $15,000 Missouri Pacific 4% bonds at the opening price in the table; $10,000 Manhattan Railway 4% bonds at the lowest price in the table; $3000 West Shore 4% bonds at the opening price in the table.

INCOMES AND INVESTMENTS

491. The income received on bonds involves the consideration of the income on the face of the bond in comparison with the income on the price paid for the bond. To illustrate: if a 6% bond is bought at 120, the annual income is \$6, but the rate of income on the price paid for the bond is 5%, brokerage omitted. From this statement it is clear that the investor would receive 5% on his investment, but it must be remembered that at the maturity of the bond, say 5 yr., he will receive only \$100 for it; hence he must deduct the difference between the face of the bond and the price paid for it, \$ 20, to get the net sum received as income. It is clear that the investor will not realize 5% on his investment.

The above statement suggests a complex problem that is fully worked out in bond tables, but these are too extended to be used in a textbook. This question is referred to in this text as a caution to those who buy bonds regarding the net income they will receive on the sum paid for the bond. It is easy for one to deceive himself on this point.

The following is from "Money and Investments," by Montgomery Rollins, to whom the author is indebted for the bond table used on page 409.

Take as an example a bond bearing 5% interest, and having exactly 10 yr. to run before maturity. If it is sold at \$108.18, that is, \$1081.80 for each one-thousand-dollar bond, the net return to the investor would be 4% per annum, which is 4% for each of the 10 yr., and is 4% upon the entire sum, \$1081.80, invested.

This brings up the point that, although in the example, the bond costs \$1081.80, at the end of 10 yr., when it matures, the holder will receive \$1000. In the meantime he will have received \$50 yearly in interest. Not all of this \$50 should be considered as income, for a sufficient amount of it should be set aside each year to liquidate the \$81.80 premium paid for the bond.

COST OF BONDS

492. A purchaser of a bond pays the market price of the bond plus the interest accrued since the last interest day. In finding the cost of bonds April 1 and October 1 will be considered as the interest days, interest being paid semiannually. If a bond were purchased after any one of these dates, the buyer would pay the market price of the bond plus the interest from the last interest day to the date of purchase. Each bond is of a denomination of $1000.

In all stock-exchange transactions in bonds a commission (brokerage) is added or deducted in sales or purchases, but when bonds are bought from a regular bond house, the price quoted is net, *i.e.*, no commission is to be added.

The examples in the text are stock-exchange transactions.

In these examples compute the interest for the exact number of days.

493. Example. What will be the cost of 10 Western Electric 5% bonds at $102\frac{3}{8}$, with brokerage added, if purchased on May 1?

SOLUTION. 1 bond costs $1023.75, without interest.
10 bonds will cost $10,237.50, without interest.
April 1 to May 1 is 1 mo.
Interest on $10,000 for 1 mo. at 5% = $41.67.
$10,237.50 + $41.67 = $10,279.17, cost plus the interest.
Brokerage, $\frac{1}{8}$% = $12.50.
$10,279.17 + $12.50 = $10,291.67, total cost.

WRITTEN EXERCISE

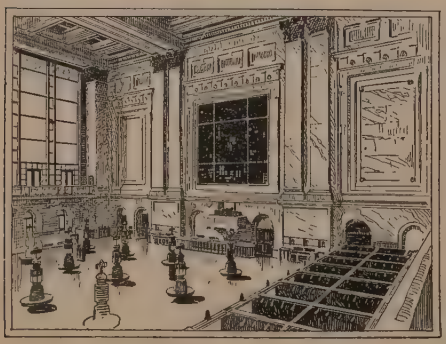
Find the total cost, including brokerage, of the following:

1. 10 Third Avenue 1st 5s at $107\frac{1}{4}$, bought on June 1.
2. 20 Union Pacific 1st 4s at 98, bought on July 16.
3. 5 Southern Railway general 4s at 76, bought on July 21.
4. 15 West Shore 4s at $93\frac{1}{2}$, bought on July 31.
5. 8 Western Maryland 4s at $79\frac{1}{2}$, bought on Oct. 1.
6. 12 U. S. Rubber 6s at $103\frac{7}{8}$, bought on Oct. 31.
7. 6 National Tube 5s at $99\frac{5}{8}$, bought on Nov. 1.
8. 10 N. Y. Telephone $4\frac{1}{2}$s at $97\frac{7}{8}$, bought on Dec. 1.
9. 10 Oregon Short Line 6s at $110\frac{3}{8}$, bought on Jan. 30.

STOCK EXCHANGES

494. A **stock exchange** is an association of brokers for the purpose of providing a common meeting place, convenience, and security for their dealings with each other. The **principal stock market** of the United States is the New York Stock Exchange, an unincorporated association of 1100 members.

There are also stock exchanges in some of the other principal cities. Next in importance to the New York Stock Exchange are the stock exchanges of Boston, Philadelphia, and Chicago. Many stocks of large issue are dealt in on several stock exchanges. Each exchange, as a rule, forms the principal market for the stocks which are controlled by local interests. Stock exchanges make public, as they occur, the record of sales and quotations of the securities in which the exchange elects to deal. In this way a holder of securities which are dealt in, or *listed*, on an exchange may know the most recent price at which his securities have sold in the market, and an intending investor may know what he may have to pay for such securities.



Interior of a Stock Exchange

A membership in a stock exchange is called a **seat**. The price of a seat varies from \$1000 to \$20,000 on local stock exchanges, to from \$30,000 to \$75,000 on the New York Stock Exchange. A stock exchange always maintains a **uniform rate of commission**. This, as has been seen, is usually $\frac{1}{8}\%$, or \$12.50 per 100 shares.

The importance of the stock exchange is shown in the initial financial results of the great European war which began in 1914. When hostilities were imminent the stock exchanges of the leading cities of Europe and America were closed. By this means reckless financial transactions were rendered much more difficult, because it made quite impossible such heavy sales of stocks and bonds as would have demoralized the money market and imposed a great hardship and loss on holders of securities, especially small holders. While the stock exchanges were closed the sale of listed securities was rendered difficult because of the lack of a common center for selling such securities.

495. The principal ways in which **stocks** are **bought** and **sold** are as follows: stocks bought or sold are deliverable on the following day unless otherwise specified; this is called "regular" delivery. However, securities are not delivered on Saturdays nor on stock exchange holidays. Transactions may be for "cash," that is, deliverable on the day of sale; "at 3 da.," that is, deliverable on the third day following the sale; "buyer's option," that is, deliverable at the option of the buyer at any time within the option period (from 4 to 60 da.); "seller's option," that is, deliverable at the option of the seller at any time within the option period.

By far the largest part of the sales are "regular." On "cash," "regular," and "at 3 da." sales no interest is paid; but on options over 3 da., interest at the legal rate on the selling price of the stock is paid by the buyer to the seller. To terminate an option of over 3 da., one day's notice is required.

496. A **margin** is a sum of money deposited with a broker to cover losses which he may sustain on behalf of his principal.

Stocks and bonds are often bought and sold on a margin, as follows:

June 8, A. M. Greyson deposited with Richard Roe & Co., his brokers, $4160, and instructed them to buy 400 shares of Atchison, Topeka and Santa Fé Railroad stock whenever they could do so at 104. On the same day the stock was bought in accordance with instructions. On June 14, pursuant to instructions, Richard Roe & Co. sold the stock at 107¼ and sent A. M. Greyson the following statement and a check for $5322.56.

New York, June 14, 19

Mr. A. M. Greyson
160 Wall Street

In account current with **RICHARD ROE & CO.**

DATE		AMOUNT	DAYS	INTEREST	DATE		AMOUNT	DAYS	INTEREST
June 8	To 400 shares A. T. & S. F. R.R. at 104	41600 00	6	41 60	June 8	By cash, margin	4160 00	6	4 16
					14	By 400 shares A. T. & S. F. R.R. at 107¼	42900 00		
8	To brokerage	50 00							
14	To brokerage	50 00			14	To interest			37 44
14	To interest	37 44							
14	To check herewith	5322 56							
		47060 00		41 60			47060 00		41 60

By the above transactions A. M. Greyson has gained $1162.56.

The amount of margins demanded by a broker depends upon the character of the stocks traded in. On stocks that have a good market 10% of the market value is usually demanded; on stocks that have little or no market

20% of the market value or more is often required. The broker, of course, pays for the stock in full. In order to do this he is frequently obliged to borrow money from a bank. This he may usually do by depositing (**hypothecating**) stock as security (see page 335).

The speculators on the stock exchange may be divided into two classes: bulls and bears. A **bull** is a speculator who buys stocks in the expectation of selling them at a higher price. A **bear** is a speculator who sells stocks which he does not own, in the expectation that he can buy them at a lower price before the date on which they must be delivered. A bull who has bought is said to be *long* of stock; a bear who has sold is said to have sold *short*, or to be *short* of stock. A bull works for advancing prices; a bear for declining prices. A bull, when selling at higher prices, is said to have *realized* his profits, or to have *liquidated* if he sells, whether the price he received is higher or lower than the price he paid. The term *liquidate* signifies the selling of securities held for long account and implies no distinction between sales at a profit or sales at a loss. A bear, when he buys stock, is said to have *covered*, no matter whether he bought at a gain or at a loss.

WRITTEN EXERCISE

(All interest computations are at 6%)

1. On June 25 I purchased, through a broker, 300 shares of Amalgamated Copper at $67\frac{1}{2}$ b. 3 (buyer's option any time within 3 da.). On June 28 the stock was delivered and, pursuant to my instructions, sold for $69\frac{3}{4}$ cash. Did I gain or lose, and how much?

2. On April 15 my broker purchased for me 500 shares Delaware & Hudson at $172\frac{1}{4}$ regular. On April 16 he sold the same at $174\frac{1}{8}$ cash. What was my gain?

3. On Sept. 15 I bought, through a broker, 250 shares Reading (preferred) at $68\frac{3}{8}$ b. 30. On Sept. 25 my broker demanded the stock and, in accordance with my instructions, sold it for $70\frac{1}{2}$ regular. Did I gain or lose, and how much?

4. On Dec. 1 D bought of me, through C, his broker, 2000 shares of Chicago, Milwaukee & St. Paul at $99\frac{1}{4}$ s. 60 (seller's option any time within 60 da.). On Dec. 17 C, pursuant to my instructions, delivered the stock which he had purchased for me on the previous day at 96 regular. Did I gain or lose, and how much?

5. Jan. 15 I deposited $4080 with my broker and instructed him to buy 400 shares of Baltimore & Ohio whenever he could do so at 92 regular. On the same day he bought the stock as directed. On Feb. 27 I ordered him to sell, and he did so at $95\frac{3}{4}$ cash. What was my net gain?

6. May 25 a speculator sent his broker a margin of $2000 with which to buy 100 shares Metropolitan Street Railway at 165 regular. The broker invested as directed. On May 27 the stock rose to $170\frac{1}{8}$ and the broker was authorized to sell. If he sold regular at this price, what was the speculator's gain? the broker's commission?

7. What is the balance due on the following account current:

New York, May 25, 19—

Mr. Charles M. Sanborn
640 Wall St, City

In account current with **RICHARD ROE & CO.**

DATE			AMOUNT		DAYS	INTEREST		DATE			AMOUNT		DAYS	INTEREST	
May	10	To 500 sh. Mich. Central at 150	75000	00	??	???	??	May	10	By cash, margin	7500	00	??	??	??
	10	To brokerage ¼%	???	??					25	By 500 sh. Mich. Central at 151	?????	??			
	25	To brokerage ¼%	???	??											

PRODUCE EXCHANGES

497. Just as there are stock exchanges in many of the large cities to supply a regular market for the purchase and sale of securities, so there are **produce exchanges** (also called **boards of trade, chambers of commerce,** etc.) to supply a regulated market for the purchase and sale of farm crops.

Produce exchanges are important accessories of commerce. They promote just and equitable principles of trade; establish and maintain a uniformity in commercial usages; and acquire, preserve, and disseminate valuable business information. The more important produce exchanges, by inspecting and grading all of the important food products, protect the public against fraud and adulterations. The cereals, for example, are

inspected and graded according to their quality. There are usually four grades of wheat, six of corn, and four of barley, oats, and rye; No. 1 wheat is the best quality, No. 4, the poorest, etc.

The **principal produce exchange** in the United States is the Chicago Board of Trade. On the floors of this exchange are bought and sold a large part of the cereals and the meat products of the Mississippi Valley and the West. The association thus practically determines the price of these commodities, not only for the United States, but for the world.

Commodities are bought and sold on the exchanges for present or for future delivery. Contracts for present delivery are called **cash** contracts; contracts for future delivery, **futures**. Speculative trading in grain and cotton is usually in "futures."

The established **brokers' commissions** on the Chicago Board of Trade are as follows: for grain, \$7.50 per 5000 bu.; for pork, \$12.50 per 250 bbl.; for lard, \$15 per 250 tierces; for ribs, \$12.50 per 50,000 lb.

The **lowest margins** received are: wheat, 5¢ per bushel; corn and oats, 3¢ per bushel; pork, \$1 per barrel; lard, \$2 per tierce; ribs, ½¢ per pound. The margins demanded are sometimes higher than the above figures.

In the accompanying table is shown the opening, highest, lowest, and closing prices of provisions for a certain day on the Chicago Board of Trade.

	Open.	High.	Low.	Clos.
Wheat—July	87	89½	87	88⅝
Wheat—Sept.	87	89	87	87⅞
Wheat—Dec.	89	89	87⅛	87¾
Corn—July	68¾	68⅞	67¾	68⅛
Corn—Sept.	67⅛	67⅛	66⅛	66⅜
Corn—Dec.	57⅞	58	57¼	57½
Oats—July	37⅝	37⅝	37¼	37⅜
Oats—Sept	36⅞	36⅞	35⅞	36¼
Oats—Dec.	38⅛	38⅛	37¼	37⅜
Pork—July	21 20	21 35	21 20	21 35
Pork—Sept.	20 05	20 17	20 05	20 10
Lard—July	10 00	10 07	10 00	10 07
Lard—Sept.	10 17	10 25	10 17	10 25
Lard—Oct.	10 22	10 27	10 22	10 27
Ribs—July	11 52	11 60	11 50	11 60
Ribs—Sept.	11 52	11 57	11 52	11 57
Ribs—Oct.	11 30	11 35	11 30	11 35

"Wheat—July" signifies wheat to be delivered in July; "Wheat—Sept." wheat to be delivered in September, etc. The usual time for future delivery is during the months of May, July, September, and December.

In the following exercise it is assumed that all transactions are effected through a broker, who charges the usual commission.

WRITTEN EXERCISE

1. What will it cost me to buy 5000 bu. September wheat at the opening price in the table?

2. C bought 10,000 bu. July oats at 35¢ per bushel and sold the same at the closing price in the table. What was his net gain?

3. B bought 15,000 bu. July corn at the lowest price and sold the same at the highest price in the table. Did he gain or lose, and how much? What per cent?

4. G bought 2250 tierces (765,000 lb.) of October lard at $7.26\frac{7}{8}$ and sold the same at the closing price in the table. Did he gain or lose, and how much?

5. F bought 1500 bbl. of September pork at the opening price and sold the same at the closing price in the table. Did he gain or lose, and how much?

6. D ordered his broker to sell 5000 bu. September corn and buy 5000 bu. December corn. If the broker sold at the highest price and bought at the lowest price in the table, what amount should he remit D?

7. A broker bought on his own account 10,000 bu. of each, September wheat, December corn, and July oats, at the opening price, and sold the same at the closing price in the table. Did he gain or lose, and how much?

8. H sold "short" 10,000 bu. September wheat at the highest price in the table. September wheat subsequently declined to $85\frac{1}{4}$ and he bought at this price to "cover his short." Did he gain or lose, and how much?

9. June 27 I deposited with my broker a margin of $200 for the purchase of 5000 bu. of September wheat at the lowest price in the table. On July 25 I ordered him to sell. He did so, receiving $89\frac{3}{8}$¢ per bushel. How much should he pay me in settlement?

10. Aug. 5 I deposited with my broker $2500 as a margin for the purchase of 5000 bbl. of September pork at the closing price in the table. On Sept. 2 I ordered him to sell at $23.07\frac{1}{2}$. He did so and remitted me a check for the amount due. What was the amount of the check?

CHAPTER XXXIV

LIFE INSURANCE

498. Life insurance companies, like fire insurance companies (page 278), are usually either *stock companies* or *mutual companies*.

There are also **assessment companies** and **fraternal beneficiary associations**. These usually depend upon monthly assessments or "calls" to pay death claims. They are required by law to hold but comparatively little, if anything, as a fund from which to pay losses.

499. Insurance rates are always a certain price per $1000 of insurance. They are payable annually, semiannually, or quarterly in advance.

500. The four leading **kinds of policies** are: ordinary life, limited life, endowment, and term.

501. An **ordinary life policy**, in consideration of premiums to be paid during the life of the insured, guarantees to pay at his death a stated sum of money.

502. A **limited life policy**, in consideration of premiums to be paid for a fixed number of years, guarantees to pay a stated sum of money at the death of the insured.

It will be observed that in an ordinary life policy the premiums are payable during the life of the insured, while in a limited life policy they are payable for a fixed number of years, when the policy becomes *paid up* (no more premiums due). The premium is higher on the latter form of policy.

503. An **endowment policy**, in consideration of premiums paid for a fixed number of years, guarantees to pay a stated sum of money to the insured at a certain time or to the *beneficiary* (one in whose favor the insurance is effected) in case of prior death.

504. A **term policy**, in consideration of premiums paid for a fixed time, guarantees to pay a stated sum of money if the insured dies within the term of insurance.

Thus, a person may insure his life for a limited number of years only. Since the company may never be called upon to pay the insurance, the premiums on these policies are low.

505. The **reserve** is that part of the premiums of a policy, with interest thereon, required by law to be set aside as a fund to be used in payment of the policy when it falls due.

The legal rate of interest on reserve funds varies slightly in different states. The higher the rate of interest, the smaller the reserve required.

506. The **surplus** of an insurance company is the excess of its assets (resources) over its liabilities.

This fund, with certain restrictions, may be used for such purposes as the company deems best. After retaining a surplus large enough to provide for contingencies, companies which issue policies on the **mutual** or **participating plan** divide the remainder of the surplus among such of its policyholders as are entitled to share in it. This is practically a return of an overcharge, but it is usually called the payment of a **dividend.**

507. Dividends may be used: (1) to reduce the next year's premium; (2) to purchase additional insurance, payable when the policy matures; (3) to shorten the time to run.

Dividends may also be left with the company, with the distinct understanding that there shall be no division of the same until the end of a certain period. As the policyholder receives no benefit unless he survives the selected period, it will be seen that the return should be somewhat larger. This plan is called *semi-tontine, distribution period, accumulated surplus, deferred dividend,* etc.

508. If a **policy is discontinued,** the insured may secure an equitable return for the reserve accumulated.

The insured usually has several options in this matter: (1) he may take the **cash value,** or practically all of the reserve value of the policy; (2) he may take a **paid-up policy** for such an amount as its reserve value will purchase; (3) he may take **extended insurance** for the face of the policy for as many years and days as its reserve value will purchase.

ANNUAL PREMIUM RATES FOR AN INSURANCE OF $1000

AGE	ORDINARY LIFE	20-PAYMENT LIFE	15-YEAR ENDOWMENT	20-YEAR ENDOWMENT
25	20.93	30.90	66.57	48.93
30	23.75	33.76	67.27	49.72
35	27.39	37.25	68.26	50.88
40	32.16	41.60	69.76	52.70
50	47.23	54.65	76.20	60.59

ORAL EXERCISE

1. What kind of a policy is that on page 423? Who is the beneficiary? the insured? What is the annual premium?

2. Should the beneficiary die in 1912, to whom would the policy be payable at the death of the insured in 1920?

3. Should the insured die after having paid one annual premium, how much would his heirs receive?

4. If the surplus earnings (dividends) on the policy amount to $1200, at the end of 10 yr., how much cash (see page 424) would the insured receive should he surrender the policy?

5. Should the insured decide to discontinue paying premiums after making five annual payments, how much paid-up insurance, exclusive of the surplus, might he receive?

6. How large a sum may the insured borrow on the policy after ten premiums have been paid?

7. If the company secures interest in advance by deducting it from the amount of the loan, and the insured should borrow $4000 for one year at 5%, what would be the amount of the check which he would receive from the company?

8. Had the insured taken out the policy when he was twenty-five years of age, what would be the annual saving, exclusive of interest, in the cost? How much would he have saved in 15 yr.? in 20 yr.?

9. If the insured should discontinue paying premiums after 5 yr. and take extended insurance, how much would the beneficiary receive should the insured die in 1914? in 1919?

10. If the insured had taken a life policy (see rates, page 421) for the same amount, instead of an endowment policy, and died after having paid ten full premiums, how much less would his insurance have cost, exclusive of dividends and interest?

11. If the insured should pay four full premiums on the policy, take extended insurance, and die 5 yr. later, how much would his beneficiary receive?

12. If the insured discontinues making payments after seven annual premiums had been paid, how much would he get in cash at the end of 20 yr. from date of issue, if living?

THE PENN MUTUAL

Number 1,000,000 Amount $10,000

LIFE INSURANCE COMPANY
OF PHILADELPHIA

AGE 35
SUM INSURED $10,000
YEARLY PREMIUM $508 80/100

In Consideration of the Application for this Policy, hereby made a part of this contract, The Penn Mutual Life Insurance Company of Philadelphia insures the life of Edward H. Catchpole of Rochester in the County of Monroe State of New York, in the sum of Ten Thousand 00/100 Dollars, and promises to pay at its Home Office, in the City of Philadelphia, unto Edward H. Catchpole his executors, administrators, or assigns, the said sum insured on the tenth day of May in the year nineteen hundred and twenty-seven or if the said insured should die before that time then to make said payment to Marjorie P. Catchpole, if she survive him; otherwise to his executors, administrators, or assigns, upon receipt of satisfactory proof of the death of the insured, during the continuance in force of this Policy, upon the following conditions, namely:

The payment in advance to the Company, at its Home Office, of the sum of Five Hundred Eight and 80/100 Dollars, at the date hereof, and of the annual premium of Five Hundred Eight & 80/100 Dollars, at or before three o'clock P.M., on the tenth day of May in every year during the continuance of this contract, or until twenty full years' premiums shall have been paid:

This Policy shall participate annually in the surplus earnings of the Company in accordance with the regulations adopted by the Board of Trustees.

The extended insurance, paid-up insurance, and loan or cash surrender value privileges, benefits, and conditions stated on the second page hereof form a part of this contract as fully as if recited at length over the signatures hereto affixed.

In Witness Whereof, The Penn Mutual Life Insurance Company of Philadelphia has caused this Policy to be signed by its President, Secretary, and Actuary, attested by its Registrar, at its Home Office, in Philadelphia, Pennsylvania, the tenth day of May 1907

W. H. Kingsley Secretary — Harry F. Weet President

Attest: A. J. Greene Registrar — Geo. G. Barker Actuary

ENDOWMENT IN 20 YEARS Regular

EXAMINED BY J. G.
Endt'l No. 3
A.D. Ed. 5, 1907
No. 1367

Table of Extension, Paid-up, and Loan or Cash Values, provided for by the Policy, if no indebtedness exists against it

AT END OF YEAR	TERM OF EXTENSION FOR THIS POLICY	These Values are for $1000 Insurance. For this Policy multiply by 10		
		PURE ENDOWMENT AT END OF EXTENSION	PAID-UP INSURANCE ON SURRENDER	LOAN OR CASH SURRENDER VALUES
3d	10 Years 279 Days	$	$ 169	$ 85 21
4th	14 " 89 "		224	126 48
5th	15 " "	49	278	169 21
6th	14 " "	131	332	213 46
7th	13 " "	210	384	259 37
8th	12 " "	286	436	306 94
9th	11 " "	359	487	356 27
10th	10 " "	429	538	407 45
11th	9 " "	497	587	456 84
12th	8 " "	562	636	508 08
13th	7 " "	624	684	561 28
14th	6 " "	684	731	616 55
15th	5 " "	742	777	674 00
16th	4 " "	798	823	733 77
17th	3 " "	851	868	796 05
18th	2 " "	902	913	861 01
19th	1 " "	952	956	928 91
20th	" "	Policy Matures		1000 00

Should any indebtedness exist it shall be deducted from the Cash Value of the Policy, and the other values shall be diminished proportionately

WRITTEN EXERCISES

1. If the insured in the foregoing policy should die just before the twelfth payment was due, how much would the estate receive above his total payments?

2. Suppose that the insured in the foregoing policy survives the endowment period, and the surplus earnings of the policy amounted to $3500. What would be the difference between the amount received and the amount paid, not reckoning interest?

3. The insured in the foregoing policy took out a $10,000 20-payment life policy at the same time he procured his endowment policy. The guaranteed cash value on the former was $2557.80 at the end of 10 yr., and the dividends for this term amounted to $83.22 per thousand. If the dividends on the endowment policy for this period amounted to $127.83 per thousand, which would have been the better investment, interest not being considered, and how much?

4. Assuming that the insured in the foregoing policy survived the endowment period and that the dividends which amounted to $350 per thousand were used to add to the value of the policy, how much less would he receive from the company than he would from investing the amount of the premiums in a savings-bank annually for 20 yr. at 4% interest? (Use table, page 323.)

5. What will be the first annual premium on a $15,000 ordinary life policy for a man 50 yr. old?

6. On his 25th birthday A took out a 20-yr. endowment policy for $5000; on his 35th birthday, a 15-yr. endowment policy for $6000; on his 40th birthday, a 20-payment life policy for $10,000. He died aged 43 yr., 6 mo. How much more did his heirs receive (dividends excepted) than he had paid the company?

7. B at the age of 25 yr. took out a 20-payment life policy for $5000. He died just before his twentieth payment became due. The company allowed $87.50 per thousand in dividends during this period, and these were used to reduce the annual premium. How much more did his heirs receive than was paid in premiums?

PARTITIVE PROPORTION, PARTNERSHIP, AND STORAGE

CHAPTER XXXV

PARTITIVE PROPORTION AND PARTNERSHIP

PARTITIVE PROPORTION

ORAL EXERCISE

1. A fails in business owing D $500, E $1500, and F $2500. If his resources are $1800, how much can he pay each of his creditors?

2. Two brothers, A and B, are engravers. A can earn $10 per day and B $5 per day. How much can they both earn in a day? What part of this amount can B earn? A?

3. They formed a partnership for one year and agreed to divide the net profits in proportion to the earning capacity of each. If the net profits for the year were $3600, what was the share of each?

4. C invests $3000, B $6000, and A $9000 in a manufacturing plant. The net profits for one year are $3600, and this sum is shared in proportion to the amount of capital invested. What amount does each receive as his share of the net profits?

5. A certain street was paved at a cost of $3000. The property owners on the street were A, who owned 200 ft. frontage, B, who owned 400 ft. frontage, and C, who owned 600 ft. frontage. If the cost of the paving was assessed on the property owners in proportion to the frontage owned, how much did each pay?

509. The process of dividing a number into parts proportional to several given numbers is called **partitive proportion.**

WRITTEN EXERCISE

1. Divide $42,770 among G, H, and I in proportion to $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$, respectively.

SUGGESTION. $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8} = \frac{2}{8}$, $\frac{4}{8}$, and $\frac{1}{8}$, respectively. Therefore, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$ stand in the same relation to each other as $\frac{2}{8}$, $\frac{4}{8}$, and $\frac{1}{8}$, or as 2, 4, and 1.

2. Divide the simple interest on $72,000 for 1 yr. 7 mo. at $3\frac{1}{2}\%$ among D, E, and F so that D's part will be twice E's part and one half of F's part.

3. An inheritance of $75,000 was divided among 3 sons and 4 daughters, so that each daughter received $\frac{1}{3}$ more than each son. How much did each son receive? each daughter?

4. A, B, and C were partners in a business. A put in $10,000, B $6000, and C $9000. Their net gain for a year was $17,500, shared in proportion to the amount of capital invested. What was each partner's share of the net gain?

PARTNERSHIP

ORAL EXERCISE

1. I invested $500 in a business and during the first year gained $1100. No withdrawals or subsequent investments having been made, what was my present worth at the close of the year?

2. Jan. 1 M invested $7500 in a factory. July 1 he found that his net loss was $1125. What was his present worth July 1, no withdrawals or subsequent investments having been made?

3. Answer problem 1 assuming that there was a withdrawal of $800 made during the year; problem 2 assuming that there was a subsequent investment of $1200 made on Mar. 1.

4. Apr. 1 B commenced business with a cash investment of $1500; Jan. 1 of the next year his present worth was $1875. What was his net gain or loss, no withdrawals or subsequent investments having been made?

5. July 1 D began business investing $25,000; Jan. 1 of the next year his net capital was $23,150. If no withdrawals or subsequent investments were made, did he gain or lose, and how much?

6. Answer problem 4 assuming that there were withdrawals amounting to $1000; problem 5 assuming that there was a subsequent investment of $5000.

7. June 1 F began business with a capital of $1750. During the 6 mo. following he lost $2750. What was the condition of his business Dec. 1?

8. Z began business on July 1 with a capital of $2500. 6 mo. later his net insolvency was found to be $1250. What was his net gain or loss?

9. A's business was insolvent $1250 on Jan. 1. From Jan. 1 to July 1 he gained $1750. What was the condition of his business July 1?

10. G gained $3750 during a certain year. He then found that his net capital was $1250. What was the condition of his business at the beginning of the year?

11. June 30, 1915, C's resources were $7500 and his liabilities $5000. June 30, 1916, his resources were $5000 and his liabilities $7500. What was his net gain or loss during this period?

12. Were the conditions in problem 11 reversed for the year stated, what would be the net gain or loss?

13. What is meant by *resources? liabilities? gain? loss?*

14. What is meant by *net gain? net loss? present worth? net capital? net insolvency?*

15. Read aloud the following, supplying the missing words: The condition of the business at the beginning + the —— —— or − the —— —— = the condition of the business at the close; and conversely, the condition of the business at the close + the —— —— or − the —— —— = the condition of the business at the beginning.

510. A **partnership** is an association of two or more persons who have agreed to combine their labor, property, and skill, or some of them, for the purpose of carrying on a common business and sharing its gains and losses.

Partnerships may be formed by either an oral or a written agreement, and in some cases by implication; but all important partnerships should be entered upon by an agreement in writing which definitely states all of the conditions relating to the business.

511. The members of a partnership are called **partners.**

Partners may be divided into four classes: (1) **Real,** or **ostensible,** those who are known to the world as partners and who in reality are such; (2) **nominal,** those who are known to the world as partners but who have no investment and receive no share of the gain; (3) **dormant,** or **silent,** those who are not known to the world but who nevertheless partake of the benefits of the business and thereby become partners; (4) **limited,** or **special,** those whose liability is limited.

Nominal partners, like real, or ostensible, partners, are liable to third parties for the debts of a business. Dormant partners are liable for the debts of the business as soon as their partnership connections become known to the world.

Ordinarily each partner is liable for all of the debts of the firm, but a special partner's liability is limited usually to the amount which he contributes to the firm's capital.

The method of forming a limited partnership is prescribed by statute. This differs somewhat in the different states. Such a partnership must usually have at least one member whose liability is not limited and who is the manager of the business.

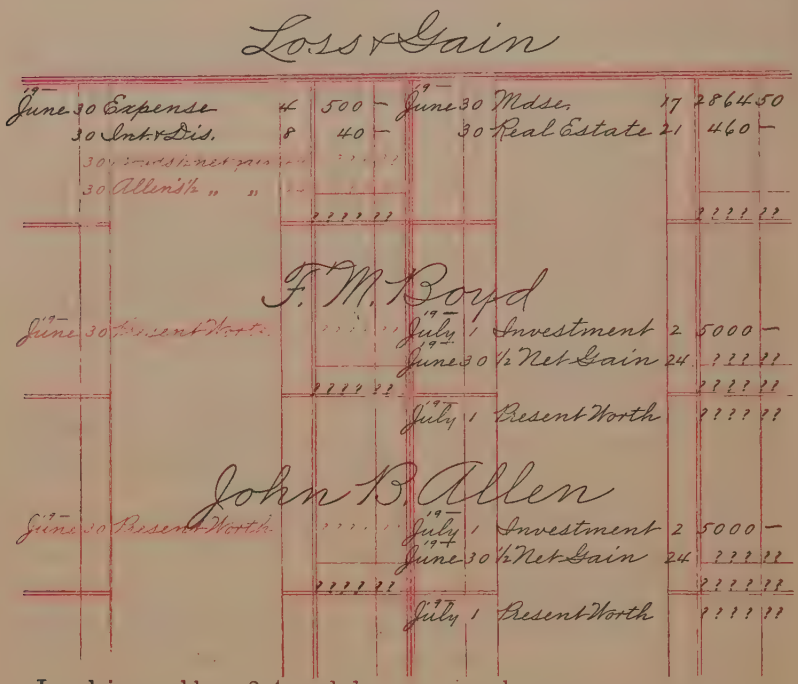
512. The **capital** of a partnership constitutes all the moneys and other properties contributed by the different partners to carry on the business.

Gains and Losses Divided Equally

513. The gains and losses of a business are divided among the partners in accordance with the agreement or contract entered into when the partnership was formed. If the partners invest equal sums and contribute equally in work, the gains are usually divided equally.

WRITTEN EXERCISE

1. Copy and complete the following ledger page:



In solving problems 2–4 use ledger paper as above.

If the student is not familiar with simple accounts, pages 41–47 should be reviewed.

2. Jan. 1, 1915, C. B. Johnson and B. H. Briggs engaged in a partnership business, each investing $3750. July 1, 1915, each partner withdrew $250. Jan. 1, 1916, their losses and gains were as follows:

Losses		Gains	
Expense	$104.75	Merchandise	$628.45
Merchandise Discounts	24.20	Interest and Discount	133.50
Real Estate	250.60	Stocks and Bonds	190.50

What was the present worth of each partner Jan. 1, 1916?

3. A, B, and C were partners for a year. Each invested $9500 and during the continuance of the partnership each withdrew $1000. The losses and gains at closing were as follows:

Losses		Gains	
Merchandise Discounts	$18.90	Merchandise	$4375.80
Expense	650.00	Interest and Discount	90.14

What was the net capital of each at closing?

4. O, P, and Q are partners sharing the gains and losses in equal proportions. O invested $8500, P $8200, and Q $8450. During their first year the gains were as follows: merchandise, $6457.10; real estate, $680.50; interest and discount, $29.90. If the cost of conducting the business was $1920.50, what was the present worth of each partner at the end of the year?

Gains and Losses Irregularly Divided

514. Sometimes the gains are divided according to certain arbitrary fractions which are not in proportion to the amount invested. In such cases the skill of a partner is frequently considered as being equal to a certain amount of capital. In some cases a certain amount is paid the heavier investor before other division of the gains or losses is made. In still other cases, a stated salary is paid to each partner before the gains or losses of the business are divided. This salary varies according to the ability of the several partners or according to the time each devotes to the business.

WRITTEN EXERCISE

1. A and B entered into partnership, each investing $7500. Because of the greater experience of A he was to be credited with $1200 before any other division of the gains or losses. The gains or losses were then to be divided equally. During the first year the gains were as follows: merchandise, $4111.10; real estate, $510. If the losses were $622.80, what was the present worth of each at the end of the year?

2. A and B entered into partnership, A investing $8000 and B $10,000. B doing no work, it was agreed that A should take $2000 from the gains before dividing, and that the net gain or loss should then be shared equally. The gains last year were $8900 and the losses $1400. What was the net gain of each?

3. C, D, and E entered into partnership Jan. 1, each investing $8500. The articles of agreement provided (1) that C should devote all his time to the business and D and E only a portion of their time; (2) that if losses occurred, they should be borne equally; (3) that if gains were realized, C should receive $\frac{1}{2}$ and D and E each $\frac{1}{4}$. During the year the gains were as follows: Merchandise, $8217.10; Stocks and Bonds, $612.50; Interest, $492.92. If the expenses were $2217.80, what was the present worth of each partner at the close of the year?

4. F and G entered into partnership, F investing $5000 and G $7500. Because of the greater skill of F it was agreed that he should be credited with $1500 a year before other division of the gains or losses. Then if losses occurred, F was to bear $\frac{2}{5}$ of them and G $\frac{3}{5}$; but if gains were realized, they were to be divided equally. During the first year the gains of the firm were as follows: Merchandise, $3129.50; Real Estate, $250; Stocks and Bonds, $575; Interest, $130.50. If the cost of conducting the business was $938.48 (exclusive of F's salary), what was each partner's net capital at the close of the year?

5. J, K, and L entered into partnership, J investing $20,000, K $10,000, and L nothing. The articles of agreement provided (1) that the gains or losses should be shared as follows: J, $\frac{3}{5}$, K, $\frac{1}{4}$, L, $\frac{3}{20}$; (2) that the capital should be kept intact; (3) that before any division of the profits was made, J should be credited with an annual salary of $1500. At the end of a year the resources were found to be $65,250 and the liabilities (not including J's salary), $16,750. What was each partner's share of the net gain? After the net gain was credited, what was the net capital of each partner?

Gains and Losses Divided According to Investment

515. Sometimes the gains and losses are divided in proportion to the amount invested; that is, according to the principles of partitive proportion.

516. Example. A and B engaged in business, agreeing to share the gains or bear the losses in proportion to the amount of capital invested. A invested \$2500 and B \$3500. They gained \$1800. What was the share of each?

Solution. \$2500 + \$3500 = \$6000, the total capital. Since the total capital is \$6000 and A put in \$2500, A's share is $\frac{2500}{6000}$, or $\frac{5}{12}$, and B's share is $\frac{3500}{6000}$, or $\frac{7}{12}$. Therefore, A should receive $\frac{5}{12}$ of \$1800, or \$750, and B should receive $\frac{7}{12}$ of \$1800, or \$1050.

ORAL EXERCISE

Find each man's gain or loss in each of the following problems:

	Investment	Gain		Investment	Loss
1.	A, \$3000; B, \$2000	\$500	6.	K, \$2000; L, \$4000	\$120
2.	C, \$1000; D, \$2000	\$150	7.	M, \$1500; N, \$2000	\$700
3.	E, \$1200; F, \$4800	\$1200	8.	O, \$1000; P, \$5000	\$600
4.	G, \$1500; H, \$4500	\$1800	9.	Q, \$1500; R, \$6000	\$750
5.	I, \$1500; J, \$7500	\$1500	10.	S, \$1750; T, \$3500	\$600

WRITTEN EXERCISE

1. A, B, and C invested \$2000, \$3000, and \$5000, respectively, in a wholesale dry goods business. During the first year the net profits were \$4155.80. What was the share of each?

2. D, E, and F invested \$2500, \$3250, and \$3500, respectively, in a manufacturing business. At the close of the first year their profits were found to be \$3774.37. What was the share of each?

3. G, H, and I formed a copartnership, G investing \$3000, H, \$2000, and I, \$1500. During the first six months their net gain was \$1829.10. How much was each man worth after his share of the net gain had been carried to his account?

4. Copy and complete the following statement:

Statement of Conditions, June 30, 19—.

		Losses and Gains				
Mdse.	total cost	$5932.50				
Mdse.	unsold	$1658.00				
Mdse.	cost of sales		4274	50		
Mdse.	total sales				5582	50
Gain on Mdse.	carried down		????	??		
			5582	50	5582	50
Gain on Mdse.	brought down				????	??
Expense	total outlays		108	—		
J. M. Robinson's	⅗ of net gain		????	??		
O. H. Bressee's	⅖ of net gain		????	??		
			1308	—	1308	—
		Resources and Liabilities				
Mdse.	unsold		1658	—		
Cash	on hand		2067	—		
Geo. M. Long	owes on account		369	—		
Nathan Berry	" " "		665	—		
F. M. Queen	" " "		466	50		
Isaac Mann & Co.	owed on account				425	50
Present Worth	of the firm				????	??
			5225	50	5225	50
		Verification				
J. M. Robinson's	net investment	$2400.00				
" " "	⅗ of net gain	800.00				
" " "	present worth		????	??		
O. H. Bressee's	net investment	$1200.00				
" " "	⅖ of net gain	400.00				
" " "	present worth		????	??		
Present Worth	of the firm				????	??
			4800	—	4800	—

Interest Allowed and Charged

517. The inequalities in investments and withdrawals are frequently adjusted by allowing and charging interest upon same. When interest is allowed on investments and charged on withdrawals, the gains and losses are usually divided equally.

518. Example. June 1, 1915, C. H. Dean and E. D. Snow formed a partnership, C. H. Dean investing $5000 and E. D. Snow $4000. They agreed that the gains and losses should be divided equally, but that, owing to the unequal investments, each partner should be allowed interest at 6 % on all sums invested and charged interest at the same rate on all sums withdrawn, said interest to be adjusted at the time of closing the books. The profits for the first six months were $1050. What was the net capital of each partner after the interest was adjusted and the net gain carried to his account?

C. H. Dean

1915					1915				
Dec.	1	Net Capital	5540	00	June	1	Investment	5000	00
					Dec.	1	Interest	15	00
						1	½ Net Gain	525	00
			5540	00				5540	00
					Dec.	1	Net Capital	5540	00

E. D. Snow

1915					1915				
Dec.	1	Interest	15	00	June	1	Investment	4000	00
	1	Net Capital	4510	00		1	½ Net Gain	525	00
			4525	00				4525	00
					Dec.	1	Net Capital	4510	00

Solution. $5000 in 6 mo. will earn $150 interest. $4000 in 6 mo. will earn $120 interest. $\$150 + \$120 \div 2 = \$135$, the average interest earned. $\$150 - \$135 = \$15$; that is, C. H. Dean's interest is $15 above the average. $\$135 - \$120 = \$15$; that is, E. D. Snow's interest is $15 below the average. Therefore to adjust the interest on the investments, credit C. H. Dean's account $15 and charge E. D. Snow's account $15. ½ of $1050 = $525, the net gain of each. Credit each account with the net gain; then C. H. Dean's net capital is $5540 and E. D. Snow's net capital $4510.

WRITTEN EXERCISE

1. Copy and complete the following statement of conditions:

Statement of Conditions, Dec. 31, 19—

	Resources			
Cash	on hand		1243.50	
Mdse.	unsold		4365.60	
Expense	items on hand		366.20	
Real Estate	estimated value		6500 —	
Accts. Rec.	total owed by customers		5387.25	
Notes Rec.	notes on hand		2833.25	20695.80
	Liabilities			
Accts. Pay.	total owed to creditors		2436.30	
Notes Pay.	outstanding		1050 —	???? ??
	Present worth of the firm			??????
	Proprietor's Accounts			
W. N. Brown's	investment	$10000		
" " "	int. due him	150		
" " "	net investment		????	
Willis M. Harts	investment	$5000		
" " "	int. due the business	150		
" " "	net investment		4850 —	
	Net investment of the firm			??????
	Net gain of the firm			2209.80
	Verification			
W. N. Brown's	net investment	$10150		
" " "	½ of net gain	1104		
" " "	present worth		??????	
Willis M. Hart's	net investment	$4850		
" " "	½ of net gain	????		
" " "	present worth		??????	
	Present worth of the firm			17209.80

2. W. H. Burgess and Otis Clapp began business July 1, 1915, the former investing $12,000 and the latter $10,000. They agreed that the gains and losses should be divided equally, but that, because of the inequality in the investments, interest at 6 % should be allowed on investments and charged on withdrawals. July 1, 1916, the firm's resources and liabilities (partners' accounts excluded) were as follows:

Resources		Liabilities	
Cash	$4150.00	Accounts Pay.	$7500.
Accounts Rec.	8150.60	Notes Pay.	4900.
Mdse.	18210.50		
Notes Rec., on hand	4250.00		
Street Railway Stock	3000.00		
Store and Lot	5200.00		
Office Fixtures	500.00		

Make a statement, as in problem 1, showing the present condition of the business.

3. Aug. 1, 1915, F. E. Greene and W. B. Linden formed a partnership for the purpose of carrying on a manufacturing business. F. E. Greene invested $8500 and W. B. Linden, $10,750. It was agreed that interest at 6% should be allowed and charged on investments and withdrawals and that the gains and losses should be divided equally. At the close of the first year the resources and liabilities (partners' accounts excluded) were as follows:

Resources		Liabilities	
Cash	$2355.20	Notes Pay.	$1158.25
Mdse.	5284.85	Accounts owed by the business	2100.00
Notes Rec.	2840.00		
Accounts owing the business	4170.50		
Office Fixtures	450.00		

Feb. 1, 1916, F. E. Greene withdrew $750 and W. B. Linden $600. Make a statement showing the condition of the business at the close of the year.

4. James B. Westfall and John L. Manning began a common business on Sept. 1, 1915, the former investing $14,500 and the latter $13,935. They agreed that interest at 6% should be

allowed and charged on investments and withdrawals, respectively, and that the gains and losses should be divided equally. Sept. 1, 1916, a trial balance of their general ledger was as follows :

	Debits	Credits
James B. Westfall		$14500.00
John L. Manning		1 935.00
Cash	$13368.64	
Merchandise	31664.00	20000.00
Office fixtures	510.50	
Horse and wagon	405.00	
Real estate	7000.00	
Expense	445.80	
Collection and exchange	12.20	
Mdse. discounts	58.50	
Accounts receivable	6852.84	
Accounts payable		8864.75
Bills payable		3000.00
Interest and discount		17.73
	$60317.48	$60317.48

The merchandise unsold was found to be worth $13,827.35 ; the real estate, $7500 ; the office fixtures, $500 ; the horses and wagons, $400; and the expense items on hand, $102.50. There was due on the merchandise account for freight, $138.50, and on the expense account for telephone service, $25. Make a statement showing the condition of the business Sept. 1, 1916. (See model, page 441.)

Gains and Losses Divided According to the Average Investment

519. That sum which, invested for a certain period, is equivalent to two or more sums invested for different periods, is called an **average investment.** The gains and losses of a business are sometimes divided in proportion to the average investment.

520. Example. April 1, 1915, A and B formed a partnership and agreed to share the gains or losses according to average net investment. A furnished $10,000 of the capital and

B $7500. July 1 A withdrew $1500 and B $500. Apr. 1, 1916, their net gain was found to be $12,800. What was the net gain of each partner?

SOLUTION

A had in $10,000 for 3 mo., when he withdrew $1500, leaving $8500 for the remaining 9 mo.

B had in $7500 for 3 mo., when he withdrew $500, leaving $7000 for the remaining 9 mo.

A's $10000 for 3 mo. = $30000 for 1 mo.
A's $8500 for 9 mo. = $76500 for 1 mo.
A's average net investment = $106500 for 1 mo.
B's $7500 for 3 mo. = $22500 for 1 mo.
B's $7000 for 9 mo. = $63000 for 1 mo.
B's average net investment = $85500 for 1 mo.

$106500 + $85500 = $192000, the firm's average net investment for 1 mo.

A's share is $\frac{106500}{192000}$, or $\frac{71}{128}$.
B's share is $\frac{85500}{192000}$, or $\frac{57}{128}$.
Therefore, A should receive $\frac{71}{128}$ of $12800, or $7100.
And B should receive $\frac{57}{128}$ of $12800, or $5700.

WRITTEN EXERCISE

1. Apr. 1 R and C formed a partnership for 1 yr., the former investing $4500 and the latter $6000. They agreed to share the gains and losses in proportion to the average net investment. Aug. 1 R invested $1500, and C withdrew $1000. On closing the books at the end of the year the net loss was found to be $1290. What was each partner's present worth after his account was charged with his share of the net loss?

2. June 1, 1915, E and F formed a copartnership for the purpose of carrying on a real estate business. E invested $25,000 and F $15,000. They agreed to share the gains and losses in proportion to the average net investment. Sept. 1, 1915, E withdrew $1000 and F $500. Dec. 1, 1915, each withdrew $1000. Mar. 1, 1916, F invested $5000. June 1, 1916, the partnership was dissolved. After all resources were converted into cash and all liabilities to outside parties paid, the amount of cash in bank was $50,890. What amount was due each partner?

WRITTEN REVIEW EXERCISE

1. Apr. 1, 1916, W. L. Cutter and O. M. Woodward formed a copartnership for the purpose of carrying on a dry goods business. W. L. Cutter invested $20,500 and O. M. Woodward $18,500. They agreed to allow interest at 6% on investments, charge interest at the same rate on withdrawals, and divide the gains and losses equally. July 1, 1916, W. L. Cutter withdrew $500. Oct. 1 O. M. Woodward withdrew $1000 and W. L. Cutter $750. At the close of the year the resources and liabilities, exclusive of partners' accounts, were as follows:

Resources		Liabilities	
Cash in bank	$2130.60	Accounts owed by the business	$7260.00
Stocks and bonds on hand	6450.00	Notes payable unredeemed	1200.00
Goods in stock	16095.00		
Notes receivable on hand	6150.00		
Office fixtures on hand	500.00		
Accounts owing the business	12260.52		

Make a statement showing the condition of the business Apr. 1. 1917.

2. July 1, 1915, A. B. Curtis and B. H. Barton formed a partnership and invested $7500, of which A. B. Curtis furnished $\frac{3}{5}$ and B. H. Barton, $\frac{2}{5}$. Jan. 30, 1916, their resources were as follows: merchandise, unsold, $2172.70; cash on hand, $2823.96; real estate on hand, $3100; account against James Noble, $840.10; account against A. H. Cook & Co., $1156.84. On the same date their liabilities were as follows: account in favor of D. M. Frost & Co., $218.60; account in favor of J. B. Neal & Co., $385. During the year the merchandise bought cost $6807.50 and the sales aggregated $7154.90. The cost of carrying on the business was $530.10. Make a statement (see page 434) showing the present condition of the business. Divide the net gain in proportion to the investments.

3. Copy and complete the following statement of conditions:

Statement of Conditions July 31, 19—

Losses & Gains					
Mdse.	total cost	$29078.22			
Mdse.	unsold	$18078.22			
Mdse.	cost of sales		????.??		
Mdse.	total sales				20150.50
Gain on Mdse	carried down		????.??		
			20150.50		20150.50
Gain on Mdse.	brought down				????.??
Wamsutta Mills Stock	estimated value	$18725—			
Wamsutta Mills Stock	cost	$15000—			3725—
Expense	cost		4150—		
C. H. Palmer's	½ net gain		????.??		
D. O. Mills's	" "		????.??		
M. W. Newbury's	" "		????.??		
			12874.62		12874.62
Resources & Liabilities					
Mdse.	unsold		18078.10		
Cash	on hand		12271.52		
Accts. Rec.	owing the business		24800—		
Wamsutta Mills Stock	estimated value		18725—		
Notes Rec.	on hand		5000—		
Accts. Pay.	owed by the business				10150—
Present Worth	of the firm		????.??		
			78874.62		78874.62
Verification					
C. H. Palmer's	investment	$25000—			
" " "	interest due him	300—			
" " "	½ of net gain	2908.21	28208.21		
D. O. Mills's	investment	$20000—			
" "	½ of net gain	2908.21	22908.21		
M. W. Newbury's	investment	$15000—			
" " "	int. due the business	300—			
" " "	net investment	14700—			
" " "	½ of net gain	2908.21	17608.20		
Present Worth	of the firm		????.??		
			68724.62		68724.62

4. Jan. 1, 1915, C. H. Smith and W. W. Osgoodby formed a copartnership for the purpose of carrying on a real estate business. C. H. Smith invested $15,000 and W. W. Osgoodby $10,000. They agreed to share the gains and losses in proportion to the average net investment. July 1, 1915, C. H. Smith withdrew $1000 and W. W. Osgoodby $750. On closing the books at the end of the year the net gain was found to be $8685. What was each partner's present worth after his account was credited with his share of the net gain?

5. Frank M. Congdon, E. H. Robinson, and O. B. Moulton are partners in a dry goods house under the firm name of E. H. Robinson & Co. On commencing business Aug. 1, 1916, Frank M. Congdon invested $17,500, E. H. Robinson $20,000, and O. B. Moulton $12,000. The articles of agreement provided: (1) that each partner should be allowed interest at 6% on investments and charged interest at the same rate on withdrawals; (2) that because of special skill and experience Frank M. Congdon should be credited $1500 before any other division of the gains and losses; (3) that then the gains should be divided equally. Aug. 1, 1917, the results of the year's business were as follows: cost of merchandise purchased, $80,872; value of merchandise on hand, $14,280.95; sales of merchandise, $78,756; cost of real estate, $18,000; cost of permanent improvements on real estate, $1200; present estimated value of real estate, $25,000; notes in favor of the firm, $11,500; interest accrued on these notes, $112; cost and present value of horses and wagons, $1250; general expenses for the year (exclusive of the amount due Congdon), $1800; traveling expenses for the year, $1200; accounts owing the firm, $20,160.90; cash on hand, $19,033.10; mortgage on the firm's real estate, $12,000; interest accrued on the mortgage, $480; notes outstanding, $3500; accounts owed by the firm, $11,260. Show in proper statements the financial condition of the partners.

CHAPTER XXXVI

STORAGE

SIMPLE STORAGE

ORAL EXERCISE

1. I stored my piano in a warehouse from June 16 to October 1 at $1 per month or fraction thereof. What sum must I pay in settlement?

2. I rented a room in a storage warehouse from Sept. 1 to Dec. 18 at $6.50 per month or fraction thereof. What amount did I have to pay?

3. What must I pay for the storage of 5000 bu. of wheat stored from Dec. 3 to Apr. 15 at 4¢ per bushel per month or fraction thereof? for the storage of 10,000 bu. of corn stored from Dec. 1 to Mar. 1 at $3\frac{1}{2}$¢ per bushel per month?

521. **Storage** is a charge made for storing goods in a warehouse.

522. The **term of storage** is the period of time for which a certain rate is charged.

The term of storage is usually, though not invariably, 30 da.; and in estimating charges, a part of a term is counted the same as a full term.

523. The **rates of storage** are sometimes fixed by an agreement between the contracting parties, sometimes by boards of trade, chambers of commerce, or associations of warehousemen, and sometimes by legislative enactment.

524. **Simple storage** is storage estimated at the time of the withdrawal of the goods from the warehouse.

ORAL EXERCISE

1. Verify the following storage bill:

Boston, Feb. 1, 19__

Mrs. D. H. Sutherland
215 Causeway St.

To Quincy Market Cold Storage and Warehouse Co., Dr.
Main Office, 133 Commercial Street

FOR STORAGE

DATE RECEIVED	QUANTITY	MERCHANDISE	STORAGE LOT NO.	DATE DELIVERED	QUANTITY	MO.	RATE	AMOUNT
May 14	490 cs.	Eggs	4546	Sept. 4	140 cs.	4	40¢	56 —
				Nov. 26	200 "	7	70¢	140 —
				Jan. 3	150 "	8	80¢	120 — 316 —

2. When were the eggs received for storage? If there are 30 doz. in a case, how many dozen were received?

3. Suppose the rate in the bill were 10¢ per case per month or fraction thereof for the first 3 mo., and 5¢ per case per month after the first 3 mo. What would this rate be for 4 mo.? for 7 mo.? for 9 mo.? for 10 mo.? for 11 mo.?

4. Using the rate in the bill, find the storage on 150 cs. eggs stored from July 1 to Jan. 14; on 500 cs. eggs stored from July 3 to June 14; on 350 cs. eggs stored from June 14 to Mar. 4; on 12,000 doz. eggs stored from June 14 to Nov. 18.

5. The storage rate on poultry is $\frac{1}{4}$¢ per pound per month. Find the storage on 1000 lb. from Jan. 10 to Feb. 6; on 800 lb. from Jan. 10 to Feb. 18; on 1200 lb. from Jan. 10 to May 27; on 1600 lb. from Jan. 10 to July 3.

6. In a certain warehouse the rate of storage on cheese is 8¢ per 100 lb., for each month or fraction thereof. At that rate find the storage on 1000 lb. cheese from May 3 to July 15; on 20,000 lb. from May 3 to Aug. 26; on 7500 lb. from May 3 to Sept. 12; on 10,000 lb. from May 3 to Oct. 6; on 5 T. from June 15 to Oct. 28; on 10 T. from June 15 to Nov. 17.

525. Example. The following memorandum of flour stored for you by the Central Storage Co.: received Nov. 1, 2000 bbl., and Nov. 16, 3000 bbl.; delivered Nov. 8, 1000 bbl., and Dec. 5, 4000 bbl. If the rate of storage was 5¢ per barrel per month or fraction thereof, what was the bill to render?

SOLUTION

RECEIPTS AND DELIVERIES			TERM	RATE	STORAGE
Nov. 1, received	2000 bbl.				
Nov. 8, delivered	1000 bbl.,	which were in storage	7 da.	5¢	$ 50
	1000 bbl.,	balance in storage			
Nov. 16, received	3000 bbl.				
	4000 bbl.,	balance in storage			
Dec. 5, delivered	4000 bbl.,	1000 of which were in storage	34 da.	10¢	100
		3000 of which were in storage	19 da.	5¢	150
		Total storage,			$ 300

WRITTEN EXERCISE

1. In a certain warehouse the storage charges on flour are 3¢ per barrel per month or fraction thereof. Nov. 1, I stored 500 bbl.; Dec. 1, I withdrew 100 bbl.; Jan. 1, I stored 600 bbl.; Mar. 1, I withdrew 1000 bbl. What was the storage on the first withdrawal? 400 bbl. of the second withdrawal was in storage for how many months? What was the total storage due Mar. 1?

2. How much is due on the following account?

Boston, Mass., Jan. 10, 19

Received from C. W. Washburn & Co.
210 Atlantic Ave.

Article Poultry Lot No. 4888

bkt. 27 bx. lb. Weight 5400 lb.

Room 6 Section 4 Rate 1/4¢ per lb. per mo.

DELIVERIES AND CHARGES									CREDITS			
DATE		QUANTITY			MONTHS	RATE	AMOUNT		AMOUNT		DATE	REMARKS
Feb.	6	5	cs.	1000#	1	1/4¢	2	22	11	50	Feb. 18	Cash
	18	9	"	1800#	2	1/2¢	2	22				
May	27	6	"	1200#	2	2¢	22	22				
July	5	7	"	1400#	2	2¢	2	22				

3. The following is a memorandum of apples stored by you for T. B. Welch & Co. : received Nov. 28, 5000 bbl., Dec. 15, 1000 bbl., and Dec. 18, 3000 bbl.; delivered Dec. 28, 2000 bbl., Feb. 1, 1000 bbl., and Feb. 10, 6000 bbl. Render a bill for the storage, charges being 5¢ per barrel per month or fraction thereof.

4. Copy and complete the following bill :

Boston, Jan. 30, 19

Messrs. John Smith & Co.
1286 Boylston St.

To EASTERN COLD STORAGE CO., Dr.
28 to 44 North Street

FOR STORAGE

LOT	DATE IN	DATE OUT	NO. MONTH	ARTICLE	WEIGHT	RATE PER 100 LB.	EXTENSION	AMOUNT
7592	'19 Jan. 10	Feb. 6	1	200 bbl. Butter	10000#	18¢		
		Apr. 9	3	100 " "	5000#	54¢		
		May 19	5	350 " "	17500#	90¢		
		June 3	5	150 " "	7500#	90¢		

AVERAGE STORAGE

526. When there are frequent receipts and deliveries of goods, it is customary for some warehouses to average the time and charge a certain rate per month of thirty days. The process is called **average storage.**

527. Example. The following is a memorandum of the receipts and deliveries of flour stored by the Eastern Storage Co. for A. M. Briggs & Co.: received Apr. 10, 2000 bbl., and Apr. 30, 3000 bbl.; delivered May 8, 1000 bbl., and June 9, 4000 bbl. The storage charge being $4\frac{1}{2}$¢ per barrel per term of 30 da. average storage, what was the amount of the bill to render?

SOLUTION. The solution of this problem is clearly shown in the following statement of account:

ACCOUNT OF FLOUR RECEIVED AND DELIVERED BY

EASTERN STORAGE CO.

For A. M. BRIGGS & CO.

DATE		RECEIPTS	DELIVERIES	BALANCE	TIME IN STORAGE	QUANTITY IN STORAGE FOR 1 DA.
19—						
Apr.	10	2000 bbl.		2000 bbl.	20 da.	40000 bbl.
	30	3000 bbl.		5000 bbl.	8 da.	40000 bbl.
May	8		1000 bbl.	4000 bbl.	32 da.	128000 bbl.
June	9		4000 bbl.	0000 bbl.	00 da.	00000 bbl.
		5000 bbl.	5000 bbl.			30)208000 bbl.

Average storage for 1 mo. = $6933\frac{1}{3}$ bbl.

$6933\frac{1}{3}$ bbl. at $4\frac{1}{2}¢ = \$312$, the amount of the bill to render.

WRITTEN EXERCISE

1. The Quincy Storage and Warehouse Co. received and delivered on account of Boynton Travers & Co. sundry barrels of apples as follows: received Dec. 1, 1915, 1000 bbl., Dec. 26, 2000 bbl.; delivered Feb. 1, 500 bbl., Mar. 1, 1000 bbl., Mar. 15, 1100 bbl., Mar. 31, 400 bbl. If the charges were 6¢ per barrel per term of 30 da. average storage, what was the amount of the bill to render?

2. The Central Storage Warehouse Co. received and delivered on account of A. S. Osborn & Co. sundry bushels of wheat as follows: received Oct. 1, 17,600 bu., Nov. 15, 3600 bu., Dec. 18, 4200 bu., Dec. 27, 4320 bu.; delivered Oct. 31, 10,000 bu., Dec. 4, 10,720 bu., Dec. 19, 4000 bu., Dec. 28, 5000 bu. If the charges were $1\frac{3}{4}¢$ per bushel per term of 30 da. average storage, what was the amount of the bill to render?

3. Metropolitan Storage Co. received and delivered on account of Chas. B. Sherman sundry barrels of flour as follows: received Nov. 15, 1915, 1800 bbl., Nov. 30, 1000 bbl., Dec. 18, 600 bbl., Jan. 30, 3000 bbl.; delivered Dec. 1, 1000 bbl., Dec. 31, 1900 bbl., Jan. 31, 600 bbl., Feb. 5, 600 bbl., Apr. 30, 2300 bbl. If the charges were $5\frac{1}{2}¢$ per barrel per term of 30 da. average storage, what was the amount of the bill to render?

WRITTEN REVIEW EXERCISES

1. I bought wheat at \$0.80 per bushel and put it in storage. If the storage charges were 2%, for how much must I sell the wheat to realize a gain of \$0.12 per bushel, and make allowance of 6% for incidentals?

2. A produce dealer bought 150 T. cabbage at \$ 5.50 per ton. He paid 90¢ per ton for storage and then sold the cabbage at a clear profit of 25%. How much did he receive per ton and what was his gain?

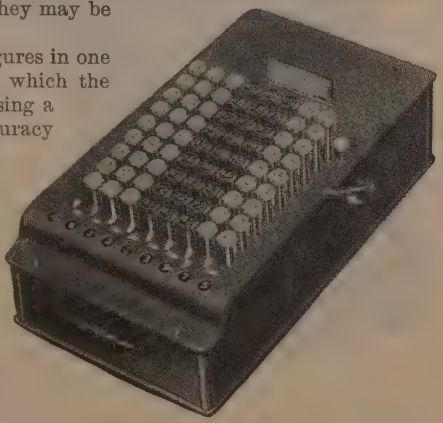
3. Nov. 1 a speculator bought 5000 bbl. apples at \$2.25 per barrel and put them in storage. Feb. 1 he withdrew them from the storage warehouse. He had them sorted and repacked, when he found that he had only 4600 bbl. of sound apples. These he sold at \$3.50 per barrel. If the storage charges were 5¢ per barrel per month or fraction thereof, and the charges for repacking were \$500, did he gain or lose, and how much? what per cent?

4. Dec. 15, 1915, A. L. Farley bought 1000 bbl. flour at \$4 and placed it with the Union Warehouse Co. for storage. Jan. 15 he bought 3000 bbl. flour at \$4.15 and placed it with the same warehouse company for storage. On Feb. 15 he withdrew 2000 bbl. from storage and sold it at \$ 5.85, on Mar. 25 he withdrew 1000 bbl. and sold it at \$ 5.62½, on Apr. 1 he withdrew 1000 bbl. and sold it at \$ 5.87½. If the storage charges were 5¢ per barrel per month or fraction thereof, and cartage and incidentals cost \$ 100, did he gain or lose, and how much?

APPENDIX A

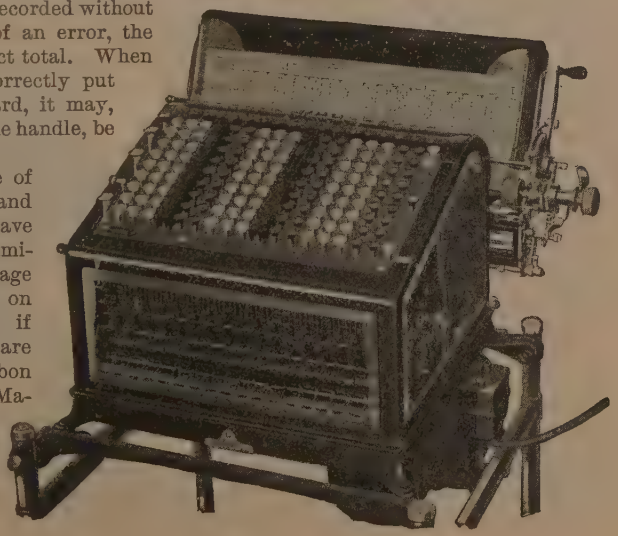
ADDING MACHINES

Machines or mechanical devices for performing arithmetical calculations are now commonly used in business offices; in banks, factories, insurance offices, and wholesale and retail houses they may be regarded as indispensable.



A machine will list and add figures in one fifth or one sixth of the time in which the work can be done by a person using a pen or a pencil, and with an accuracy that a person cannot equal. The operations of subtraction, multiplication, division, and trade discount may be as readily performed as those in addition.

The machine writes figures as rapidly as a typewriter, and as legibly; figures are recorded by simply touching the keys. The figures written down are added automatically, and at any time, by the mere operation of a handle, will be recorded without the possibility of an error, the absolutely correct total. When an item is incorrectly put into the keyboard, it may, before pulling the handle, be corrected.

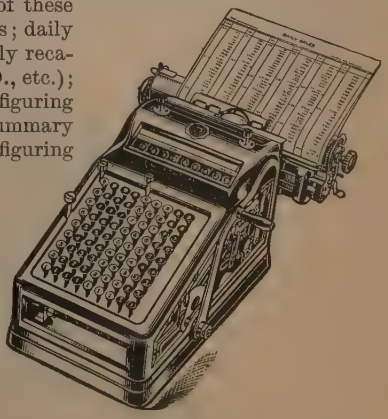


Machines are of different sizes, and some machines have paper carriages similar to the carriage on a typewriter; on these carriages, if desired, results are printed and carbon copies made. Machines may be furnished with an electric drive, thus avoiding the handle pull.

Machines can be equipped for adding dollars and cents; feet and inches; dozens and gross; hours and minutes; tons and hundred weights; pounds and bushels; grains and pennyweights; English pounds, shillings, and pence, or any other kind of foreign money; dates and amounts; or any kind of figures.

Machines may be equipped with the *unlimited split* device for dividing the keyboard into two or more sections, for listing and adding two or more sets of figures at one operation. Thus they may also be equipped with devices for automatically listing and adding across the sheet or form in two or more columns. There is a *duplex* adding machine with two sets of wheels, to accumulate two separate totals at the same time. With a machine of this type, totals of groups of items may be secured and a grand total of the group totals accumulated at the same time.

Adding-subtracting machines add debits, subtract credits, and automatically compute the difference and print it. There are special adding machines for handling monthly statements and for ledger posting and cost accounting; a pay-roll machine that, with one operation, prints the employees' numbers and the amount of pay on the pay-roll sheet and pay envelopes.



The following are some of the uses of these machines in offices: proving daily postings; daily ledger balance; daily cash balance; daily recapitulation of sales (as cash, credit, C.O.D., etc.); checking invoices and freight bills; figuring discounts; computing commissions; summary of day's receipts and disbursements; figuring estimates; making out pay envelopes; analysis of outstanding accounts; analysis of accounts payable; balancing petty cash account; footing ledger accounts before taking the trial balance; taking off the trial-balance figures (debits and credits); reconciling cashbook balance with bank balance, listing the number and the amount of each outstanding check; making monthly statements giving month, date, total of debits, total of credits, balance and special terms; compiling statements of cost of production; footing inventories and calculating extensions; posting customers' ledger.

The cuts show various types of calculating machines.

APPENDIX B

TABLES OF MEASURES

MEASURES OF CAPACITY

Liquid Measure	*Dry Measure*
4 gills = 1 pint	2 pints = 1 quart
2 pints = 1 quart	8 quarts = 1 peck
4 quarts = 1 gallon	4 pecks = 1 bushel
= 231 cubic inches	= 2150.42 cubic inches

Barrels and **hogsheads** vary in size; but in estimating the capacity of tanks and cisterns 31.5 gal. are considered a barrel, and 2 bbl., or 63 gal., a hogshead.

A **heaped bushel**, used for measuring apples, corn in the ear, etc., equals 2747.71 cu. in. A **dry quart** equals 67.2 cu. in., and a **liquid quart** 57.75 cu. in.

MEASURES OF WEIGHT

Avoirdupois Weight	*Troy Weight*
16 ounces = 1 pound	24 grains = 1 pennyweight
100 pounds = 1 hundredweight	20 pennyweights = 1 ounce
2000 pounds = 1 ton	12 ounces = 1 pound

Apothecaries' Weight	*Comparative Weights*
20 grains = 1 scruple	1 lb. troy or apothecaries' = 5760 gr.
3 scruples = 1 dram	1 oz. troy or apothecaries' = 480 gr.
8 drams = 1 ounce	1 lb. avoirdupois = 7000 gr.
12 ounces = 1 pound	1 oz. avoirdupois = 437½ gr.

The ton of 2000 lb. is sometimes called a **short ton**. There is a ton of 2240 lb., called a **long ton**, used in all customhouse business and in some wholesale transactions in mining products.

In weighing diamonds, pearls, and other jewels, the unit generally employed is the **carat**, equal to 3.2 troy grains. The term "carat" is also used to express the number of parts in 24 that are pure gold. Thus, gold that is 14 carats fine is $\frac{14}{24}$ pure gold and $\frac{10}{24}$ alloy.

Miscellaneous Weights

1 keg of nails = 100 pounds	1 barrel of salt = 280 pounds
1 cental of grain = 100 pounds	1 barrel of flour = 196 pounds
1 quintal of fish = 100 pounds	1 barrel of pork or beef = 200 pounds

A cubic foot of water contains about 7½ gal. and weighs 62½ lb., avoirdupois.

MEASURES OF EXTENSION

Long Measure

12 inches	= 1 foot
3 feet	= 1 yard
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod
320 rods, or 5280 feet	= 1 mile

Surveyors' Long Measure

7.92 inches	= 1 link
25 links	= 1 rod
4 rods, or 100 links	= 1 chain
80 chains	= 1 mile

City lots are usually measured by feet and decimal fractions of a foot; farms, by rods or chains.

Miscellaneous Long Measures

4 inches	= 1 hand
6 feet	= 1 fathom
120 fathoms	= 1 cable length
1.15 miles, nearly,	= 1 knot, or
1 nautical or geographical mile	

Square Measure

144 square inches	= 1 square foot
9 square feet	= 1 square yard
$30\frac{1}{4}$ square yards	= 1 square rod
160 square rods	= 1 acre
640 acres	= 1 square mile

The **hand** is used in measuring the height of horses at the shoulder. The **fathom** and **cable length** are used by sailors for measuring depths at sea. The **knot** is used by sailors in measuring distances at sea. Three knots are frequently called a **league.**

Surveyors' Square Measure

625 square links	= 1 square rod
16 square rods	= 1 square chain
10 square chains	= 1 acre
640 acres	= 1 square mile
36 square miles	= 1 township

Cubic Measure

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
128 cubic feet	= 1 cord
1 cubic yard	= 1 load (of earth, etc.)
$24\frac{3}{4}$ cubic feet	= 1 perch

The square rod is sometimes called a **perch.** The word **rood** is sometimes used to mean 40 sq. rd. or $\frac{1}{4}$ A. In the government surveys, 1 sq. mi. is called a **section.**

The perch of stone or masonry varies in different parts of the country; but it is usually considered as 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, or $24\frac{3}{4}$ cu. ft.

Angular Measure

60 seconds	= 1 minute
60 minutes	= 1 degree
90 degrees	= 1 right angle
360 degrees	= 1 circumference

Angular (also called circular) measure is used principally in surveying, navigation, and geography for measuring arcs of angles, for reckoning latitude and longitude, for determining locations of places and vessels, and for computing difference of time.

A minute of the earth's circumference is equal to a geographical mile. A degree of the earth's circumference at the equator is therefore equal to about 69 statute miles.

MEASURES OF TIME

60 seconds	= 1 minute	12 months	= 1 year
60 minutes	= 1 hour	360 days	= 1 commercial year
24 hours	= 1 day	365 days	= 1 common year
7 days	= 1 week	366 days	= 1 leap year
30 days	= 1 commercial month	100 years	= 1 century

September, April, June, and November have 30 da. each; all of the other months have 31 da. each, except February, which has 28 da. in a common year and 29 da. in a leap year.

Centennial years that are divisible by 400 and other years that are divisible by 4 are leap years.

In running trains across such a broad stretch of country as the United States, it is highly important to have a uniform time over considerable territory. Recognizing this, in 1883, the railroad companies of the United States and Canada adopted for their own convenience a system of **standard time.** This system divides the United States into **four time belts,** each covering approximately 15° of longitude, $7\frac{1}{2}°$ of which are east and $7\frac{1}{2}°$ west of the governing meridian. The region of **eastern time** lies approximately $7\frac{1}{2}°$ each side of the 75th meridian, and the time throughout this belt is the same as the local time of the 75th meridian. Similarly, the regions of **central, mountain,** and **Pacific time** lie approximately $7\frac{1}{2}°$ each side of the 90th, 105th, and 120th meridians, respectively, and the time throughout each belt is determined by the local time of the governing meridian of that belt. There is just one hour's difference between adjacent time belts. Thus, when it is 11 o'clock A.M. by eastern time, it is 10 o'clock A.M. by central time, 9 o'clock A.M. by mountain time, and 8 o'clock A.M. by Pacific time. Since railroad companies change the time at important stations and termini, regardless of the longitude of such stations and termini, the boundaries of the time belts are quite irregular.

MEASURES OF VALUE

United States Money	*English Money*
10 mills = 1 cent	4 farthings = 1 penny
10 cents = 1 dime	12 pence = 1 shilling
10 dimes = 1 dollar	20 shillings = 1 pound sterling
10 dollars = 1 eagle	= $4.8665

The term "eagle" is seldom used in business. The mill is not a coin, but the name is frequently used in some calculations. In Canada the units of money are the same as in the United States. 1 far. = $\frac{5}{9}¢$; 1*d.* = $2\frac{2}{9}¢$; 1*s.* = $24\frac{1}{3}¢$.

French Money	*German Money*
100 centimes = 1 franc = $0.193	100 pfennigs = 1 mark = $0.238

MISCELLANEOUS MEASURES

Counting by 12	*Counting Sheets of Paper*
12 things = 1 dozen	24 sheets = 1 quire
12 dozen = 1 gross	20 quires = 1 ream
12 gross = 1 great gross	= 480 sheets

BUSINESS ABBREVIATIONS

A . . . acre
Apr. . . April
Aug. . . August
bbl. . . barrel; barrels
bdl. . . bundle; bundles
bg. . . bag; bags
bkt. . . basket; baskets
bl. . . bale; bales
bu. . . bushel; bushels
bx. . . box; boxes
cd. . . cord; cords
ch. . . chain; chains
c.i.f. . . carriage and insurance free
Co. . . company; county
C.O.D. . collect on delivery
Coll. . . collection
Cr. . . creditor; credit
cs. . . case; cases
ct. . . cent; cents; centime
cu. ft. . cubic foot; cubic feet
cu. in. . cubic inch; cubic inches
cu. yd. . cubic yard; cubic yards
cwt. . . hundredweight
d. . . . pence
da. . . day; days
Dec. . . December
doz. . . dozen; dozens
Dr. . . debtor; debit; doctor
E. . . . east
ea. . . each
e.g. . . *exempli gratia*, for example
etc. . . *et cætera*, and so forth
far. . . farthing; farthings
Feb. . . February
f.o.b. . . free on board
fr. . . franc; francs
ft. . . foot; feet
gal. . . gallon; gallons
gi. . . gill; gills
gr. . . grain; grains
gro. . . gross
hhd. . . hogshead; hogsheads
hf. cht. . half chest; half chests
hr. . . hour; hours
i.e. . . *id est*, that is
in. . . inch; inches
Jan. . . January
kg. . . keg; kegs
l. . . . link; links
lb. . . pound; pounds

Mar. . . . March
Mdse. . . merchandise
Messrs. . . *Messieurs*, Gentlemen; Sirs
mi. . . . mile; miles
min. . . . minute; minutes
mo. . . . month; months
Mr. . . . Mister
Mrs. . . . Mistress
N. . . . north
No. . . . number
Nov. . . . November
Oct. . . . October
oz. . . . ounce; ounces
p. . . . . page
pc. . . . piece; pieces
per. . . . by the; by
per cent. . *per centum*, by the hundred
pk. . . . peck; pecks
pkg. . . . package; packages
pp. . . . pages
pr. . . . pair; pairs
pt. . . . pint; pints
pwt. . . . pennyweight; pennyweights
qr. . . . quire; quires
qt. . . . quart; quarts
rd. . . . rod; rods
rm. . . . ream; reams
Rm.(or M.) *Reichsmark*, Mark
s. . . . . shilling; shillings
S. . . . . South
sec. . . . second; seconds
sq. ch. . . square chain; square chains
sq. ft. . . square foot; square feet
sq. mi. . . square mile; square miles
sq. rd. . . square rod; square rods
sq. yd. . . square yard; square yards
T. . . . . ton
tb. . . . tub; tubs
Tp. . . . township; townships
viz. . . . *videlicet*, namely; to wit
via . . . by way of
wk. . . . week; weeks
wt. . . . weight; weigh
yd. . . . yard; yards
yr. . . . year; years.

BUSINESS SYMBOLS

a/c	account	=	equal; equals	$1\frac{3}{4}$	one and three fourths
a/s	account sales		foot; feet; minutes	℔	per; by
+	addition	C	hundred	%	per cent; hundredth; hundredths
( ), ¯	aggregation	″	inch; inches; seconds	£	pounds sterling
&	and	×	multiplication	∵	since
.....	and so on	#	number, if written before a figure; pounds, if written after a figure	−	subtraction
@	at; to			∴	therefore
c/o	care of	$1\frac{1}{4}$	one and one fourth	M	thousand
¢	cent; cents	$1\frac{2}{4}$	one and two fourths; one and one half	$5/6$	5 shillings 6 pence; five sixths
√	check mark				
°	degree				
÷	division				
$	dollar; dollars				

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